B.Sc. (Honours) examination 2023

Semester-IV

Subject: Physics

Paper: Mathematical Methods III

Course: CC8

Time: 3 Hours **Full Marks: 40**

(Questions are of values as indicated in the margin)

Answer any FOUR questions.

1. [1+1+2+3+3]

(a) Define the Fourier Transform of a function $f(x)$.

(b) State and prove the shifting property of Fourier transform.

(c) State and prove the convolution property in the context of the Fourier transform.

(d) Obtain the Fourier transform of the function $f(x) = \frac{1}{x^2+a^2}$, where a is a real positive constant.

(e) Define the sine Fourier transform of a function $f(x)$ and then obtain the sine Fourier transform of the function $f(x) = \frac{e^{-ax}}{x}$, where a is a real positive constant.

2. [1+4+2+3]

(a) Define the inverse Fourier transform of the function $f(k)$.

(b) You are given the function $f(x) = e^{-|x|}$. Using Fourier transform, and the inverse Fourier transform technique show that $\int_0^\infty \frac{dx}{1+x^2} = \frac{\pi}{2}$ and $\int_0^\infty \frac{x\sin(xt)}{1+x^2} dx = \frac{\pi}{2} e^{-t}$.

(c) Obtain the sine and cosine Fourier transform of the second derivatives of a function in terms of the initial conditions and the sine and cosine transform of the function.

(d) The temperature $T(x,t)$ in a semi-infinite bar $(0 < x < \infty)$ is governed by the equation, $\frac{\partial T(x,t)}{\partial t} = c^2 \frac{\partial^2 T(x,t)}{\partial x^2}$.

Given the conditions, $T(x,0) = 0$, $\frac{\partial T(0,t)}{\partial x} = \alpha$, and $\frac{\partial T(x,t)}{\partial x} \rightarrow 0$ for $x \rightarrow \infty$, determine $T(x,t)$ using appropriate Fourier transform.

3. [4+2+2+2]

(a) Find Laplace's transform of the following functions.

i. $cos^2 t$

ii. $sin^3 t$

iii. $e^{-3t}(2\cos 5t - 3\sin 5t)$

iv. $e^{-3t}cos^2 t$

(b) If the Laplace transform of $f(t)$, $\mathcal{L}[f(t)] = \bar{f}(s)$, then show that

$$\mathcal{L}\left[\frac{f(t)}{t}\right] = \int_s^\infty \bar{f}(s)ds$$

(c) If the Laplace transform of $f(t)$, $\mathcal{L}[f(t)] = \bar{f}(s)$, then show that

$$\mathcal{L}\left\{\int_0^t f(u)du\right\} = \frac{\bar{f}(s)}{s}.$$

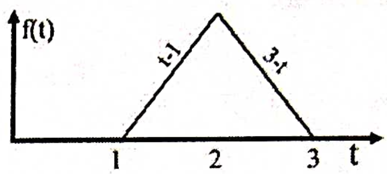
(d) Consider a function $f(t)$ which is periodic with a period T, *i.e.*, $f(t+T) = f(t)$. Then show that the Laplace transform of the function is,

$$\mathcal{L}[f(t)] = \frac{\int_0^T e^{-st} f(t)dt}{1-e^{-sT}}.$$

4. [1+3+3+3]

(a) Obtain Laplace's transform of unit step function, $\theta(t-a)$.

(b) Using the unit step function, write the mathematical form of $f(t)$ in the figure and obtain its Laplace transform.



(c) Using the Convolution theorem, obtain the inverse Laplace transform of the function. $\bar{f}(s) = \frac{1}{(s^2+1)(s^2+9)}$.

(d) A particle of mass' m' can oscillate about the position of equilibrium under the effect of a restoring force mk^2 times the displacement. It started from rest by a constant force F which acts for time T and then ceases. Using the Laplace transform technique, obtain the amplitude of the subsequent oscillation.

5. [2+2.5+1+2+1+1.5]

(a) Find all the distinct values of the complex variable, z satisfying the following equation,

$$z^6 + 64 = 0.$$

(b) Define poles and branch points with examples. Also, construct a complex function having a Pole of order three at $z = 2$ and a Branch point and $z = 3$.

