B.Sc. Examination, 2023
(Semester-VI)
PHYSICS
Course Code: CC-13
Electromagnetic Theory

TIME: 3 HRS. FULL MARKS: 40

Attempt *four* questions. **Question No. 1 is compulsory.**
Questions are of value as indicated in the margin.

1. Answer any five. [2 × 5]

   (a) Write down the Maxwell's equations in integral form for (linear) material media and deduce the boundary conditions for magnetic field.

   (b) Consider $V = 0$ and $\vec{A} = A_0 sin(kx - \omega t)\hat{y}$, where A_0, ω, and k are constants. Find $\vec{E}$ and $\vec{B}$, and check that they satisfy Maxwell's equations in vacuum. What condition must you impose on ω and k.

   (c) Show that, under the Lorentz gauge condition, the Maxwell's equations lead to the non-homogeneous wave equations for scalar and vector potentials.

   (d) Write a short note on Poynting vector. What is the unit of Poynting vector ?

   (e) What is displacement current. How does this displacement current resolve the paradox of charging a capacitor.

   (f) Find the fields, and the charge and current distributions, corresponding to $V(\vec{r}, t) = 0$, $\vec{A}(\vec{r}, t) = -\frac{1}{4\pi\varepsilon_0}\frac{qt}{r^2}\hat{r}$. Use the gauge function $\lambda = -\frac{1}{4\pi\varepsilon_0}\frac{qt}{r}$ to transform the potentials and comment on the result.

2. For the given plane wave solutions:
   $$\vec{E}(z,t) = E_0 \, cos(kz - \omega t)\hat{x} \text{ and } \vec{B}(z,t) = \frac{E_0}{c} cos(kz - \omega t)\hat{y}.$$

   (a) Calculate the average value of energy density, energy flux density, momentum density and the intensity of electromagnetic wave. [2+2+1+1]

   (b) What is plasma? For a dilute plasma, deduce an expression between the refractive index of plasma and its frequency. [4]

3. (a) Consider an infinite parallel-plate capacitor, with the lower plate (at $z = -d/2$) carrying the charge density $-\sigma$, and upper plate (at $z = +d/2$) carrying the charge density $+\sigma$. Determine all the components of Maxwell's stress tensor T_{ij} in a region between the plates. Display your answer as a 3×3 matrix. Also determine the force per unit area on the top plate. [4+2]

   (b) A plane electromagnetic wave is propagating in a lossless dielectric media. The electric field is given as $\vec{E}(x, y, z, t) = E_0(\hat{x} + \alpha\,\hat{z})\, exp\big[ik_0\big(-ct + (x + \sqrt{3}\,z)\big)\big]$, where c is the speed of light in vacuum, E_0, k_0 and α are constants. Calculate the relative dielectric constant ε_r and α (Given: $\mu_r \approx 1$). [2+2]

4. (a) State the work-energy (or Poynting's) theorem of electrodynamics and derive the following expression: $\frac{\partial}{\partial t}\left(\mathcal{U}_{mech} + \mathcal{U}_{em}\right) = -\vec{\nabla} \cdot \vec{S}$. [2+5]

(b) A cylindrical wire of length L and conductivity σ carries a current I that is uniformly distributed over its cross-section area A. Show that $\int(\vec{E} \cdot \vec{J})\, d\tau$ over the length of wire is equal to $I^2 \frac{L}{\sigma A}$. [3]

5. (a) What do you mean by skin depth ? Derive an expression of skin depth for linear conducting media. Obtain the expression of skin depth for good conductor. [1+5+1]

(b) Calculate the skin depth for an electromagnetic wave moving through copper if the frequency of the wave is 1MHz. Also calculate the speed of electromagnetic wave in copper. Take the conductivity of copper to be $\sigma = 5.76 \times 10^7 \Omega^{-1} m^{-1}$ and its relative permeability $\mu_r \approx 1$. [3]

6. (a) Calculate the reflection and transmission coefficients for the normal incidence of ele-cromagnetic wave at an interface of two different transparent linear dielectric media and confirm that $R + T = 1$. [7]

(b) An electromagnetic wave $\vec{E} = 6\, Cos(\omega t - kz)\hat{x}$ travelling in air falls normally on a glass surface (of refractive index 1.5). Calculate (i) the amplitude of reflected and transmitted waves, and (ii) the percentage of energy reflected and transmitted at the interface. [3]

7. (a) Find out the polarization of the resulting wave of two electromagnetic waves having electric fields $\vec{E}_1 = \hat{x}\, a_1\, Cos(kz - \omega t)$ and $\vec{E}_2 = \hat{y}\, a_2\, Cos(kz - \omega t + \theta)$. What would be the state of polarization for $\theta = n\pi$ (where $n = 0, 1, 2, 3, ...$) and $\theta = (n + 1/2)\pi$? The symbols have their usual meaning. [6]

(b) State the law of Malus. [2]

(c) What is optical anisotropy ? [2]

B.Sc. Examination, 2023

(Semester - VI)

Physics (Core)

Course Code: CC-14

Statistical Mechanics

Time : Three Hours | Full Marks : 40

Questions are of value as indicated in the margin.
Answer **question no. 1 and any three** from rest of the questions.

1. Answer any *five* questions. 2×5

   (a) What is the physical significance of microstate? Calculate the density of states for a system with one particle having energy E in a three-dimensional volume V.

   (b) Find the entropy of a system of N, non-interacting, localized, spin-$\frac{1}{2}$ particles at a temperature T.

   (c) Consdier four particles in a system having four energy levels $E, 2E, 3E$ and $4E$ (where $E > 0$) at a temperature T. Obtain the partition function.

