B. Sc. (Honours) Examination, 2023
Semester - I
Physics (Honours)
Paper: CC-1 (Mathematical Physics-I)

Time: 3 Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Answer any **four** *questions.*

1. (a) If $\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$, $\vec{B} = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{C} = 3\hat{i} - 4\hat{j} + 2\hat{k}$, find the projection of $\vec{A} + \vec{C}$ in the direction of $\vec{B}$.

   (b) Show that $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}\,(\vec{\nabla}.\,\vec{A}) - \nabla^2\vec{A}$.

   (c) Show that $\nabla^2 r^n = n(n+1)r^{n-2}$

3+4+3=10

2. (a) Show that the force field $\vec{F} = (y^2z^3 - 6xz^2)\,\hat{i} + 2xyz^3\,\hat{j} + (3xy^2z^2 - 6x^2z)\,\hat{k}$ is a conservative force field. Find out the corresponding scalar function or potential $V(x, y, z)$ such that $\vec{F} = -\vec{\nabla}V$.

   (b) if $\vec{A} = (2x - y + 4)\hat{i} + (5y + 3x - 6)\hat{j}$, evaluate $\oint \vec{A}\,.\,d\vec{r}$ around a triangle with vertices at $(0, 0, 0)$, $(3, 0, 0)$, $(3, 2, 0)$.

(2+3)+5=10

3. (a) Verify Stokes' theorem for $\vec{A} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ above the xy-plane and C is the boundary.

   (b) A series LR circuit has an emf of 5 volts, a resistance of 50 ohms, an inductance of 1 henry, and no initial current. Find the current in the circuit at any time t.

6+4=10

4. (a) Solve the initial value problem $\frac{dy}{dx} + y = \sin x$, $y(\pi) = 1$.

   (b) Solve: $x\,dy + (x^2 - y)\,dx = 0$.

   (c) Find the particular integral of the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = xe^x$ using the method of variation of parameters.

3+3+4=10

5. (a) Obtain a general solution of the following differential equations:

   (i) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = e^x \cos x$.

   (ii) $\frac{d^2y}{dx^2} - y = (1 + x^2)e^x$.

(b) Find the Wronskian of the set $\{\sin 3x, \cos 3x\}$.

(4+4)+2=10

6. (a) What are curvilinear coordinates? Find expressions for the element of area in orthogonal curvilinear coordinates.

(b) Find the unit vectors $\hat{e_r}$, $\hat{e_\theta}$, $\hat{e_\phi}$ of a spherical coordinate system in terms of $\hat{i}, \hat{j}, \hat{k}$. Also check that $\hat{e_r} \cdot \hat{e_r} = 1$, $\hat{e_\theta} \cdot \hat{e_\phi} = 0$ and $\hat{e_r} \times \hat{e_\theta} = \hat{e_\phi}$.

(c) Evaluate the divergence of $\vec{F} = 2\rho^2 \sin\phi\, \hat{e_\rho} + 3\rho^2 \sin\phi\, \hat{e_\phi} + 7z\, \hat{e_z}$.

3+5+2=10

7. (a) Explain the terms mutually exclusive events and equally likely events with examples.

(b) Show that the variance of a binomial distribution is given by npq, where the symbols have their usual significance.

(c) Show that $\delta(kx) = \frac{1}{|k|}\delta(x)$, where k is any non-zero constant.

3+5+2=10

B.Sc (Honours) Examination, 2023

Semester-I (CBCS)
Paper Code: CC-2
Paper Name: Mechanics (Theory)

Time : Three Hours **Full Marks : 40**

Answer any four questions

1. (a) Define center of mass for a system of particles (consider both discrete and continuous cases).
(b) Show that the linear momentum, angular momentum and kinetic energy of a system of particles with respect to an external origin (O) can be expressed by,

$$\vec{P}_{\text{sys}}(O) = \vec{P}_{CM}(O),$$

$$\vec{L}_{\text{sys}}(O) = \vec{L}_{CM}(O) + \vec{L}_{\text{sys}}(CM)$$

and

$$T_{\text{sys}}(O) = T_{CM}(O) + T_{\text{sys}}(CM),$$

respectively; where the symbols have their usual meaning.
(c) Write the Newton's law for a system of particles, and show that the linear momentum of the system will be conserved if external force is zero.
[2+6+2]=10

2. (a) Define central force.
(b) Show that the motion of a particle (reduced particle) under a central force is confined on a two dimensional plane.
(c) Then considering plane polar coordinates (r and θ), find the equation of motion of a particle moving under a central force and identify the centripetal force.
(d) With the help of equations of motion, show that the path of a particle moving under inverse square force will be a conic section.
(e) Find the eccentricity of the conic section in terms of the particle's energy.
(f) Then show that the conic section is an ellipse when the particle's energy is negative (analytically and graphically). [Hints: For analytical purpose, you need to show that the eccentricity is less than unity; for graphical purpose, you need to show that there exists two turning points]
[1+2+2+2+1+2]=10

