Use separate answer script for each unit

B. Sc. (Honours) Examination-2020

Semester-V (CBCS)
Mathematics
Paper: CCMA 11
(Analysis-V and Differential Equations-III)

Time: Three Hours **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I (Full Marks: 30)
(Analysis-V)

Answer any three questions. $3 \times 10 = 30$

1. (a) If $f\colon [a,\infty) \to \mathbb{R}$ is such that f is bounded and integrable on $[a, X]$ for all $X > a$ and if $\int_a^\infty f$ is absolutely convergent. Then show that $\int_a^\infty f$ is convergent. Moreover, show that the converse is not true. **(5)**

(b) State and prove the Dirichlet's test for convergence of improper integral and then show that $\int_0^\infty (\sin x/x)\, dx$ is convergent. **(5)**

2. (a) Prove that for $n > 0$, $\sqrt{\pi}[\Gamma(2n)] = 2^{2n-1}\Gamma(2n)\Gamma(n + (1/2))$. **(4)**

(b) Prove that for $p > -1$, $\int_0^{\pi/2} \sin^p x\, dx \times \int_0^{\pi/2} \sin^{p+1} x\, dx = \pi/[2(p+1)]$. **(3)**

(c) If (f_n) is a sequence of functions on $[0, 1]$ such that

$$f_n(x) = \begin{cases} 0 & \text{if } 0 \leqslant x < \frac{1}{n+1} \\ \sin^2(\frac{\pi}{x}) & \text{if } \frac{1}{n+1} \leqslant x \leqslant \frac{1}{n} \\ 0 & \text{if } \frac{1}{n} < x \leqslant 1. \end{cases}$$

Then show that (f_n) converges to a continuous function, but not uniformly. **(3)**

3. (a) Let (f_n) be a sequence of Riemann integrable functions converging uniformly to f on a closed and bounded interval $[a, b]$. Then show that f is Riemann integrable on $[a, b]$ and $\int_a^b f_n \to \int_a^b f$ as $n \to \infty$. **(5)**

(b) Let (f_n) be a sequence of functions converging uniformly to f on $[0, \infty)$. Also assume that $\int_0^\infty f$ and $\int_0^\infty f_n$ exist, $n \in \mathbb{N}$. Is it true that $\int_0^\infty f_n \to \int_0^\infty f$ as $n \to \infty$? **(2)**

(c) Assume that $f\colon \mathbb{R} \to \mathbb{R}$ is differentiable and f' is uniformly continuous on $\mathbb{R}$.
Verify that $n\,[f(x + (1/n)) - f(x)] \to f'(x)$ uniformly on $\mathbb{R}$. **(3)**

4. (a) Let R be the radius of convergence of $\sum_{n=1}^\infty a_n x^n$ and let $[a, b]$ be a closed and bounded subset of $(-R, R)$. Then prove that $\sum_{n=1}^\infty a_n x^n$ converges uniformly on $[a, b]$. **(5)**

(b) Let (f_n) be a sequence of real-valued and uniformly continuous functions, converging uniformly to f on (a, b). Then show that $f\colon (a, b) \to \mathbb{R}$ is uniformly continuous. **(3)**

(c) Find the radius of convergence of the power series $\sum_{n=1}^\infty n^n x^{n^3}$ and then determine its interval of convergence. **(2)**

5. (a) Let $f\colon \mathbb{R} \to \mathbb{R}$ be periodic function with period 2π such that

$$f(x) = \begin{cases} 0 & \text{if } -\pi < x \leqslant 0 \\ x & \text{if } \quad 0 < x \leqslant \pi. \end{cases}$$

Find the Fourier series of f and then prove that $(\pi^2/8) = \sum_{n=1}^\infty (1/(2n-1)^2)$. **(5)**

(b) Assuming the validity of differentiating under the integral sign, show that

$$\int_0^{\pi/2} \log(\alpha\cos^2\theta + \beta\sin^2\theta)\, d\theta = \pi \log\left((\sqrt{\alpha} + \sqrt{\beta})/2\right). \quad \textbf{(5)}$$

Unit-II (Differential Equations-III)
(Full Marks: 30)
Answer question number 1 and ***any two*** from the rest.

