M.Sc. Semester III Examination, 2023

Physics

Course: MPCC-32

Nuclear & Particle Physics (Core)

Time: 3 Hours Full Marks: 40

Answer Any Four Questions

1. (a) Write down the required mass formulas and subsequently deduce the Gell-Mann-Okubo mass formula (you have to mention the specific assumption) for the $\frac{1}{2}^+$ (as per J^π notation) baryon. [2+2]

(b) Following Gell-Mann Okubo mass formula (equal spacing rule) for $3/2^+$ (as per J^π notation) baryon, predict the mass of Ω^-. Given: the masses (in MeV/c^2) of Δ, Σ, Ξ are 1232, 1385, and 1533, respectively. [Instruction: *you have to use the average of the calculated values of the two spacings following the given set of data*]. [2]

(c) Draw the triangular array (as per the Eightfold Way) for the representation of the Baryon Decuplet. [2]

(d) Draw the necessary curve representing the presence of resonance peaks in a particular reaction involving the elementary particles. Mention the underlying method for extracting the mean lifetimes of the particles those are responsible for origin of the concerned resonance peaks. [1+1]

2. (a) Write down the special features of neutral Kaon particles. [2]

(b) Establish the nature of the reaction, $\Lambda^0 \rightarrow \pi^- + p^+$ [2]

(c) If ^{16}O and ^{12}C at rest were combined to form ^{28}Si in its ground state, there would be an energy released of Q = 16.75 MeV. Similarly, formation of ^{28}Si in its ground state through the capture of proton by ^{27}Al would release the energy of Q = 11.59 MeV.

With what excitation energy would the compound nucleus be formed when ^{12}C is bombarded by ^{16}O ions of 50 MeV? What energy proton bombarding ^{27}Al would be required to form the same compound nucleus at the same excitation? What is your expectation regarding the decay channels as per the predictions of compound nucleus theory of nuclear reaction. [1+1+1]

Estimate the highest angular momentum with which you would expect to form ^{28}Si in each reaction? [Assume the proton is a point particle and the other nuclei have radii $1.3 \times A^{1/3}$ fm; Given: $\hbar c = 200$ MeV fm; 1 amu = 10^3 MeV/c^2] [1+1]

(d) Draw the necessary curves illustrating the features of a direct reaction. [1]

3. (a) Draw the necessary curve that represents the schematic view for the angular distribution of protons emitted from the (d,p) stripping reaction. Comment on the feature of differential cross-section from the curve. [1+1]

(b) In the $^{12}C(d,p)^{13}C$ reaction, the ground, first excited, second excited, and third excited states are formed with the l_n values of 1, 0, 1, and 2, respectively. [l_n represents the angular momentum carried out by the emitted neutrons]. Determine the parity of the states.

[2]

(c) In the process of β-delayed proton emission (^{27}S is the parent nuclide, ^{27}P is the intermediate nuclide), the various excited states of ^{26}Si (^{26}Si is the final product nuclide) are formed. Calculate the energy (along with the associated uncertainty) of the excited level of ^{26}Si when the proton carries a kinetic energy of 913(9) keV emitting from ^{27}P that lies at the excitation of 4506(13) keV. Given: $S_p(^{27}P)$ = 807(9) keV. [*The value within bracket represents the uncertainty associated with the corresponding physical quantity; the symbols have their usual significance*]

[2]

(d) "In the case of α-scattering process, as the impact parameter (b) becomes smaller, the angle of scattering (θ) increases" – Justify the statement by deriving the necessary mathematical expression. How you will use the result to estimate the size of a nucleus? [3+1]

4. (a) Show, from energy considerations, that a nucleus undergoing positron decay may also decay via electron capture but the opposite is not true.

(b) What are allowed and forbidden beta transitions?

(c) Write down the selection rules for allowed Fermi and allowed Gamow-Teller transitions.

(d) ^{64}Cu nucleus decays by β^-, β^+ and electron capture with a half-life of about 13 hrs. Write down the nuclear reactions for these three processes and identify the daughter elements in each case.

[3+2+2+3]

5. (a) Examine the possibilities of isomeric transitions between nuclei ^7Be and ^7Li, given $M(^7Be) = 7.016004$ u and $M(^7Li) = 7.016929$ u.