   (d) Consider a system of N identical, massless, relativistic, free particles in a volume V at a temperature T. If the energy of the i-th particle with momentum $\vec{p_i}$ is given by $E_i = c|\vec{p_i}| = cp_i$, where c is velocity of the particle, find an expression for the partition function.

   (e) Why can't we observe the Bose-Einstein condensation phenomena in two dimensions?

   (f) Find the condition when quantum statistics will be applicable on a system.

2. (a) The energy of a one-dimensional harmonic oscillator is given by $E_N = (n + \frac{1}{2})\hbar\omega$. Find the number of phase points between two successive energy levels. 3

   (b) Consider a classical ideal gas with energy E having N number of particles in a one-dimensional volume V.

      i. Calculate the number of microstates of the system. 4

      ii. Obtain the equation of state for this system. 3

3. (a) For a discrete probability distribution p_i, express the entropy in terms of p_i. 2

   (b) Show that the energy fluctuation of a system is proportional to $\sqrt{c_v}$, where c_v is the specific heat at constant volume. 4

   (c) State and prove the equipartition theorem. 4

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4. (a) Obtain an expression for the partition function and average energy of a system of N localized, one-dimensional classical harmonic oscillators at a temperature T. 5

(b) What would be the average energy if the energy is quantized. 2

(c) Write down the Planck's formula for the blackbody radiation. 1

(d) Hence, obtain the Rayleigh-Jeans formula and Wien's displacement law from it. 2

5. (a) Find the partition function for a classical system of N-identical, localized, static, non-interacting, spin-$\frac{1}{2}$ particles in an external magnetic field H at a temperature T. 4

(b) Find an expression for the magnetization, entropy, average energy and spcfic heat of the system. 4

(c) Define the negative temperature. 2

6. Suppose an ideal Fermi gas contains N number of particles in a volume V at a temperature T. The Fermi-Dirac distribution function is given by $< n_\epsilon >= \frac{1}{z^{-1}e^{\beta\epsilon}+1}$, where, z is the fugacity and other symbols have their usual meanings.

(a) Show that the internal energy, volume and pressure of the gas are related as $P = \frac{2}{3}\frac{U}{V}$. 5

(b) Express the Fermi momentum and the Fermi energy for an ideal Fermi gas in terms of particle density. 4

(c) What is the physical significance of the Chandrasekhar's limit on the mass of a white dwarf star? 1

7. (a) Discuss the Debye specific heat model for solid. 6

(b) Show that in the high temperature limit the specific heat is given by $3R$ per mole. 2

(c) Will there be any difference if we wish to obtain the low temperature specific heat for a liquid? 2

B. Sc. (Hons) Examination, 2023
Semester - VI
DSE-10 (Astronomy & Astrophysics)

Time: 4 hours **FM - 60**

Questions are of value as indicated in the margin.
Answer any five from the following questions.

1. (a) Define the terms apparent magnitude, absolute magnitude, and bolometric magnitude. 1+1+1

(b) The parallax angle for Sirius is $0.379''$. Find the distance to Sirius in units of parsec. Determine the distance modulus for Sirius. Determine the absolute bolometric magnitude of Sirius and compare it with that of the Sun. What is the ratio of Sirius's luminosity to that of the Sun? (the apparent bolometric magnitude of Sirius is -1.53). 1+2+2+2

(c) What is an ecliptic? Write down the right ascension and declination of the Sun when it is located at the autumnal equinox. 1+1

2. (a) Derive Kepler's 3rd law for binary stars. Halley's comet has an orbital period of 76 years. What is the semi-major axis of comet Halley's orbit? Use the orbital data of comet Halley to estimate the mass of the Sun. 3+4

(b) The effective temperature and luminosity of an $O6$ star are $T_{eff} \approx 45,000$ K and $L \approx 1.3 \times 10^5\ L_\odot$, respectively. Calculate the radius of the star in terms of solar radius ($\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}). 2

(c) Describe how the Doppler-shifted spectral line is used to calculate the radial velocity of any astronomical object. 3

3. (a) Calculate the gravitational potential energy of a star of mass M and radius R. 4

(b) What is Jeans criterion regarding the formation of protostars? Show that for a homogeneous cloud with temperature T and density ρ, Jeans length is $R_J = \left(\frac{15\,kT}{4\pi\, G\, \mu\, m_H\, \rho}\right)^{\frac{1}{2}}$. 2+4

(c) Calculate the free-fall time for a molecular cloud of density $\sim 2 \times 10^{-18}$ g cm^{-3} ($G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}). 2

4. (a) Discuss Virial theorem for stars. 4

(b) Show that the average temperature of a star with mass M and radius R can be written as $\bar{T} \approx 4 \times 10^6 \left(\frac{M}{M_\odot}\right)\left(\frac{R}{R_\odot}\right)^{-1}$ K. 4

(c) Assuming a mass loss rate of $10^{-7} M_\odot$ yr^{-1} and a stellar wind velocity of 80 km s^{-1} from a star, estimate the mass density of the wind at a distance of 100AU from the star. 4

5. (a) Discuss the nuclear reactions in the CNO cycle. What is the role of carbon, nitrogen, and oxygen in this series of nuclear reactions? Also, discuss the triple alpha process of helium burning. 2+1+2

(b) Estimate the hydrogen burning lifetime of the Sun, assuming only inner 10% of the Sun's mass becomes available for burning. 2

(c) Starting with the Lane-Emden equation and imposing the necessary boundary conditions, give a solution for the $n = 0$ polytrope. Describe the density structure associated with this polytrope. 4+1

6. (a) What are the different energy transport mechanisms that operate in stellar interiors? Derive the temperature gradient equation for radiative transport. 2+3

(b) Starting from the basic stellar structure equations, show that $L \propto M^3$ (where symbols have their usual meanings). 3