(c) Show that a complex function $f(z)$ has to be continuous at z_0 if the derivative of the function exists at z_0.

(d) Check if the function $f(z) = e^{z^2}$ satisfies Cauchy-Riemann's conditions.

(e) Evaluate the line integral $\int (x^2 - iy^2)\, dz$ in a complex plane along the parabola $y = 2x^2$ from (1, 2) to (2, 8).

(f) Expand the complex function $f(z) = \frac{z+1}{(z-3)(z-4)}$ about $z = 2$ and find the region of validity.

6. [4+1+4+1]

(a) State and prove Jordan's Lemma.

(b) State Cauchy's residue theorem in complex analysis.

(c) By applying Cauchy's residue theorem, show that $\int_0^\infty \cos x^2\, dx = \int_0^\infty \sin x^2\, dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}$.

(d) Show that the value of integrating an analytic function in a complex plane is independent of the path connecting the initial and the final point.

B.Sc. (Honours) in Physics, (Sem-IV) 2023-Examination
Department of Physics
VISVA BHARATI
Course-CC-10 [Analog Systems and Applications (Theory)]

Time- 3 Hours | Full marks- 40

Answer question no. Q1 and any three from the rest.

Q1. 2×5 = 10

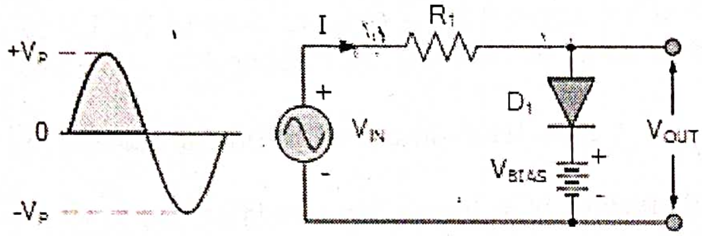
Answer any five from the following

(a) Define the Fermi energy level of a semiconductor and a metal. Where will be the Fermi energy levels of two dissimilar materials in contact at equilibrium? 2

(b) A silicon p-n junction diode operates at 27°C. If the applied forward bias voltage is increased, the current, I is tripled. Calculate the increase in the bias voltage. Assume $I >> I_S$. 2

(c) What do you mean by ripple factor, conversion efficiency of a rectifier? 2

(d) In the following circuit if V_p = 5 V, and V_γ= 0.7 V, and V_{BIAS}= 2 V, Draw the output waveform. Calculate the maximum output voltage, V_{max} in the positive half cycle. Also calculate how much voltage amplitude will be clipped off.



2

(e) Define Early effect or base-width modulation and Thermal run away of transistor? 2

(f) What do you mean by virtual ground of an inverting OPAMP amplifier? What is the difference between virtual and actual ground. 2

(g) Select the correct answer in the following 2

(g1) Semiconductor best suited for Light emitting diode (LED) fabrication is

(i) $GaAs_{1-x}P_x$ (ii) Si (iii) Ge

(g2) The Fermi level of n-type semiconductor lies

(i) Near the conduction band edge (ii) Near the valence band edge (iii) At the middle of the forbidden gap

(g3) Temperature co-efficient of the avalanche breakdown is (i) zero, (ii) positive (iii) negative.

(g4) The Zener breakdown voltage is (i) higher (ii) lower (iii) equal to the avalanche breakdown voltage.

Q2. (a) What is the static and dynamic characteristics, load line and transfer characteristic of a diode?

5. (a) What is an Operational Amplifier (OPAMP)? State the characteristics of an ideal OPAMP. How does the characteristics of a practical OPAMP differ from those of the ideal OPAMP?

(b) Describe with proper circuit diagram and input output voltage waveform, the operation of an OPAMP as a voltage comparator when a reference voltage V_R, is applied in the inverting terminal. How this voltage comparator can be modified to work as a zero-crossing detector?

(c) An inverting OPAMP amplifier has an input resistor of 20 kΩ and a feedback resistor of 100 kΩ. If the input voltage is 5 V, find the output voltage and input current.

(1+ 2 + 1)+ 4+ 2= 10

6. (a) Determine the output voltage, V_O in the circuit of Figure 2 in terms of V_1 and V_2.