(b) Give an account of the observed β-ray spectrum. Explain the role of neutrino hypothesis in understanding the spectrum. What is Kurie plot ?

c) Write short notes on any two of the following nuclear processes:
(i) Fine structure of alpha decay (ii) Parity violation in β decay (iii) GM counter (iv) Van de Graaf Accelerator

(d) Using Bethe-Bloch formula, explain how the stopping power depends on energy for different charged particles. Hence, define a typical Bragg curve.

[3+3+2+2]

. (a) Write down the Schrodinger wave equation for the ground state of deuteron with zero orbital angular momentum. Hence obtain a relation between the depth and range of nuclear potential.

(b) It is found that deuteron possess both magnetic dipole moment and electric quadrupole moment. Write a note on its implication.

(c) For deuteron, the maximum value of the spin, $S = 1\hbar$. Find out the possible values of the orbital angular momentum quantum number l ?

(d) If the binding energy of deuteron be taken to be 6 MeV, calculate roughly the depth of the potential well, assumed square.

[5+2+1+2]

M.Sc. Examination,2023
Semester-III
Physics
Course: MPEC-35(E-08)
(Condensed Matter Physics-I)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin
Answer **any four** of the following questions

1.a) Assuming Shur's Lemma (you don't have to prove Schur's Lemma) derive the Grand Orthogonality Theorem. Give a geometrical interpretation. 4+1

b) Using the above, derive the two orthogonality relations for character tables. What do they imply? 2+2+1

2. a) What is the regular representation? Show that the regular representation contains each irreducible representation a number of times equal to the dimension of that representation. 2+3

b) How do we construct the group of the Shrődinger Equation? Show that this really results in a group that is isomorphic to the symmetry group of the system under consideration. 2+3

3. From semi-classical considerations, show the steps leading up to the phenomenon of Azbel-Kaner resonance. Your discussion should include why it is so difficult to observe Cyclotron Resonance directly. 10

4. Write down the Lagrangian density for the non-relativistic Fermionic fields. Obtain the Hamiltonian of the system. Write down the quantization rule for the non-reltivistic Fermions. Show that these particles obey Fermi-Dirac statistics. 1+2+1+6

5. a) Define normal order product? What do you mean by contraction? Show that the time ordered product of the two fermionic operators $\phi_A(k,t)$ and $\phi_B(k',t')$ can be expressed in terms of normal order product and contraction. 1+1+3

b) Let c_k and $c_k^\dagger$ be the annihilation and creation operators for the electrons for the momentum eigen state $|k>$. Obtain the Hamiltonian for a system of interacting elections in terms of c_k and $c_k^\dagger$ 4

c) Write down the Greens' function for the interacting electronic system in the interaction picture. 1

6. a) State and explain Bloch theorem. 1

b) Consider free electrons in presence of a periodic potential. Assuming that the strength of the periodic potential to be small show that for certain values of the wave vector k there are energy gaps. Derive the general formula for the gaps and show in particular that the gap in energy at the top of the first Brillouin zone is proportional to the strength of the periodic potential. 6

c) The method of orthogonalised plane waves (OPW) is very often used to make band structure calculation for electronic states in metals. It explains why the nearly free electron approximation can be used although the actual lattice potential acting on the electrons is not weak. Discuss the essential physical ideas and mathematical formalism involved in this method. 3

7. a) When an an external potential is applied on an electronic system the effective potential acting on the electrons is not the applied potential V, but is divided by the dielectric constant $\epsilon(q,\omega)$ which is a function of wave length and frequency of the applied perturbing potential V. Obtain the Lindhard's expression for the dynamic dielectric constant $\epsilon(q,\omega)$. Hence show that at a particular frequency ω_p, called plasma frequency, the system is self exciting. 8

b) Show that a static external field of long wave length is screened out almost entirely by the flow of electrons. 2

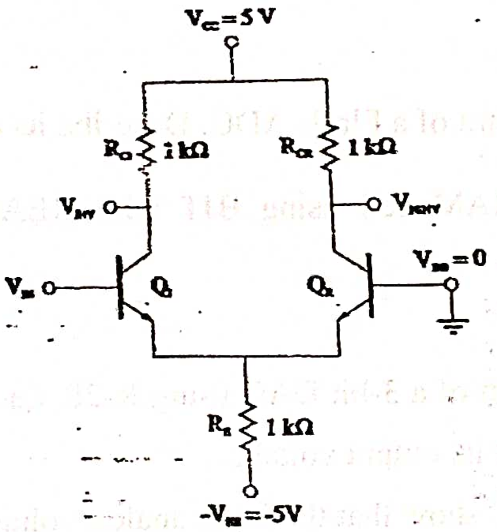
M.Sc. Examination, 2023
Semester III
Physics
Course: MPEC35
(Digital Integrated Circuits and Microprocessors – E23)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin

Answer **any four** questions

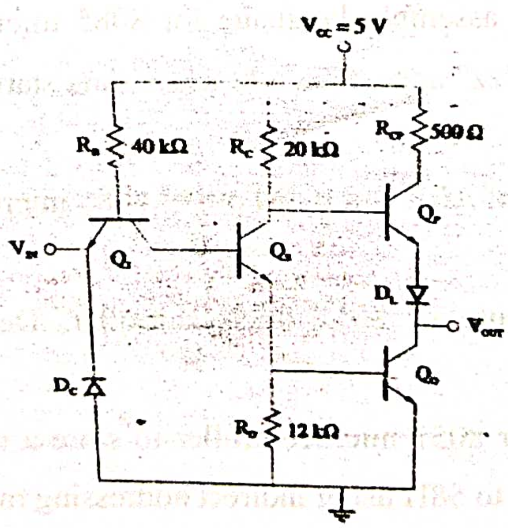
1. a) Draw the voltage transfer characteristics of the given ECL inverter.



Assume V_{BE}= 0.7V, V_{BE}(Sat)= 0.8V and $\beta = 50$.

b) How can you convert a standard TTL inverter to LTTL and HTTL

c) Draw the voltage transfer characteristics of the TTL circuit. Assume V_D= 0.7V, V_{BE}= 0.7V, V_{BE}(Sat)= 0.8V and $\beta = 50$.

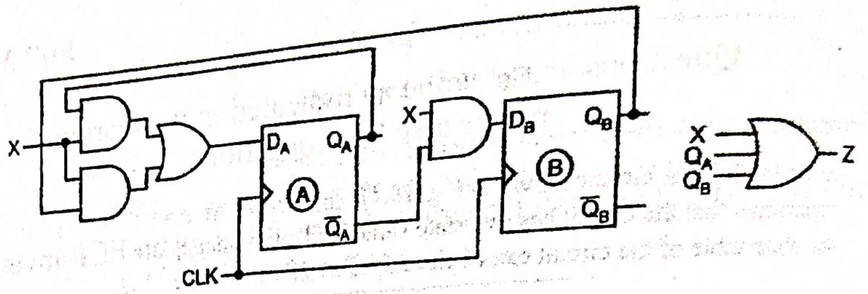


4+2+4

2. a) Implement the following expression with only NAND gates

$F = \Pi M (0, 5, 6, 8\text{-}11, 16, 20, 24\text{-}27, 29, 31)$

b) Obtain the state table and state diagram for the circuit shown in the following figure



4+6

3. a) Draw the circuit diagram of a Flash-ADC. Describe its operation.

b) Design a one-bit SRAM cell using BJT with READ and WRITE option. Explain its operation. 4+6

4 a) Draw a circuit diagram of a 3-bit DAC using R-2R Ladder network and derive the general expression of its output voltage.

b) For a Dual slope ADC show that the input analog voltage is proportional to the count recorded in the binary counter. 4+6

5. a) Explain how the concept of microprogram in introduced in control unit of a typical microprocessor.

b) Write a program in assembly language for 8085 microprocessor to find how many times a number "zz" appears in a block of data starts from 3101H. The size of the block is stored at 3100H.

c) Explain the function of ALE and IO/M pin of 8085 microprocessor.

4+4+2

6. a) Draw the architecture of 8051 microcontroller. Describe the structure of internal RAM of 8051.

b) Write a program for 8051 microcontroller to store a number "FF" into RAM memory locations 50H to 58H using indirect addressing mode.

5+2+3

7. Write notes on **(any two)**: **5+5**

a) EPROM device

b) CCD memory device

c) Programmer model of 8085 microprocessor

d) Architecture of peripheral IC 8255

M.Sc. Semester III Examination, 2023
Physics
Course: MPEC-35 (E-11)
(Nuclear Physics – I)

Time: 3 Hours | Full Marks: 40

Answer Any Four Questions

(a) What is the principal advantage of bipolar shaped pulses over mono-polar shaped pulses ?