(c) Derive an expression for the Eddington limited luminosity. Calculate the value of it for a 90 $M_\odot$ star, assuming that the major contribution to the opacity comes from the electron scattering. 2+2

7. (a) What do you mean by the resolving power of a telescope? In this context, discuss the Rayleigh criterion. What are the advantages of using the reflecting type telescopes? 1+2+3

(b) Discuss how the distances of the galaxies can be measured from the Cepheid variable stars. 3

(c) A galaxy has a visible mass of 10^{11} $M_\odot$ and a flat rotation curve extending to 25 kpc at the level of 150 km s^{-1}. What is the ratio of its dark-matter mass to its visible mass? 3

B. Sc. (Honours) Examination, 2023
Semester – VI
Physics
Course: DSE-06 (Classical Dynamics)

Time : 4 Hours **Full Marks : 60**

Questions are of value as indicated in the margin
Answer any five questions
Symbols bear their usual meanings

1. a) State the basic postulates of Special Theory of Relativity.
b) What are cyclic co-ordinates? Show that the generalized momentum conjugate to a cyclic co-ordinate is conserved.
c) Show that the norm of the four momentum vector p^{μ} remains preserved.
d) Show that for a central force, the motion is confined to a plane.
e) What do you mean by stable and unstable equilibrium? Explain with examples.
f) Obtain the transformation relation for the contravariant tensor $T^{\alpha\beta}$.

2+2+2+2+2+2=12

2. a) Obtain the Euler-Lagrange's equation of motion for Atwood's machine system.
b) Consider the Lagrangian for a central force field in polar co-ordinates as

$$L = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 - V(r)$$

(i) From this obtain the equations of motion for the system.
(ii) Show that these equations lead to the conservation of total energy of the system.
c) Show that the line element $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$ remains invariant under Lorentz transformation.

3+(2+3)+4=12

3. a) An object is placed at the top of a hill. Find the equation of the path along which it should slide under the action of gravity so that it reaches the bottom of the hill in the shortest possible time.
b) Show that the angular momentum of a system of particles can be expressed as the sum of angular momentum of motion of the centre of mass and the angular momentum of motion about the centre of mass.
c) Explain relativistic length contraction using space-time diagrams.

4+4+4=12

4. a) Using Euler-Lagrange's equation of motion, show that for a translational coordinate, the component of total force along the direction of translation is equal to the time rate of change of component of total linear momentum along the direction of translation.
b) A moving clock runs slow. Explain this statement in the context of time dilation.
c) State Hamilton's principle. Obtain Euler Lagrange's equations of motion from Hamilton's principle.

4+4+(1+3)=12

5. a) A particle is moving under the action of the central force $f(r) = -\frac{k}{r^2}$. Represent graphically the effective potential $V_{eff}(r)$ and discuss about the shape of orbits for energy levels E > 0 and E < 0.

b) A spaceship moving away from the earth with velocity $0.5c$ fires a rocket whose velocity relative to the space is $0.5c$ away from the earth. Calculate the velocity of the rocket as observed from the earth.

c) Establish Kepler's second law of planetary motion from the Euler-Lagrange's equations of motion for a particle moving in a central force field.

d) Consider a system of symmetric linear triatomic molecule having equal masses. Find out the eigen frequencies of oscillations. Also find the eigenvector corresponding to any one of the eigen frequencies.

3+2+3+4=12

6. a) Find out the Hamiltonian for a simple pendulum moving in a plane and hence obtain the Hamilton's equations of motion for the system.

b) Starting from the equation of conic section, establish the relation between the eccentricity and total energy of the system as $\epsilon = \left[1 + \frac{2l^2 E}{mk^2}\right]^{1/2}$.

c) A mass m moves in a circular orbit of radius under the influence of a central force whose potential is $= -\frac{k}{r^n}$. Show that the circular orbit is stable under small oscillations if $n < 2$.

d) A spherical container of fixed radius R is filled with a gas. Identify the nature of constraints.

4+3+4+1=12

7. a) Obtain the expression for velocity of flow of an incompressible fluid of viscosity η through a horizontal narrow tube of length l and radius a. Hence show that the volume of fluid flowing through the tube per unit time is given by

$$V = \frac{\pi(P_1 - P_2)a^4}{8\eta l}$$

where P_1 and P_2 are pressures at the two end of the tube.

b) Find the law of force for a particle which describes the path $r = \frac{2k}{1+cos\theta}$ where k is a constant.

c) The point of support of a simple pendulum (of length l and mass of the bob m) is moving vertically according to the relation where $y = a\, cos\omega t$ where a and ω are constants. Obtain the Euler-Lagrange's equation of motion for the system.

(3+2)+3+4=12

B.Sc. Examination, 2023
(Semester-VI)
PHYSICS
Course Code: DSE-05
Advanced Mathematical Physics-II

TIME: 4 HRS. FULL MARKS: 60

Attempt *five* questions. **Question No. 1 is compulsory.**
Questions are of value as indicated in the margin.

1. Answer any four questions. [4×3]

   (a) Using the variation of calculus, show that the shortest distance between two points in a plane is a straight line.

   (b) Show that

   $$\left[p_i,\, (\vec{J}\cdot\vec{J})\right] = 2(\vec{r}\cdot\vec{p})\,p_i - 2(\vec{p}\cdot\vec{p})\,r_i, \qquad \left[\vec{A}\cdot\vec{J},\, \vec{B}\cdot\vec{J}\right] = \vec{J}\cdot(\vec{A}\times\vec{B}),$$

   where $\vec{A}$ and $\vec{B}$ are the arbitrary constant vectors and $\vec{r}$, $\vec{p}$ and $\vec{J}$ are the position, momentum and angular momentum, respectively.