1. Answer ***any five*** questions.
(a) Find a partial differential equation (PDE) by eliminating arbitrary function ϕ from the equation $xz = \phi(x^4 + x^2y^2)$.
(b) Distinguish between linear, semilinear and quasilinear types of first order PDE in two independent variables.
(c) Solve the PDE $py + qx = 0$, together with the auxiliary condition that $z(0, y) = e^{-y^2}$.
(d) Under which conditions Charpit's method can be applied to solve first order nonlinear PDE?
(e) Show that there is no singular integral for PDE of the type $f(p, q) = 0$.
(f) Let $z = z(x, y)$ be the complete integral of the PDE $pq = xy$ passing through the points $(0, 0, 1)$ and $(0, 1, \frac{1}{2})$ in the xyz-space. Find the value of $z(x, y)$ evaluated at $(2, -2)$.
(g) Find the particular integral of the PDE
$$(2D - 3D^{'})^2 = tan(3x + 2y); D \equiv \frac{\partial}{\partial x}, D^{'} \equiv \frac{\partial}{\partial y}.$$
(h) What do you mean by a well-posed problem for a PDE? [5 × 2=10]

2. (a) For the PDE
$$px(z - 2y^2) = (z - qy)(z - y^2 - 2x^2),$$
find the general integral and the integral surface through the curve $x^2 = y^6 = z^3$.
(b) Find the complete integral of the following PDE by Charpit's method:
$$p^2 + q^2 - 2px - 2qy + 1 = 0.$$
[5+5]

3. (a) Show that the PDEs $2(z + xp + yq) = yp^2$ and $p = \frac{2020}{y^2}$ are compatible on a specified domain D and solve them.
(b) Find the singular and general integrals of the following PDE:
$$z = px + qy + \sqrt{1 + p^2 + q^2}.$$
[5+5]

4. (a) Using Cauchy's method of characteristics, find an integral surface of the PDE $pq = 1$ passing through the straight line $x_0 = 2s$, $y_0 = 2s$, $z_0 = 5s$.
(b) Solve:
$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial x} = z + e^{-x}.$$
[5+5]

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B. Sc.(Honours) Examination-2020

Semester-V (CBCS)
Mathematics
Course: DSE-2
(Linear programming Problem, Game theory and Mathematical Statistics)

Time: Three Hours **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I (Full Marks : 40)
(Linear programming Problem, Game theory)

Answer ***any four*** questions.

1. (a) A factory is engaged in manufacturing three products A, B, C which involve lathe work, grinding and assembling. The cutting, grinding and assembling times required for one unit of A are 2, 1, 1 hours, respectively. Similarly they are 3, 1, 3 hours for one unit of B and 1, 3, 1 hours for one unit of C. The profits on A, B, and C are Rs. 2, Rs. 3, Rs. 4 per unit respectively. Assuming that there are available 300 hours of lathe time, 200 hours of grinding time and 240 hours of assembling time, pose a linear programming problem in terms of maximizing the profit on the items manufactured. [4]

   (b) Solve graphically the following L.P.P.
   Maximize $Z = 4x_1 + 7x_2$
   subject to
   $$12x_1 + 7x_2 \leq 42$$
   $$5x_1 + 4x_2 \leq 20$$
   $$2x_1 + 3x_2 \geq 6$$
   $$x_1 \geq 0,\ x_2 \geq 0\ .$$
   Also, mention the vertices of the feasible region. [6]

2. (a) Find all the basic solutions of the following equations by idendifying in each case the basic vectors and basic variables :
   $$x_1 + x_2 + x_3 = 4$$
   $$2x_1 + 5x_2 - 2x_3 = 3$$
   . [5]

   (b) Given a feasible solution $x_1 = 2, x_2 = 4, x_3 = 1$ to the set of equations:
   $$2x_1 - x_2 + 2x_3 = 2$$
   $$x_1 + 4x_2 = 18$$
   $$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.$$
   Reduce this to a basic feasible solution. [5]