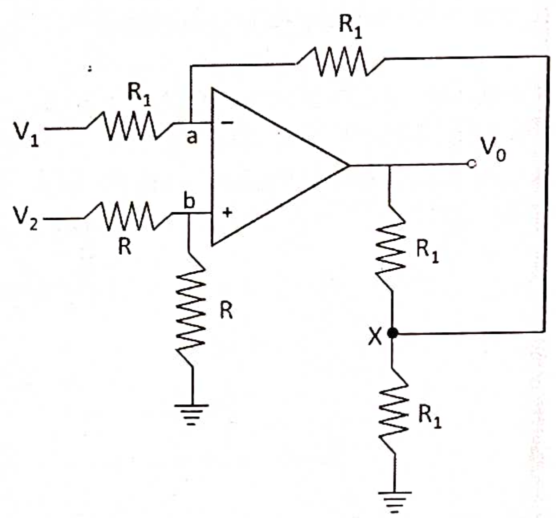


Figure: 2

(b) For the following circuit in figure 3, express the output voltage, V_O in terms of V_1 and V_2 at any instant of time, t when the capacitor is charged with capacitance C.

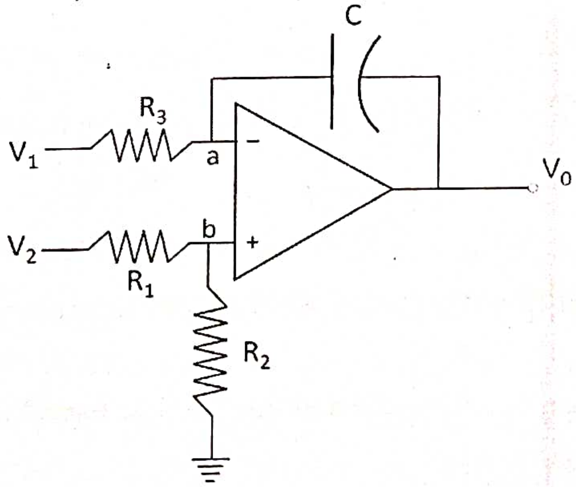


Figure: 3

5+5 = 10

END

Undergraduate Examination, 2023

Semester-IV (CBCS)

Physics Generic Elective Course

GEC-4 (Waves and Optics)

Time : Three hours **Full Marks : 40**

Questions are of value as shown within brackets. Answer any FOUR questions

Q.1 Set up the the equation for forced vibrations with external periodic force "F Cos(pt)" and damping constant "k" . Write the steady state of solution of the system. Hence find out the energy of the system and the condition for energy resonance. (1+1+2)

(b) Write the solution for a particle undergoing simple harmonic oscillations . Show both analytically and graphically that for such a system sum of kinetic energy and potential energy is always a constant. (3)

(c) The motion of a particle undergoing simple harmonic motion is described by the displacement function, $x(t) = A \cos(\omega t + \phi)$. If $x(t) = 1$ cm and initial velocity $v(t) = \omega$ cm/sec at t=0, find out the amplitude A and initial phase angle ϕ. (3)

Q.2(a) State Fourier's theorem. Show how you shall calculate the Fourier coefficients A_0, A_n and B_n for a periodic function $y(\omega t)$ where ω is the angular frequency . Hence write down the general mathematical form of the Fourier series expansion for $y(\omega t)$, in terms of Fourier coefficients of the periodic function. (4)

Q. 2 (b) Write the equation of motion for transverse vibrations in a string when the vertical displacement of the string is $y(x,t)$, with T = tension in the string and m= mass per unit length. Obtain the solution $y_n(x,t)$ for the " n " th order vibration , when the string is fixed at both ends i.e. at x = 0 and x = l; where "l" is the length of the string . Also find out frequency f_n of the nth mode of transverse vibrations of a stretched string, and also the conditions for occurrences of nodes and antinodes. (4+2)

Q.3 (a) Find out the equation of motion of plane progressive acoustic waves travelling through the medium and show that the velocity is equal to $c = \sqrt{K/\rho}$ where K is the bulk modulus and ρ is the density. What is the relation between the phase velocity and group velocity of the travelling wave. (4 +1)