   (c) Show that a relation R on a set $A = \{1, 2, 3, 4, 5\}$ given by

   $$R = \{(a, b) : |a - b| \text{ is even}\},$$

   is an equivalence relation.

   (d) Prove that the set $G = \{0, 1, 2, 3, 4, 5\}$ is an Abelian group under the operation of addition modulo 6.

   (e) A family has three children. What is the probability that all the children are girls provided that at least two of them are girls.

2. (a) If H is the Hamiltonian and f is any arbitrary function of positions, momenta and time, then show that

   $$\frac{df}{dt} = \frac{\partial f}{\partial t} + \left[f,\, H\right]$$

   Further show that if f is an integral of motion and explicitly independent of time, then $\left[f,\, H\right] = 0$. [2+2]

   (b) Prove the following relation between Lagrange and Poisson brackets

   $$\sum_{k=1}^{2n} \{F_k,\, F_i\}\left[F_k, F_j\right] = \delta_{ij},$$

   where F_k are $2n$ independent functions of $q_1, q_2, ...q_n, p_1, p_2, ...p_n$. [4]

   (c) A pendulum of mass m is attached to a block of mass M with a massless string of length l. The block is sliding on a horizontal and frictionless surface. Set up the Lagrangian for this system and show that the time period of pendulum for small oscillation is $T = 2\pi\sqrt{\dfrac{lM}{g(M+m)}}$. [2+2]

3. (a) If of $Q - \{1\}$ be the set of all rational numbers except 1 with binary operation '$*$' defined by $a * b = a + b - ab$, $\forall a, b \in Q - \{1\}$. Show that $Q - \{1\}$ is an Abelian group. Further solve the equation $5 * x = 3$ in $Q - \{1\}$. [3+1]

(b) If a and b are the elements of a group G, then the equation $ya = b$ has a unique solution in G. [4]

(c) A non empty subset H of a group G is a subgroup iff [4]

$$a \in H, b \in H \Rightarrow ab^{-1} \in H.$$

4. (a) Define the cyclic group. Find out all the generators of given cyclic group [1+3]

$$Z = (\{0, 1, 2, 3, \}, +_4).$$

(b) Find all the cosets of $3Z$ in the group $G = (Z, +)$. [4]

(c) If H is a subgroup of a group G and $g \in G$, then show that [2+2]
(i) $gHg^{-1} = \{ghg^{-1} : h \in H\}$ is a subgroub of G, and
(ii) If H if finite, then $O(H) = O(gHg^{-1})$.

5. (a) State the Bayes' theorem. Given three identical bags. Each of them contains two coins. In first bag, both are gold coins. In second bag, both are silver coins and in third bag, there is one gold coin and one silver coin. A person chooses a bog at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the bag is also of gold ? [1+3]

(b) A random variable x has the following probability distribution: [2+2]

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$.

(i) Find the value of k. (ii) If $P(x \leq a) = \frac{1}{2}$. Find the value of a

(c) A continuous random variable x has the following density function

$$f(x) = \begin{cases} 2e^{-2x}, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

Calculate the mean and variance. [2+2]

6. (a) Find the best values of a and b so that $y = ax^b$ fits the data given in the table [2+2]

x	2	7	13	22	28
y	9	14	26	70	130.

(b) Prove that the variance of Binomial probability distribution is equal to npq where n, p and q are the number of trials, probability of success and probability of failure, respectively. [4]

(c) Show that under what conditions, the Binomial distribution leads to the Poisson distribution. [4]

7. (a) Show that a subgroup H of a group G is a normal subgroup iff [4]

$$H \lhd G \Longleftrightarrow xHx^{-1} = H, \ \forall x \in G.$$

(b) Define the homomorphism and isomorphism. Show that the following mapping

$$f : (R^{+}, X) \longrightarrow (R, +), \text{where} f(x) = \ln x, \ \forall x \in R^{+} \text{ to } R$$

is an isomorphism. [1+1+2]

(c) Out of 2400 families with 5 children each, How many families would be expected to have (i) 3 girls, (ii) 5 boys, (iii) either 2 or 3 girls, and (iv) at least 2 girls. [1 × 4]

B. Sc. (Honours) in Physics Examination 2023

Class-BSc

Semester- VI

Course-DSE12 A (Nanomaterials and Application)

Time- 3:00 Hours   Full marks- 40

Answer any four questions

1. (a) What do you mean by quantum well, quantum wire and quantum dot?

(b) Derive the expression for the density of states for a quantum well. How experimentally a quantum well structure can be fabricated in laboratory?

(c) Draw schematic diagram of the plots of the density of states with respect to energy for the bulk material, quantum well, quantum wire and quantum dot.

2+ (5+1) + 2 = 10

2. (a) What is nanoparticle? At which length scale nanoparticles manifest different property compared to their bulk counterpart and why?

(b) Describe comparatively how the valence energy band level of a bulk metal is modified with its size - large metal cluster, small metal cluster.

(c) What is quantum size effect? What is the result of the quantum size effect on the color of the metal nanoparticles?

(1 + 2) +4+1+2 = 10

3. (a) How one can determine the energy band gap of semiconductor nanoparticles or clusters and excitonic energy levels experimentally?

(b) What is photo fragmentation, Coulombic explosion and molecular clusters?

(c) How the nanoparticles are synthesized using RF plasma method.

(d) How many numbers of atoms constitute smallest nanoparticle in face-centered cubic structure. What is structural magic numbers?

3 +3+2+2 = 10

4. (a) What do you mean by epitaxy or epitaxial growth? What is the difference between homoepitaxy and hetero-epitaxy?

(b) Describe the different types of growth modes in epitaxial growth.