(b) What are the different NIM logic levels used in nuclear electronics ?

(c) Sketch a schematic drawing of nuclear electronics for a two-body scattering experiment and find the standard master trigger logic to run the experiment.

(d) Sketch a configuration of two integral discriminators and an anti-coincidence unit that will perform the function of a single-channel analyzer.

(e) A Wilkinson-type ADC has a conversion gain of 4096 channels and a maximum conversion time of 15 microseconds. At what frequency must the oscillator operate ?

[1+1+2+4+2]

(a) Given a graph of the gas amplification curve for an ion chamber, identify the different regions of the curve and characterize the detectors operated in each of the useful regions of the curve.

(b) A particle is passing through a G-M tube, operating under certain voltage and with a given size of wire. How the quenching of the discharge will occur ?

(c) In a certain G-M tube the central wire has a diameter of 0.18 mm and is 1200 volts positive with respect to the cylindrical cathode whose inner radius is 1.8 cm. Express the maximum and the minimum values of the electric field along the radius.

[4+2+4]

3. (a) Describe the working principle of scintillation counter with special reference to the following points: construction, characteristics and efficiency ?

(b) Why does the pulse height from a G-M tube continue to increase with applied voltage even after a full discharge is obtained ?

(c) Calculate the capacitance of a silicon detector with the following characteristics: area 1.5 cm^2, dielectric constant 12, depletion layer 50 microns. What potential must be developed across the capacitance by the absorption of a 4.5 MeV aplha-particle which produces one ion pair for each 3.5 eV expended ?

[4+2+4]

4. (a) Draw a diagram to explain how the voltage is multiplied in the Cockcroft-Walton voltage generator.

(b) What are the merits and demerits of linear accelerators compared to cyclotrons ?

(c) How do the relativistic effects limit the acceleration of electrons to high energies in cyclotrons ?

(d) Protons of energy 1.5 MeV are injected into a 60 MeV linear accelerator powered by a 300 MHz rf-supply. Find the approximate lengths of the first and the last drift tubes.

[2+2+2+4]

5. (a) Write down the construction and working principle of van de Graaff accelerator.

(b) Write down the working principle of a linear accelerator.

(c) A cyclotron with dees of diameter 1.2 meter has a magnetic field of 0.9 Wb/m^2. Calculate the energies to which protons and deuterons are accelerated.

[4+2+4]

6. (a) If the energy resolution of a particular NaI(Tl) scintillation detector is 9% for ^{137}Cs gamma-rays (0.662 MeV), estimate its energy resolution for the 1.17 & 1.33 MeV gamma rays from ^{60}Co.

(b) In what way a scintillation detector is superior to a gas detector ?

(c) Calculate the scintillation efficiency of Anthracene if 1 MeV of particle energy loss creates 20300 photons with average wavelength of 447 nm.

(d) The gain per dynode of a 12-stage PM tube varies as $V^{0.9}$ where V is the interdynode voltage. If the tube is operated at an overall voltage of 0.6 kV, how much voltage fluctuation can be tolerated if the gain is not to change by more than 2.5% ?

[3+1+3+3]

7. (a) The following is a portion of a gamma-ray pulse height spectrum recorded using a germanium detector and an MCA. Channel Number: 711, 712, 713, 714, 715, 716,717, 718, 719, 720, 721, 722, 723, 724, 725,726, 727, 728; corresponding counts:238, 241, 219, 227, 242, 280, 409, 736, 1190, 1625, 1739, 1412, 901, 497, 308,256, 219, 230. It can be assumed that the data consist of a constant background plus a Gaussian-shaped peak. Plot the data and estimate the constant background level, net number of counts under the peak using direct summation, centroid of the peak and the FWHM.

(b) Draw a schematic diagram of the CAMAC dataway showing all the details of signals and their functions with symbols.

[6+4]

M.Sc. Examination, 2023

(Semester-III)

PHYSICS

Paper: MPEC-35 (E-02)

Quantum Field Theory Theory

TIME: 3 HRS. FULL MARKS: 40

Attempt *four* questions. **Question No. 1 is compulsory.**
Questions are of value as indicated in the margin.