Q.3(b)Suppose a plane progressive wave in a medium of density $\rho = 0.8$ gm/c.c. is given by,
$y = 5 \sin(200\pi t + 0.6\pi x)$.
Hence find out the wave length, frequency, velocity and Bulk Modulus of the medium . (3)

Q.3(c) What do you understand by musical interval? Hence define an ***octave*** in the musical scale . If "m/n " and " n/p " are the interval between successive notes how their inter-relationship can be expressed in the logarithmic scale. (2)

Q.4 (a) Neatly draw the diagram of Fraunhofer diffraction at a single slit and explain. (2.5)

(b) Derive the expression of '*n*' th minima of the of the single slit experiment. (2.5)

(c) Define the reverberation time for a room. Give the relevant expression. (2)

(d) Write the Sabine's formula for the decay of energy intensity in a room of volume V and absorption coefficient α. Hence find out the reverberation time T in terms of the other parameters. (3)

5. (a) What are Newton's rings? Explain how such rings are formed by suitable diagram. Deduce an expression for the diameter of the '*n*'th dark ring. (1+2+2=5)

(b) If a monochromatic light fall at centre of the convex lens normally what are characteristics of the rings formed by this method and what is colour of the central ring? (2)

(c) A convex lens is placed on a slab of plane glass and is illuminated by sodium light of wave length 5.890×10^{-5} cm. The diameter of the 10 th ring is measured and is found to be 0.406 cm. What is the radius of curvature of the lower face of the lens? (3)

6. (a) State and prove the Stoke's theorem for the reflection of light at the interface between the two media? (3)

(b) Write down the conditions of getting permanent interference. (1.5)

(c) Compare between the Fresnel's bi-prism and Lloyd's mirror fringes. (2)

(c) Why the light emitted from two candles do not ordinarily form interference fringes? (1)

(d) A parallel beam of sodium light strikes a film of olive oil ($\mu=1.6$) on water. When viewed at an angle of 30 ° from the normal, the 8^{th} dark band of the system is seen. What is the thickness of the film? $\lambda = 5890$ Å. (2.5)

7. (a) Neatly draw the diagram of Fresnel's bi-prism experiment and explain . (2.5)

(b)Derive the fringe width of the bi-prism experiment. (2)

(c) What do you mean by diffraction of light? (1)

(d) Write down distinction between interferences and diffraction. (2)

(e) What are the two different classes of diffraction phenomena? (1)

(f) Compare between a zone plate and a convex lens. (1.5)

B.Sc. (Honours) Examination, 2023
Physics
Basic Instrumentation Skills
(SECC-II)

Time : 2 Hours Full Marks: 20

Answer any four questions from the rest. Questions are of value as indicated in the margin.

Unless otherwise specified symbols carry their usual meanings.

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1. You have used an experimental set up to measure a quantity n times and obtained the following values $\{v_1, \ v_2, \ ... \ v_n\}$.

   (a) How can you relate this set of values with t, the true value of the quantity?

   (b) How can you test if this experimental set up has a systematic bias? 4+1

2. (a) Describe the working principle of a PMMC device with a labelled schematic diagram.

   (b) Draw the schematic diagram of a set-up for measuring multiple current ranges that includes a PMMC device. Explain how this set-up works. 3+1+1

3. (a) Draw the schematic diagram of a Maxwell bridge to measure an inductance.

   (b) Derive the required expression.

   (c) The relative uncertainty on the values of the resistances used is 1% and that of the capacitances is 2%. Estimate the uncertainty on the measured value of the inductance. 1+2+2

4. Discuss the use of an oscilloscope to measure various quantities related to different types of waveforms. Use suitable diagrams of its display to illustrate. 5

5. Draw the schematic diagram of a Successive Approximation Register (SAR) ADC. Explain how it works. 2+3

6. Discuss briefly the advantages and disdvantages of digital multimeters compared to analog multimeters. 5

B.Sc. (Honours) Examination, 2023

Semester - IV

Physics (Honours)

Paper: CC-IX

(Elements of Modern Physics)

Time : Three Hours **Full Marks : 40**

Questions are of value as indicated in the margin.