(c) What is lattice mismatch and how it is measured? What do you understand by lattice-matched, pseudomorphic and dislocated growth?

3 +3+4 = 10

5. (a) What is the Vegard's law for the determination of the lattice parameter of compound semiconductors?

(b) AlN and InN have 'a' lattice parameters a_{AlN} = 3.112 Å and a_{InN} = 3.538 Å respectively. Determine the percentage of In content in $Al_xIn_{1-x}N$ at which the $Al_xIn_{1-x}N$ can be grown epitaxially on GaN (a_{GaN} = 3.189 Å) substrate with least lattice mismatch.

(c) Discuss the working principle of Field emission scanning electron microscope mentioning the main components. Discuss the processes that occur when the high energy primary electron beam hits the sample.

(d) What is excitons? What is the difference between Mott-Wannier excitons and Frenkel excitons?

$1 + 2 + 5 + 2 = 10$

6. (a) Describe the basic working principle and main components of molecular growth expitaxy technique.

(b) What is the difference between molecular beam epitaxy and solid phase epitaxy?

(c) Discuss how the lattice energy stored per unit area at the interface behaves with the lattice mismatch or misfit for strained growth and dislocated growth.

(d) Discuss how the lattice energy stored per unit area at the interface behaves with the film thickness for strained growth and dislocated growth.

$4 + 2 + 2 + 2 = 10$

7. (a) Discuss the working principle of Scanning tunneling microscopy mentioning the main components. Name the type of mode-of-operation of Scanning tunneling microscopy and discuss each briefly.

(c) Write down the Vegard's law for estimation of Energy band gap of compound semiconductors, defining all the parameters. Calculate the energy band gap of $In_{0.18}Al_{.82}N$, given that E_g of InN is 0.77 eV and that of AlN is 6.10 eV and the bowing parameter is 2.5 eV.

(c) Which type of hybridization occurs in the neighbouring carbon atoms in fullerene and graphene and what is the reason of good conductivity in these materials?

$5 + 3 + 2 = 10$

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B. Sc. (Honours) in Physics Examination, 2024
Semester – VI
Course Code: DSE-06
Course Name: Classical Dynamics

Time: 4 Hours **Full Marks: 60**

Questions are of value as indicated in the margin
Answer any five questions
Symbols bear their usual meanings

1. a) Discuss with examples the 'Rheonomic' and 'Scleronomic' constraints of motion.
b) Find the velocity at which the kinetic energy of an electron becomes twice its rest mass energy.
c) Are the two Lagrangians $L_1 = (q + \dot{q})^2$ and $L_2 = (q^2 + \dot{q}^2)$ equivalent? Give reason for your answer.
d) Show that the norm of a four velocity vector remains preserved.
e) Define Reynold's number.
f) Obtain the transformation relation for the tensor $A_{\gamma}^{\alpha\beta}$.

2+2+2+2+2+2=12

2. a) (i) Explain the principle of virtual work and D'Alembert's principle.
(ii) Hence obtain Euler-Lagrange equations of motion from D' Alembert's principle.
b) Establish the relativistic law of addition of velocities.
c) Show that the motion of a two body system under central force can be reduced to an equivalent one body problem.

(1+1+4)+2+4=12

3. a) Show that the shortest distance between two points is a straight line.
b) Consider the Lagrangian for a central force field in polar co-ordinates as

$$L = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 - V(r)$$

(i) From this obtain the equations of motion for the system.
(ii) Show that the total energy of a planetary system depends only on the length of the semi-major axis 'a' of the elliptic orbit and is given by $E = -\frac{k}{2a}$.
c) Explain relativistic time dilation using space-time diagrams.

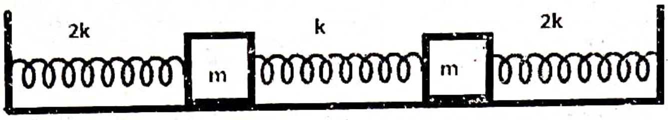
4+(2+3)+3=12

4. a) Consider a dumbbell of length l having two equal masses of value m at both ends such that one of the masses is fixed at the origin. Obtain the Lagrangian for the system and hence obtain the Euler-Lagrange's equations of motion for the system.
b) Establish the relation $E^2 = p^2c^2 + m_0^2c^4$.
c) An astronaut wants to go to a star 15 light years away from the Earth. With what velocity the spacecraft should move relative to the Earth so that the Earth-star distance measured by the astronaut on board is 3 light years?

d) Prove that $\delta L = \frac{d}{dt}(p_i \delta q_i)$ for a conservative system where symbols have their usual meaning.

5+2+2+3=12

5. a) Two blocks of equal masses are tied with springs as shown in the figure below. They execute small oscillations on a frictionless surface. Find the normal frequencies of oscillation of the system.



b) Starting from the equation of conic section for an inverse square law force, establish the relation between the eccentricity and the total energy of the system as $\epsilon = \left[1 + \frac{2l^2 E}{mk^2}\right]^{1/2}$.

c) Suppose a particle A having rest mass m_A decays into two particles B and C having rest masses m_B and m_C. Find the energies of the decayed particles in terms of rest masses of the particles.

5+3+4=12

6. a) A particle is moving under the action of the central force whose potential is given by $V(r) = -\frac{k}{r}$. Represent graphically the effective potential $V_{eff}(r)$ and discuss about the shape of orbits for energy levels E > 0 and E < 0.

b) Obtain the Hamiltonian for a simple pendulum moving in a plane and hence find out the Hamilton's equations of motion for the system.

c) Find the law of force for a particle which describes the path $r = a\exp(b\theta)$ where a, b are constants.

d) What are the advantages of Hamiltonian dynamics over Lagrangian dynamics?