1. Answer any five questions. [2×5]

   (a) Consider the Lorentz transformation $x^\mu \to x'^\mu = \Lambda^\mu_{\ \nu} x^\nu$. For the infinitesimal Lorentz transformation (with $\Lambda^\mu_{\ \nu} = \delta^\nu_\mu + \epsilon^\nu_{\ \mu}$), show that $\epsilon_{\mu\nu} = -\epsilon_{\nu\mu}$.

   (b) Define the Fock space and hence construct a state of n-particles with momentum $\vec{p}$.

   (c) For the given Lagrangian density in D-dimensions of spacetime

   $$\mathcal{L} = \frac{1}{2}\partial_\mu\phi(x)\partial^\mu\phi(x) - \frac{m^2}{2}\phi^2(x) - \frac{\lambda}{4!}\phi^4(x),$$

   calculate the mass dimension of $\phi(x)$, m and λ (in natural units $\hbar = c = 1$).

   (d) Prove that $\gamma_\mu \not{A}\not{B}\gamma^\mu = 4\,A\cdot B$ and $(\vec{\sigma}\cdot\vec{A})(\vec{\sigma}\cdot\vec{A}) = i\vec{\sigma}\cdot(\vec{A}\times\vec{B}) + \vec{A}\cdot\vec{B}$.

   (e) Show that for the given orthonormal functions $f_k(x) = \dfrac{e^{-ik\cdot x}}{\sqrt{(2\pi)^3\,2k_0}}$ and $f_k^*(x) = \dfrac{e^{+ik\cdot x}}{\sqrt{(2\pi)^3\,2k_0}}$ satisfy the relation: $\int d^3x f_k^*(x) i\overleftrightarrow{\partial_0} f_{k'}(x) = \delta^3(\vec{k}-\vec{k}')$.

   (f) Define the covariant derivative D_μ. Show that $[D_\mu,\ D_\nu] = iqF_{\mu\nu}$ where q is the electric charge as a coupling constant.

2. (a) Check the Hermiticity of the Dirac's Lagrangian density. [3]

   (b) Show that the propagator for Dirac field in momentum space is [3]

   $$S_F(k) = \frac{\not{k} + m}{k^2 - m^2 + i\varepsilon}$$

   (c) Show that Dirac's Lagrangian density remains invariant under the continuous global gauge transformations $\delta\psi(x) = -ie\theta\psi(x)$ and $\delta\bar{\psi}(x) = +ie\theta\bar{\psi}(x)$. Derive an expression for the corresponding four-vector current J^μ and verify $\partial_\mu J^\mu = 0$. [4]

3. (a) Define the Pauli-Lubanski four-vector W^μ and show that [4]

   $$W^2 = W^\mu W_\mu = -\frac{1}{2}M_{\mu\nu}M^{\mu\nu}P^2 + M_{\mu\alpha}M^{\nu\beta}P^\mu P_\nu.$$

   (b) Show that $W^2|\vec{p}=0, m, s, s_z\rangle = -m^2 s(s+1)$ where $|\vec{p}=0, m, s, s_z\rangle$ is a state vector for a particle of mass m, momentum $\vec{p}$, spin s and s_z is the z-component of spin. [6]

4. (a) Compare the Schrödinger picture, Heisenberg picture and Interaction picture. Derive the Dyson's formula $U(t) = \mathcal{T}\Big(e^{-i\int d^4x \mathcal{H}_I(x)}\Big)$ where $\mathcal{T}$ stands for the time-ordered product. [2 + 4]

(b) What is Wick's theorem ? For the real scalar field theory, show that [1+3]

$$\mathcal{T}\Big(\phi(x)\phi(y)\Big) = :\phi(x)\phi(y): + iG_F(x-y)$$

5. (a) Show that source free Maxwell's theory in 4D of spacetime remains invariant under local gauge transformations. Also derive the Euler-Lagrange equation of motion. [1+1]

(b) Calculate the momenta conjugate to the four-vector field A_μ. Is your result consistent with the canonical quantization scheme? [2]

(c) Show that the photon propagator for the Maxwell's theory does not exists if the gauge is not fixed. Derive the expression for the photon propagator in the Feynman gauge. [3 + 3]