3. Use Charne's Big M method to solve the following L.P.P. [10]
   Maximize $Z = 2x_1 - 3x_2$
   subject to
   $$-x_1 + x_2 \geq -2$$
   $$5x_1 + 4x_2 \leq 46$$
   $$7x_1 + 2x_2 \geq 32$$
   $$x_1 \geq 0,\ x_2 \geq 0$$

4. Solve the following transportation problem using North-West Corner method and Vogel's Approximation Method (VAM) to determine the initial basic feasible solution. Also, determine the unique optimal solution and corresponding cost associated with VAM : [10]

Destination

Origin		D_1	D_2	D_3	D_4	a_i
	O_1	6	1	9	3	70
	O_2	11	5	2	8	55
	O_3	10	12	4	7	90
	b_j	85	35	50	45	

5. (a) A company has 5 jobs to be done. The following matrix shows the return in rupees of assigning ith machine (i=1,2,...,5) to the jth job (j=1,2,..,5). Assign the five jobs to five machines so as to maximize the expected profit.

Machines

Jobs	M_1	M_2	M_3	M_4	M_5
J_1	5	11	10	12	4
J_2	2	4	6	3	5
J_3	3	1	5	14	6
J_4	6	14	4	11	7
J_5	7	9	8	12	5

[5]

(b) Solve the following game using graphical method:

Player C

Player R	C_1	C_2	C_3	C_4
R_1	8	5	-7	9
R_2	-6	6	4	-2

[5]

6. (a) Use dominance to reduce the payoff matrix and solve the game with the following payoff matrix:

Player B

Player A	B_1	B_2	B_3	B_4	B_5	B_6
A_1	0	0	0	0	0	0
A_2	4	2	0	2	1	1
A_3	4	3	1	3	2	2
A_4	4	3	7	-5	1	2
A_5	4	3	4	-1	2	2
A_6	4	3	3	-2	2	2

[5]

(b) Use the algebraic method to solve the game problem with the following payoff matrix:

Player B

Player A	B_1	B_2	B_3
A_1	-1	2	1
A_2	1	-2	2
A_3	3	4	-3

[5]

Unit-II (Full Marks: 20)
(Mathematical statistics)
Answer ***any Four*** questions.

1. Define sampling distribution of statistic. Show that the statistic $Y = nS^2/\sigma^2$ has a χ^2-distribution with $(n-1)$ degrees of freedom (S^2 is the sample variance). Using it show also that for a Normal (μ, σ) population

$$\sigma(s^2) = \sqrt{\frac{2}{(n-1)}}\sigma^2.$$

[5]

2. If $X_1, X_2, \cdots X_n$ is a random sample from a distribution with density function

$$f(x, \theta) = \begin{cases} (1-\theta)x^{-\theta} & \text{if } 0 \ < \ x \ < \ 1, \\ 0 & \text{otherwise,} \end{cases}$$

what is the maximum likelihood estimator of θ? [5]

3. Define unbiased estimator. Let $X_1, X_2, \cdots X_n$ be a random sample from a population with mean μ and variance $\sigma^2 \ > \ 0$. Is the sample variance S^2 an unbiased estimator of the population variance σ^2? [5]

4. Let $X_1, X_2, \cdots X_n$ be a random sample from a population X with probability density function

$$f(x, p) = \begin{cases} p^x(1-p)^{(1-x)} & \text{if } x \ = \ 0, 1, \\ 0 & \text{otherwise,} \end{cases}$$

what is the $100(1-\alpha)\%$ approximate confidence interval for the parameter p? [5]

5. State and prove Neyman-Pearson theorem on finding the best critical region for simple hypothesis. Apply it to find the form of the best critical region for testing
$H_0 : \theta = 1$ against $H_1 : \theta = 2$, where

$$f(x, \theta) = \begin{cases} (1+\theta)x^{\theta} & \text{if } \ 0 \ \leq \ x \ \leq \ 1 \\ 0 & \text{otherwise} \end{cases}$$

and X_1, X_2, X_3 denote three independent observations. [5]

6. Construct an approximate test for the null hypothesis $H_0 : p = p_0$ for the Binomial (n, p) population. [5]

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B.Sc. (Honours) Examination-2020

Semester-V (CBCS)
Mathematics
Course: DSE-1
(Mechanics-II)

Time: Three Hours **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I (Full Marks: 40)
(Dynamics of a rigid body)
Answer ***any four*** questions.