(2+2)+(2+2)+3+1=12

7. a) Show that the Euler's equation of motion for steady flow of a fluid of density ρ in dynamical equilibrium is given by

$$\frac{\partial \vec{v}}{\partial t} + \frac{1}{2}\vec{\nabla} v^2 - \vec{v} \times (\vec{\nabla} \times \vec{v}) = \vec{F} - \frac{1}{\rho}\vec{\nabla} p$$

where $\vec{v}$ is the velocity of the fluid and $\vec{F}$ is the body force.

b) Consider the Hamiltonian $H = q_1 p_1 - q_2 p_2 - a q_1^2 + b q_2^2$ where a, b are constants and q_i, p_i are generalized coordinates and momenta respectively. Show that

(i) $q_1 = Ae^t$ and (ii) $q_1 q_2$=constant.

c) Show that the angular momentum of a system in a central force field is always conserved.

d) State Hamilton's principle.

4+(2+2)+2+2=12

B.Sc (Honours) Examination, 2024

Semester-VI

Paper Name: Statistical Mechanics (Theory)

Paper Code: CC-14

Time : Three Hours **Full Marks : 40**

Answer any **four** questions

1. Consider a system of non-interacting classical ideal gas particles having temperature T, volume V and number of particles N.

   (a) Write the Hamiltonian of the system.

   (b) Find the canonical partition function and the free energy of the system (without the Gibbs factor).

   (c) Thereafter, determine the entropy of the system, and show that the entropy is not additive in nature.

   (d) Do the same problem of finding entropy, by including the Gibbs factor, and show that the entropy becomes additive due to the appearance of the Gibbs factor. **[1+3+3+3]=10**

2. (a) Consider a canonical ensemble of temperature T, Volume V and number of particles N. Find the probability of finding the system within energy E to $E + dE$ (let symbolize it by $p_c(E)dE$).

   (b) Show that $p_c(E)$ has a maximum at a finite E.

   (c) Show that at the limit of large number of particles, $p_c(E)$ converges to the probability distribution of microcanonical ensemble of temperature T, volume V and number of particles N. **[3+2+5]=10**

3. (a) Prove the inter-relation between canonical and grand canonical partition function, i.e.

$$\mathcal{Z}(T, V, \mu) = \exp\left[e^{\beta\mu} Z_1\right] ,$$

   where $\mathcal{Z}(T, V, \mu)$ is the grand canonical partition function and Z_1 is the one-particle canonical partition function (and the other quantities have their usual meaning).

(b) Consider a system of non-interacting classical ultra-relativistic gas particles of temperature T, volume V, number of particles N and chemical potential μ. Find Z_1, and thereafter by using the above relation, find $\mathcal{Z}(T, V, \mu)$. **[5+5]=10**

4. Consider two identical bosons of temperature T (although it is hard to define temperature for a two particle system, still you may consider it for the sake of a problem) and three one particle states of energy 0J, 1J and 2J respectively.

(a) How many states are possible for the system ?

(b) Find the canonical partition function and the total average energy of the system.

(c) Henceforth determine the density operator (in the matrix form) of the system. Show the limiting behaviour of the density matrix at $T \to 0$ and at $T \to \infty$. **[1+3+6]=10**

5. Consider a system of photons of temperature T, enclosed within a container of volume V. the frequency of the photons ranges from $\omega = 0$ to $\omega \to \infty$.

(a) If the one particle Hamiltonian is given by,

$$\hat{H} = c\hat{P} \ ,$$

(where the quantities have their usual meaning), then find the one particle density of states.

(b) Determine the amount of energy of the photons within frequency range ω to $\omega + d\omega$.

(c) Now let make a hole of unit area in the container. Then find the radiative power of the photons (from the hole) within frequency interval ω to $\omega + d\omega$. Thereafter show that the total radiative power (over the entire range of frequency) is proportional to quartic power of the temperature, and the proportionality constant depends on three fundamental constants of nature c, k_B and $\hbar$.

[3+3+4]=10

6. Consider a system of ideal Bose gas of temperature T, volume V and chemical potential μ.

(a) How do you express the state of the system in number representation ?

(b) Determine the grand canonical partition function.

(c) Find — (i) the probability of the case when all the particles lie at higher energy levels, and (ii) find the probablity of the case when all the particles lie at the ground state. Thereafter demonstrate the Bose-Einstein condensation.

[1+3+6]=10

B.Sc. Examination, 2024
(Semester-VI)
PHYSICS
Course Code: DSE-05
Advanced Mathematical Physics-II

TIME: 4 HRS. MAXIMUM MARKS: 60

Attempt *five* questions. **Question No. 1 is compulsory.**
Questions are of value as indicated in the margin.

1. (a) Using the variation of calculus, show that the shortest distance between two points in a plane is a straight line. [3]

   (b) Evaluate the following Poisson brackets

$$\left\{|\vec{r}|,\ |\vec{p}|\right\} \quad \text{and} \quad \left\{(\vec{A}\cdot\vec{J}),\ (\vec{B}\cdot\vec{J})\right\},$$

   where $\vec{A}$ and $\vec{B}$ are the arbitrary constant vectors and $\vec{r}$, $\vec{p}$ and $\vec{J}$ are the position, momentum and angular momentum, respectively. [3]

   (c) What is Transitive relation? Check, whether the relations $R_1 = \{(2,3),(3,1),(2,1)\}$ and $R_2 = \{(1,2),(2,2)\}$ are Transitive or not. [3]

   (d) Prove that the fourth roots of unity form an Abelian group under multiplication. [3]

2. (a) Set up the Lagrangian and obtain the Euler-Lagrange equation of motion for a simple pendulum. Also find out the Hamiltonian of the system. [3]

   (b) Prove the Jacobi's identity. [6]