6. (a) Show that under the Lorentz transformations $x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x_\nu$ and $\psi(x) \to \psi'(x') = S(\Lambda)\psi(x)$, the covariance of Dirac equation demands the condition $\gamma^\nu = S^{-1}(\Lambda)\gamma^\mu \Lambda_\mu{}^\nu S(\Lambda)$. For the infinitesimal Lorentz transformation, further, show that the above condition lead to $\Big[\gamma^\nu,\, \sigma_{\alpha\beta}\Big] = 2i\Big(\delta^\nu_\alpha \gamma_\beta - \delta^\nu_\beta \gamma_\alpha\Big)$. [3 + 3]

(b) Consider the real massive scalar field theory described by the Lagrangian density

$$\mathcal{L} = \frac{1}{2}\,\partial^\mu\phi(x)\,\partial_\mu\phi(x) - \frac{m^2}{2}\,\phi^2(x).$$

Derive the expressions for $a(\vec{k})$ and $a^\dagger(\vec{k})$ as inverse Fourier transformations of $\phi(x)$ and $\Pi(x)$. Assuming the standard equal time commutation relations between $\phi(x)$ and $\Pi(x)$, verify the commutation relation: $\Big[a(\vec{k}),\, a^\dagger(\vec{k}')\Big] = \delta^3(\vec{k}-\vec{k}')$. [4]

7. (a) Write down the Lagrangian density for the dynamically closed system of electron, positron and photon and show that it respects the local $U(1)$ gauge transformations. [3]

(b) State the Feynman diagram's rules for this system. [3]

(c) Using the properties of covariant derivative, show that, under the local $U(1)$ gauge transformations, the gauge field $A_\mu(x)$ transforms as

$$A_\mu(x) \to A'_\mu(x) = U(x)A_\mu(x)U^{-1}(x) + \frac{i}{q}\Big(\partial_\mu U(x)\Big)U^{-1}(x),$$

where $U(x)$ is an unitary operator (i.e., $U^\dagger(x) = U^{-1}(x)$). [4]

M.Sc. Examination, 2023

Semester-III

Physics

Course: MPEC-35

(Quantum Electronics I (E-19))

Time: Three Hours **Full Marks:40**

Questions are of value as indicated in the margin.

Answer ***any four*** questions

1. Show that the photon number distribution in a coherent state is Poisonian and is given by

$$P_n(\alpha) = \exp(-|\alpha|^2)\frac{|\alpha|^{2n}}{n!}$$

Establish that the coherent state is a state with minimum uncertainty (4+6)

2. Discuss the gain saturation of homogeneous broadening. Obtain the gain saturation of a ruby laser of $\lambda = 0.6943\,\mu m$ and $\Delta\nu = 2 \times 10^{11}\,Hz$ at 300^0k. (Given $\tau = t_{spon} = 3 \times 10^{-3} sec$, $n = 1.77$ and $\eta = 1$). (6+4)

3. Show that the Optical Bloch equations for a two level system with frequency gap ω_0 is given by

$$\begin{aligned} \frac{d\rho_{21}}{dt} &= -i\omega_0\rho_{21} + i\frac{\mu E(t)}{\hbar}(\rho_{11} - \rho_{22}) \\ \frac{d\rho_{22}}{dt} &= -i\frac{\mu E(t)}{\hbar}(\rho_{21} - \rho_{12}) \\ \frac{d}{dt}(\rho_{11} - \rho_{22}) &= 2i\frac{\mu E(t)}{\hbar}(\rho_{21} - \rho_{12}) \end{aligned}$$

How the collisional-decay terms are included in the equation of motion of density matrix elements for a two level atomic system? (8+2)

4. By using self consistent approach of Lamb, discuss the oscillation of a gas laser. (10)

5. Establish the relation between the Einstein A and B coefficients. Show that for monochromatic field frequency, the induced transition rate can be expressed as $(W_{21})_i = \frac{c^3\rho_\nu}{8\pi n^3 h\nu^3 t_{spon}} g(\nu)$. (7+3)

6. Show that the vector representation of a two-level atom coupled to a radiation field leads to the following form

$$\frac{d\vec{r}}{dt} = \vec{\omega}(t) \times \vec{r}$$

where the symbols are of usual meaning. Discuss the physical origin of $\vec{\omega}$ and $\vec{r}$. (7+3)

7. Under Rotating Wave Approximation, obtain the Jaynes-Cummings Hamiltonian of a two-level system. By using Jaynes Cummings model, obtain the atomic inversion for an ideal two-level system. (6+4)

M. Sc. Physics Semester - III, 2024

General Theory of Relativity, EC-04

MPEC – 35

Time : 3hrs **FM : 40**

Symbols bear their usual meanings

Marks for each question is given in the margin.