1. (a) Define the momental ellipsoid of a rigid body at a point. When two rigid bodies are said to be equi-momental? [3+2]

   (b) Suppose that $ABCD$ is a uniform parallelogram of mass M. At the middle points of the four sides are placed particles of mass each equal to $M/6$, and at the intersection of the diagonals a particle of mass $M/3$. Show that these five particles and the parallelogram are equi-momental systems. [5]

2. Find the moment of inertia of the area of the lemniscate $r^2 = \alpha^2 \cos 2\theta$ about a straight line through the origin and perpendicular to its plane.
   Show further that the principal axes at the node of a half-loop of the lemniscate are inclined to the initial line at angles $\frac{1}{2}\tan^{-1}\frac{1}{2}$ and $\frac{\pi}{2} + \frac{1}{2}\tan^{-1}\frac{1}{2}$. [5+5]

3. (a) What is meant by a reversed effective force? Discuss the motion of a rigid body about its centre of inertia. [1+4]

   (b) A rough uniform board of mass m and length $2l$, rests on a smooth horizontal plane, and a man of M walks on it from one end to the other. Show that the distance through which the board moves is $2Ml/(m + M)$. [5]

4. (a) Define the radius of gyration and the kinetic energy of a rigid body about an axis of rotation on its plane. [2+2]

   (b) A uniform rod of length $2a$ is freely movable about a smooth pin, fixed through one of its ends, on a rough plane inclined at an angle α with the horizon. If the rod, held in a horizontal position in the plane, is allowed to fall from this position and the angle through which it falls from rest becomes small then show that the inclination α is given by $\alpha = \tan^{-1}\mu$, where μ is the coefficient of friction. [6]

5. (a) Distinguish between simple and compound pendulums. Define the simple equivalent pendulum and find its length. [3+2]

   (b) A sphere of radius r is suspended by a fine wire from a fixed point at a distance s from the centre. If a is the amplitude of vibration, show that the time T of a small oscillation is given by

$$T = \frac{\pi}{10g}\sqrt{5 + \frac{2r}{s^2}}\left(5 - \cos^2\frac{a}{2}\right).$$

[5]

6. (a) A right circular cone of semi-vertical angle θ can turn freely about an axis passing through the centre of its base and perpendicular to its axis. If the cone starts from rest with its axis horizontal, then find the ratio between the weight of the cone and the thrust on the fixed axis. [3+2]

(b) Let a fine thread, fixed at one of its ends, be wounded on a reel whose axis is horizontal and which falls in a vertical line. If the unwound part of the thread is vertical and the reel is a solid cylinder of mass m, show that the ratio between the tension of the thread and the acceleration of the centre of the reel is $m : 2$. [5]

Unit-II (Full Marks: 20)

(Hydrostatics)

Answer ***any two*** questions.

1. (a) Discuss the notion of "stress" and hence define "shearing stress" for a fluid in equilibrium. [1+1]

   (b) Show that in a homogeneous fluid at rest under gravity, the difference of the pressures between any two points, not necessarily in the same vertical line, is proportional to the difference of their depths below the free surface. [3]

   (c) A small uniform circular tube, whose plane is vertical contains equal quantities of fluids whose densities are ρ and σ ($\rho < \sigma$), and do not mix. If they together fill half the tube, show that the radius passing through the common surface makes with the vertical an angle $\tan^{-1} \frac{\sigma-\rho}{\sigma+\rho}$. [5]

2. (a) If a fluid be in equilibrium under the force of gravity only then show that its free surface is a horizontal plane and so are the surfaces of equi-pressure and equi-density. [4]

   (b) A given volume V of liquid is acted upon by forces which are proportional to x/a^2, y/b^2 and z/c^2 towards a fixed point along the axes. Show that the equation of the free surface is an ellipsoid. [3]

   (c) Prove that if the density of a liquid at rest under gravity varies as the square root of the pressure, the density increases uniformly with the depth. [3]

3. (a) Define the thrust of a heavy homogeneous liquid on a plane surface of area S, and find its expression. [1+2]