3. (a) Define rotation between two inertial frames in 3-dimensional space.
(b) Express a general rotation (matrix) in terms of three Euler angles.
(c) Define an infinitesimal rotation between two inertial frames, and find the corresponding matrix (you may use the matrix for finite rotation from the previous question).
(d) Show that two finite rotations are not commutative, while two infinitesimal rotations are.
[1+4+2+3]=10

4. Consider two frames (S_1 and S_2) moving with a uniform relative velocity.
(a) Write down the coordinate transformation (the Galilean transformation) between the two frames.
(b) How does the Galilean transformation modify if S_2 moves with an uniform linear acceleration $\vec{\alpha}$ w.r.t. S_1 ? In this case, determine the transformation rules for velocity and acceleration of a particle, and hence, find the pseudo force.
(c) Once again, find the transformation rules for coordinate and velocity of a particle (in terms of Euler angles) if S_2 rotates with an uniform angular velocity ($\vec{\omega}$) w.r.t. S_1 about a common origin.
[1+3+6]=10

5. (a) Consider a damped harmonic oscillator with natural frequency ω_0:
(i) Write the equation of motion.

(ii) Directly from the equation of motion, show that the damping term always causes the decrease of energy of the particle (w.r.t. time).

(b) Consider a damped driven harmonic oscillator with natural frequency ω_0 and the frequency of the periodic force is ω:

(i) Write the equation of motion.

(ii) Solve the equation and find the position of the particle at some instant t.

(iii) Discuss the three cases: slow driving, fast driving and resonance.

(iv) Show, that at the resonance condition, the energy of the particle remains conserved (i.e. the amount of energy loss due to damping gets exactly compensated by the driving force). $[(2\times1)+(4\times2)] = 10$

6. (a) Two spaceships approach each other, each moving with the same speed as measured by a stationary observer on the Earth. Their relative speed is $= 0.70c$. Determine the velocities of each spaceship as measured by the stationary observer on Earth.

(b) The path of a particle moving under a central force is given by: $r = 2a\cos\theta$. Find the force law.

(c) For a damped harmonic oscillator, the damping factor $\gamma = 0.1$ (in SI unit). Find the fractional rate of change of energy of the oscillator. **[4+3+3]=10**

B.Sc. (Honours) Semester-I Examination, 2023 (CBCS)
Physics
Paper: Mechanics
Paper Code: GEC1

Time: Three Hours **Full Marks: 40**

Answer any four questions. Marks are of value as indicated.

Q.1(a) Explain how shearing strain is measured. State and explain the theorem of shearing strain with the relevant diagram.

(b) Derive an expression for torsional rigidity of an uniform solid cylinder of length "l" and radius "r". What will happen if the cylinder is hollow of external radius "r_1" and internal radius "r_2" ?

(c) Derive an interrelationship between Y, K, and σ where symbols have their usual meaning.

(1+2)+(3+1)+3=10

Q.2(a) Explain the dynamical method of measuring shearing modulus deriving the relevant formula. A disc of 10 cm radius and mass 1kg is suspended in a horizontal plane by a vertical wire attached to the center. If the diameter of the wire is 1mm and it's length is 1.5 meter and time period of oscillations is 5 second, find the rigidity modulus of the material of the wire.

(b) Define gravitational "potential" and "intensity". Obtain the expression for the gravitational potential for a unit mass which is situated at a distance "r" from a gravitating mass " M".

(3+3)+(2+2)=10

Q.3(a) What are the characteristics of the motion of a particle in a central force field ? Prove that the areal velocity is constant for a particle moving in a central force field.

(b) Obtain the expressions for "escape velocity" and " parking velocity of geostationary satellites". What should be the velocity in km per hour of an earth satellite revolving round the earth at an altitude of 1600 kms along a circular trajectory? (Given, Radius of the earth = 6400 km, Mass of earth = 6x 10^{24} kg, and G = 6.67 x 10^{-11} m^3 /kg/sec^2).

(2+3) +(3+2) =10

Q.4(a) Write the differential equation for a simple harmonic oscillator and obtain it's solution. Show that the total energy is constant in case of a simple harmonic oscillator.

(b) In case of a damped harmonic oscillator what do you understand by logarithmic decrement? In a spring mass system undergoing damped harmonic oscillations amplitude decreases from 10cm to 2.5cm in 200 seconds. Find the damping constant "b", and the time in which amplitude falls to 1/e times the initial value.

(3+2)+(2+3) =10

Q.5(a) Is the angular displacement " θ" a vector ? Justify. Obtain the relation between angular velocity " ω" and linear velocity "v" of a rigid body undergoing a fixed axis rotation.

(b) Write the expression of moment of inertia for a fixed axis rotation and hence define radius of gyration for a system of particles. Find out the moment of inertia of a circular disc, about an axis perpendicular to the disc.

(2+3)+(2+3)=10

Q.6(a) State and prove the parallel axis theorem. Find out the moment of inertia of a cylindrical body about an axis passing through it's c.g. and perpendicular to the axis of the cylinder.

(b) State the fundamental postulates of theory of Einstein's special theory of relativity. Write Lorentz space time transformation equations. Hence explain " time dilation" and obtain it's expression.

(2+3)+(2+1+2)=10