Answer any four

1. (a) Outline the postulates of Einstein's theory of gravitation.
   (b) Show that ordinary derivatives of tensors defined in general manifold do not transform as tensors. Find the expression for the covariant derivative.
   (c) Show that the covariant derivative of metric tensor vanishes. 3+2+3+2

2. (a) Define a geodesic in a general spacetime.
   (b) Find the equation for a geodesic. (You can choose any method you prefer, your definition should match your method of derivation.)
   (c) Define one meter of time. Find the mass of the sun in geometrized unit. 2+4+(2+2)

3. (a) Define a Killing vector.
   (b) *If all the components of $g_{\mu\nu}$ are independent of x^μ for some fixed index, the conjugate momentum p_μ is a constant along particle trajectory* – Prove.
   (c) Prove that a particle moves in a trajectory where the direction of the momentum remains unchanged. (3+4+3)

4. (a) In physics, how do you define an inertial frame. Explain the concept of a *locally inertial frame.*
   (b) A lift is freely falling in the gravitational field of a planet. What is the nature of the frame attached to it?
   (c) If a tensor equation is valid in a locally inertial frame what will be your conclusion regarding its validity in a general reference frame? Explain with reason.
   (d) State the equivalence principle. (2+2+2+2+2)

5. (a) Consider the following equation

$$[\nabla_\alpha, \nabla_\beta] = Z^\lambda_{\mu\alpha\beta} T_\lambda$$

   and show that in general spacetime covariant derivatives do not commute. Find the expression for $Z^\lambda_{\mu\alpha\beta}$.
   (b) You can identify this tensor $Z^\lambda_{\mu\alpha\beta}$ with the curvature (Riemann) tensor $R^\lambda_{\mu\alpha\beta}$. Explain why is it called *curvature tensor.* 4+4+2

6. (a) What is assymptotic flatness?

(b) Write down the expression for the stress-energy tensor of a perfect fluid in curved spacetime.

(c) Einstein's equation is given by

$$G_{\mu\nu} = \kappa T_{\mu\nu}$$

Find the expression for κ. 2+2+6

7. (a) Find the radial equation for a massive particle in a static spherically symmetric metric.

(b) Explain the mercury perihelion shift of $43''$/century. 5+5

M.Sc.(Physics) Semester-III Examination 2023
[MPEC-35] - Nonlinear Dynamics I (E25)

Time: Three Hours **Full Marks: 40**

- Answer **any four** of the following questions. Questions are of value as indicated in the margin.
- Symbols have their usual meanings unless specified otherwise. Symbols used in a question can not be changed while writing the answer for the same.

1. The motion of a damped harmonic oscillator is described by,

$$\ddot{x} + b\dot{x} + \omega^2 x = 0, \ \ b, \omega \in \mathbb{R}$$

   (a) Rewrite the equation as a two dimensional linear system. 1

   (b) Classify the fixed point at the origin and sketch the phase portrait. Consider all possible cases that can occur depending on the relative sizes of the parameters. 6

   (c) How do your results relate to the standard notions of overdamped, critically damped, and underdamped oscillations? 3

2. (a) Write down the equation of a simple pendulum and find its equilibrium points. 2

   (b) classify each equilibrium point by following the linear stability analysis. Give a qualitative description of the phase trajectories around these equilibrium points. 3

   (c) Find a constant of motion for a simple pendulum by using the method of quadrature. 2

   (d) Find an exact solution in terms of elliptic function. 3

3. (a) Explain the idea of a limit cycle with examples. 3

   (b) Verify the truth in the statement with proper explanation: 'A limit cycle can not occur in a Hamiltonian system'. 2

   (c) State and explain Poincaré-Bendixson theorem with examples. 3

   (d) Determine whether the following system is dissipative or not: 2

$$\dot{x} = \sigma\left(y - x\right), \ \dot{y} = rx - y - xz, \ \dot{z} = xy - bz, \ \ \sigma, r, b \in \mathbb{R}^{+}$$