   (b) ABC is a triangular lamina with the side AB in the surface of a heavy homogeneous liquid. A point D is taken in AC such that the thrusts on the areas ABD and DBC are equal. Prove that $AD : AC = 1 : \sqrt{2}$. [3]

   (c) A lamina in the shape of a quadrilateral $ABCD$ has its side CD in the surface of a liquid and the sides AD, BC vertical and equal to α, β respectively. Show that the depth of its centre of pressure is

$$\frac{1}{2}\frac{(\alpha^2+\beta^2)(\alpha+\beta)}{\alpha^2+\alpha\beta+\beta}.$$

[4]

B.Sc.(Honours) Examination-2020

Semester-V(CBCS)
Mathematics
Core Course: CC-12
(Numerical Analysis)

Time: Three Hours **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer ***any six*** questions.

1. Define approximate number and errors involved there. What do you mean by rounding of numbers. State basic rules for rounding of number. State and prove the theorem on relationship between relative error of an approximate number and the number of correct digits involved there. [1+2+1+2+4]

2. Describe the method of chord for the determination of an isolated root of an algebraic or a transcendental equation. Provide an estimate of accuracy of the approximate solution. Whether this method may be regarded as an iterative scheme? Find an isolated root of the equation $\sin x = 1$ correct upto three significant digits, if exists. [3+2+1+4]

3. What do you mean by separated roots of an algebraic or a transcendental equation. State the basic steps of Graffe's method for finding real distinct roots of an algebraic equation of degree six. Use this method to find real roots of the equation $x^4 - 3x^2 + 2 = 0$, if exists. [2+3+5]

4. Describe Gaussian method for the determination of root a system of linear simultaneous equations in n-variables. Present this in the form of iteration for a system of five variables. State how the results of the table may be used for the evaluation of the determinant of the coefficient matrix involved with the system of linear equations. [4+4+2]

5. Describe Seidel's method for the determination of root of a system of linear equations. Verify whether the system of equations

$$\begin{aligned}
296x - 482y - 395z + 242t &= 720\\
475x - 316y - 407z + 253t &= 521\\
282x - 286y - 315z + 448t &= 266\\
364x - 421y - 643z + 342t &= 634
\end{aligned}$$

has unique solution. Use this method to solve the system of equations correct upto two decimal places, if yes. [3+2+5]

6. Define finite difference of a function $f \in \mathcal{C}^n(\mathbb{R})$. Establish the relation between derivative and difference of a function. Present how an error in the data propagates in the difference table. Define generalized power of x. Establish the recurrence relation for $\Delta^k x^{[n]}$ in case of $k \leq n$ and $k > n$. [1+3+2+1+3]

7. State the problem of interpolation of a function $f \in \mathcal{C}^n(\mathbb{R})$ and interpret your observation geometrically. Write down formula for Newton's forward and Lagrange's interpolating polynomials including an estimation of error for equally spaced and unequal spaced data. [2+2+2+2+2]

8. What do you mean by inverse interpolation. Which of the Newton's or Lagrange's interpolation formula may be useful for the inverse interpolation? Find the value of x corresponding to the value of $y = 2$ by using the data [2+1+7]

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
y	1.00000	1.64872	2.718	4.48169	7.38906

9. Derive formulas for approximation of first and second derivatives of a function $y \in \mathcal{C}^{n(\geq 2)}(\mathbb{R})$ in case of explicit rule of correspondence between dependent and independent variables are not known, instead given at $n+1$ equally spaced nodes $(x_0, y_0), \cdots, (x_n, y_n)$. Estimate error in the approximation in terms of given data. Hence derive the formulae for $y'(x_1)$ and $y''(x_0)$. [3+3+2+2]

10. What do you mean by mechanical quadrature and cubature for numerical integration. When are they called closed- and open-type? Give an example for each of them. Derive Newton-Cote's quadrature formula in its general form. Prove the relation $H_i = H_{n-i}$ among the Cote's coefficients. [2+2+3+3]

Reference of books etc.

*PAPER-SETTER*____________________

*Examiner in Subject*__________ *Paper*__________ *Examination*__________

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