Answer **question no. 1 and any three** from rest of the questions.

<u>**Useful Informations:**</u> (Notations have their usual meanings.)

$c = 3 \times 10^8$ m/s, $h = 6.62 \times 10^{-34}$ J-s, 1 nm $= 10^{-9}$ m, 1 eV $= 1.6 \times 10^{-19}$ Joule, $m_e = 9.1 \times 10^{-31}$ kg and $\Gamma(n) = \int_0^\infty x^{n-1} e^{-x} dx$ $(n > 0)$.

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1. Answer any *five* questions. 2×5

   (a) If the peak of the blackbody radiation curve appear as red ($\sim$ 7000 Å) at a temperature 5000°C, approximately at which temperature it would appear as blue ($\sim$ 5000 Å)?

   (b) What will be the kinetic energy of the emitted photoelectron when a metal plate with work function 2 eV, is exposed by a light of wavelength 500 nm?

   (c) If an X-ray beam of wavelength 0.012 nm is used as an incident beam in a Compton scattering experiment, what would be the wavelength of the beam scattered at an angle $\pi/6$?

   (d) Show that time dependent Schrödinger equation is a linear equation. Can we use it in quantum mechanics if the Schrödinger equation is a not a linear equation?

   (e) Calculate the energy of a particle of mass m, represented by a time-independent wave function $\psi(x) = Ae^{-x^2/a^2}$, in a particular region of space if the potential energy is given by $V(x) = \frac{\hbar^2}{ma^4}(2x^2 - a^2)$, where A and a are constant.

   (f) Explain why an electron cannot exist within the nucleus of an atom.

2. (a) What is the Planck's formula for the blackbody radiation? 1

   (b) Explain how this formula can solve the ultra-violet catastrophe problem. 2

   (c) Show that in the low frequency range it leads to the classical Rayleigh-Jeans formula. 2

   (d) Discuss the shortcomings of the classical theory to explain the photoelectric effect. Considering photon as particle how can you explain the observed phenomena of photoelectric effect? 5

P.T.O.

3. (a) What would be the de-Brogile wavelength of a 54 eV electron? **2**

(b) State Ehrenfest's principle in quantum mechanics. Prove that $\frac{d}{dt} < x >= \frac{<p_x>}{m}$. **4**

(c) Suppose the speed of an electron of mass m_e is 300 m/s. Find the uncertainty in the measurement of the position coordinate if you simultaneously measure the speed of the electron. **4**

4. (a) A particle of mass m is represented by the wave function $\psi(x,t) = \mathcal{N} e^{-a[(mx^2/\hbar)+ibt]}$, where a, b and N are real, positive constants. Find the normalization constant $\mathcal{N}$ in terms of m, a and/or b. **4**

(b) Show that the function $exp[\frac{i}{\hbar}(px - Et)]$, with p and E as the momentum and energy of the particle, may represent a wave function for a free particle. **2**

(c) Instead of $i\hbar\frac{\partial}{\partial x}$ can we define the momentum operator as $\hbar\frac{\partial}{\partial x}$? Explain why. **1**

(d) Why a wave function $\psi(x)$ and its derivative $\frac{d\psi(x)}{dx}$ both have to be *finite, single valued* and *continuous*? **3**

5. (a) Derive the equation of continuity for a one-dimensional quantum mechanical system in terms of the probability density and probability current density. **4**

(b) Consider the motion of a particle under the influence of a step potential $V(x)$, defined as $V(x) = 0$ for $x < 0$ and V_0 elsewhere. Discuss the motion of the particle if it is having energy $E < V_0$. **6**

6. (a) What do you mean by magic numbers and packing fraction in nuclear physics? Can you explain the origin of magic numbers? **3**

(b) How does the binding energy per nucleon of light, medium and heavy nuclei vary with mass number? **2**

(c) Derive expressions for the volume, surface and Coulomb terms in the semi-empirical mass formula. **5**

7. (a) Briefly discuss the importance of tunneling phenomena in radioactive α-particle decay process. **3**

(b) Explain the necessity of introducing an antineutrino in nuclear beta decay. **3**

(c) State the main assumption of the liquid drop model and the shell model. **4**