   (c) State and prove the Poisson theorem. [3]

3. (a) If G be the set of all positive rational numbers equipped with the binary operation $*$ such that $a * b = \dfrac{ab}{7}$, $\forall\ a, b \in G$, then show that $(G, *)$ is an Abelian group. Solve the algebraic equation: $3 * y = 3^{-1}$. [3+1]

   (b) Show that $f : (\mathbb{R}, +) \longrightarrow (\mathbb{C}_0, \times)$ where $f(x) = e^{ix}$, $\forall x \in \mathbb{R}$ is a group homomorphism. Also compute the kernel. [1+3]

   (c) If H is a subgroup of G, then show that the order of any element $a \in H$ is same as the order of $a \in G$. [2]

   (d) Show that the set $H = \{0, 2, 4\}$ is a subgroup of the group $G = \{0, 1, 2, 3, 4, 5\}$ under addition modulo 6. [2]

4. (a) Define the cyclic group. Find all the generators of the cyclic group $(G = \{1, 2, 3, 4, \}, \times_5)$. [1+3]

   (b) Find all the cosets of $3\mathbb{Z}$ in the group $G = (\mathbb{Z}, +)$. [4]

   (c) Show that any two left or right cosets of a subgroup are either identical or disjoint. [4]

5. (a) State the Bayes' theorem. In a factory, machines A, B and C manufacture 20%, 35% and 45% toys, respectively. Of their outputs, 5%, 4% and 2% are defective toys. A toy is selected at random from the product and it is found to be defective. What is the probability that it is manufactured by machine B? [2+4]

(b) A continuous random variable x has the following density function

$$f(x) = \begin{cases} k\,(2x - x^2), & 0 < x < 2; \\ 0, & \text{otherwise.} \end{cases}$$

Find k, $P(x \geq 1)$, mean and variance. [4]

(c) A fair coin is tossed 10 times. What is the probability of getting at least six heads?[2]

6. (a) Find the best values of a and b so that $y = ab^x$ fits the data given in the table [4]

x	2	4	6	8	10
y	4.08	11.08	30.13	81.89	222.62

(b) Show that mean and variance for Poisson distribution are same. [4]

(c) Prove that, under certain limits, the Binomial distribution reduces to the Poisson distribution. [4]

7. (a) State the Hamilton's principle and derive the Euler-Lagrange equation of motion. [4]

(b) If the mapping f is homomorphism from group $G \longrightarrow G'$ with kernel K, then show that K is normal subgroup of G (i.e. $K \triangleleft G$). [4]

(c) Define isomorphism and isomorphic group. Show that the group $(\{+1, -1\}, \times)$ is isomorphic to the group $(\{0, +1\}, +_2)$. [1+1+2]

B. Sc. (Hons) Examination, 2024

Physics

Semester - VI

Paper: DSE-10 (Astronomy & Astrophysics)

Time: 4 hours FM - 60

Questions are of value as indicated in the margin.
Answer any five from the following questions.

1. (a) Define parsec and express it in meters. Given 1 AU=1.5×10^8 km.
(b) What is the bolometric magnitude? Establish the relation between the absolute bolometric magnitude and the luminosity.
(c) The star Sirius has an apparent magnitude of -1.46 and lies at a distance of 2.64 pc from the Earth. Calculate its luminosity as compared with the Sun. The absolute magnitude of the Sun is 4.77.
(d) What are zenith, nadir, and meridian? Explain how right ascension and declination are used to locate objects on the celestial sphere.
(1+1)+(1+2)+2+(2+3)=12

2. (a) Derive the general form of Kepler's 3rd law for binary stars. The period of the Martian moon Phobos is 0.3189 days and the radius of the orbit 9370 km. What is the mass of Mars (in terms of the Solar mass)?
(b)How the surface temperature of a star can be measured? What would be the radiant flux from the Sun at a distance of 10 pc?
(c) The laboratory wavelength of Hβ line is 4861.3 Å. Its wavelength measured in the spectra of some celestial objects is 4857.64 Å and 4865.81 Å, after making corrections for the motion of the Earth around the Sun. Calculate the radial velocities of the objects in km/s. Comment on the results.
(2+2)+(2+2)+4=12

3. (a) Calculate the gravitational potential energy of a star with mass M and radius R that has a density profile $\rho(r) = \rho_c (1 - r/R)$, where ρ_c is the central density. Give your answer in terms of M and R.
(b) What is an isothermal collapse? How does the fragmentation of the collapsing cloud occur? How long does the fragmentation process of the cloud continue?
(c) Establish the condition of hydrostatic equilibrium of a star.
4+(1+2+2)+3=12

4. (a) Estimate the pressure at half the Solar radius from the condition for the hydrostatic equilibrium. Suppose the density is constant and equal to the average density ($\sim 1.4 \times 10^3$ Kg m^{-3}).
(b) Discuss the path of a photon from the centre of a star to its surface. Calculate the average time it would take for the photon to escape from the

Sun if the mean free path remained constant for the photon's journey to the surface.

(c) Derive the luminosity gradient equation of a star. From the basic stellar structure equations, show that $L \propto T_{\rm eff}^{6}$ (where symbols have their usual meanings).

3+(3+2)+(2+2)=12

5. (a) Discuss two major ways in which hydrogen is converted into helium inside a star. What is the triple-alpha process for helium burning?

(b) Assume that a star remains 10^9 years in the main sequence and burns 10% of its hydrogen. Then the star will expand into a red giant, and its luminosity will increase by a factor of 100. How long is the red giant stage, if we assume that the energy is produced only by burning the remaining hydrogen?

(c) Derive an expression for the convective temperature gradient inside the star. Find the condition for the gas bubble to keep rising inside the stars.