4. (a) Describe the pitchfork bifurcation. 6

(b) Find the value of r at which bifurcation occurs in the first-order system $\dot{x} = r - x - e^{-x}$ and classify the nature of bifurcation in terms of the standard nomenclature. Sketch the bifurcation diagram of fixed points x^* vs. r. 4

5. The Logistic Map is defined by the equation:

$$x_{n+1} = ax_n(1 - x_n), \quad n \in \mathbb{Z}^{\geq 0}$$

(a) Find the equilibrium points and their stability. 4

(b) Show that a two cycle solution exists for a Logistic Map. 4

(c) Find an exact solution of the Logistic Map for $a = 4$. 2

6. Write short notes on any two of the following topics: 5+5

(a) van der Pol Oscillator

(b) Quasiperiodic Attractor

(c) Chaos & Lyapunov exponent

M.Sc. Examination, 2023

Semester-III

Physics

Course: MPCC 31

(Atomic and Molecular Physics)

Time: Three Hours **Full Marks:40**

Questions are of value as indicated in the margin.

Answer ***any four*** questions

1. What you mean by the lifetimes of atomic levels? Why the $2s$ state of the hydrogen atom is a metastable one? Explain how the electrostatic field could be used to quench the metastable state of the hydrogen atom. (2+2+6)

2. Draw a comparative differences between homogeneous and inhomogeneous broadening. Obtained the normalized Doppler broadened lineshape function. Obtain the expression of the FWHM of the said lineshape function. (2+6+2)

3. Explain the meaning of *exchange degeneracy.* By using the independent particle model, calculate the ground state energy of a two electron atom and hence the ionization potential. How far the independent particle model is helpful to explain the experimental data? Justify that the orthohelium in its ground state is not a reality. (1+4+1+4)

4. What a Lamb Shift is ? Why these shifts are unavailable in the relativistic theory of hydrogen atom due to Dirac? How the experiments of Lamb-Retherford is used to obtain the Lamb shift? (1+2+7)

5. Show that $(\vec{L} - \vec{S} + 3\frac{(\vec{S}.\vec{r})\vec{r}}{r^2}).\vec{J} = \vec{L}^2$,where the symbols are of usual meaning. Discuss the origin of $21cm$line of hydrogen atom. Discuss the significance of this particular line. (3+4+3)

6. (a)The spacing between molecular vibration levels is $1000\,cm^{-1}$. Show that the population in the first vibrational level $v = 1$ is approximately 1% of the $v = 0$ level.
(b)The spectrum of HCl shows a very intense absorption at $2886\,cm^{-1}$, a weaker one at $5668\,cm^{-1}$ and a very weak at $8347\,cm^{-1}$. Find the vibrational frequency, anharmonic constant and the force constant of HCl. (5+5)

7. Write down the Schrodinger equation of hydrogen molecular ion. Explan the meaning of LCAO. Explain the bonding and antibonding orbitals and their physical significances of a hydrogen molecular ion. (1+2+7)

M.Sc. Examination,2023
Semester-III
Physics
Course: MPCC-33
(Relativistic Mechanics and Field Theory)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin
Answer **any four** of the following questions

1. a) Obtain the Lorentz transformation when the two inertial frames move with the relative velocity v. 3

b) Show that Maxwell equations are invariant under the Lorentz transformation. 3

c) Suppose the Lagrangian of a relativistic particle is invariant under translation and rotation. Obtain the corresponding conserved quantities. 4

2. a) Write down the action of a relativistic particle interacting with the electromagnetic field. Set up the equation of motion of the above mentioned relativistic particle interacting with electromagnetic field.(Given the elements of electromagnetic field tensor ($F_{01} = E_x$, $F_{02} = E_y$,$F_{03} = E_z$, $F_{12} = -B_z$, $F_{13} = B_y$, $F_{23} = -B_x$), where $\vec{E}$ and $\vec{B}$ are the elctric and magnetic field) 1+4

b) A particle with mass m and energy E approaches to an identical particle at rest. They collide with each other in such a way that they both scatter at an angle θ relative to the incident direction. Express the angle θ as a function of E and m. 5

3. a) Define covariant derivative of a contravariant vector. Evaluate the covariant derivative of a covariant metric tensor. Determine Christoffel symbol in terms of covariant metric tensor. 1+1+2

b) Show that for flat space-time Riemann curvature tensor vanishes. 6