(4+1)+3+4=12

6. (a) Derive the equation of radiative transfer and find a general solution to this equation.

(b) What is an Airy disk? Discuss the resolving power of a telescope. What is the grazing angle for X-ray reflection?

(c) The distance between the components of a binary star is $1.38''$. What should the diameter of an optical telescope be to resolve the binary?

5+(2+2+1)+2=12

7. (a) What is electron degeneracy pressure? What is the maximum possible mass of a white dwarf?

(b) The Sun has a rotation period of about 27 days. If the Sun collapsed to become a Neutron star with a radius of 10 km conserving its angular momentum, what would be the expected rotation period?

(c) Define the event horizon of a black hole.

(d) Explain how Cepheid variables can be used for measuring distances of the galaxies.

(e) How the presence of dark matter was confirmed from the rotation curve of a galaxy?

(1+1)+2+2+3+3=12

($k_B = 1.380649 \times 10^{-23}$ m^2 kg s^{-2} K^{-1}, $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}, $m_H = 1.67 \times 10^{-27}$ kg.)

B.Sc. Examination, 2024

(Semester-VI)

PHYSICS

Course Code: CC-13

Electromagnetic Theory

TIME: 3 HRS. MAXIMUM MARKS: 40

Attempt *four* questions. **Question No. 1 is compulsory.**
Questions are of value as indicated in the margin.

1. (a) Consider an interface in XY-plane which separates two linear dielectric media I and II with dielectric constants $\varepsilon_I = 4$ and $\varepsilon_{II} = 3$, respectively. The electric field in region I at the interface is given by $\vec{E}_I = 4(\hat{x}+\hat{z})+3\hat{y}$. Calculate the electric field $\vec{E}_{II}$ at the interface in region II. [2]

   (b) A current I is created by a narrow beam of positive charges moving with constant velocity $\vec{v} = v\hat{z}$ in vacuum. Calculate the Poynting vector and its direction outside the beam at a radial distance r from the axis of beam (assume that r is much larger than the width of the beam). [2]

   (c) Using the Maxwell's equations, derive the continuity equation and provide its physical interpretation. [2]

   (d)The vector potential $\vec{A} = k\,e^{-at}r\,\hat{r}$ (where k and a are constants) corresponding to electromagnetic field is changed to $\vec{A}' = -k\,e^{-at}r\,\hat{r}$. If this transformation represents the gauge transformation of vector potential, then calculate the corresponding change $V' - V$. [2]

   (e) Write down the Maxwell's equations in their integral form in material medium. Show that the parallel component of $\vec{H}$-field is discontinuous across a surface carrying a free surface current density. [2]

2. (a) A plane wave is incident obliquely at the boundary between two dielectric material media. The electric field lies in the plane of incidence. Find the amplitudes of reflected and transmitted waves. [8]

   (b) Show that at a certain angle of incidence (i.e. Brewster's angle) the amplitude of the reflected wave vanishes. [2]

3. (a) What is Brewster's law? Show that when a light ray is incident at the Brewster's angle, the reflected ray is perpendicular to the refracted ray. [2+2]

   (b) Find out the polarization of the resulting wave of two linearly polarized waves defined by $\vec{E}_1 = A_1 Cos(kz - \omega t)\hat{x}$ and $\vec{E}_2 = A_2 Cos(kz - \omega t + \theta)\hat{y}$. What would be the state of polarization for $\theta = n\pi$ (with $n = 0, 1, 2, 3, ...$) and $\theta = (n+1/2)\pi$? The symbols have their usual meaning. [6]

4. (a) The charge distribution inside a material of conductivity σ and permittivity ε at initial time $t = 0$ is $\rho(r, 0) = \rho_0$ (constant). Calculate the charge density $\rho(r, t)$ at subsequent times t. [2]

(b) Suppose you embedded some free charges in a piece of glass with index of refraction around 1.5 and resistivity $10^{-12}\Omega$-m. About how long would it take for the charge to flow to the surface? (Given: $\varepsilon_0 = 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$ and $\mu_0 = 4\pi \times 10^{-7} NA^{-2}$). [2]

(c) An electromagnetic wave travels through salty water, which has conductivity $4\,\Omega^{-1}m^{-1}$. The frequency of wave is $5\,MHz$ and it moves along z-direction. At this frequency, the relative permittivity and relative permeability of water are 74 and 1, respectively. Find the skin depth, wave speed and wavelength. [6]

5. (a) State the Poynting's theorem. [2]

(b) Show that the Lorentz force on a distribution of charges in a given region of volume $\mathcal{V}$ is

$$\vec{F} = \oint_S \overleftrightarrow{T} \cdot d\vec{a} - \varepsilon_0\,\mu_0 \frac{d}{dt} \int_{\mathcal{V}} \vec{S}\, d\tau,$$

where $\overleftrightarrow{T}$ defines the Maxwell's stress tensor. [5]

(c) Consider an infinite parallel plate capacitor with the lower plate (at $z = -d$) carrying surface charge density $-\sigma$ and the upper plate (at $z = +d$) carrying charge density $+\sigma$. Determine all the components of Maxwell's stress tensor in the regions
(i) $+d > z > -d$ (ii) $z > +d$, and (iii) $z < -d$. [3]

6. (a) What do you understand by specific rotation? 80 gm of impure sugar when dissolved in a litre of water gives an optical rotation of 9.9° when placed in a tube of length 20 cm. if the specific rotation of sugar is 66° per decimeter per unit concentration, find the percentage purity of the sugar sample. [2+3]

(b) State the law of Malus. A polarizer and an analyzer are set in such a way that the intensity of the emergent light is maximum. What percentage of the maximum intensity of light is transmitted from the analyzer if either is rotated by 30°? [2+3]