Use separate answer script for each unit

B. Sc. (Honours) Examination-2020

Semester-VI
Mathematics
Course: CC-13
(Analysis VI)

Time: Three Hours **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I (Full Marks: 30)
(Complex Analysis)

Answer ***any three*** questions.

1. (a) If $\lim_{z\to\alpha} f(z) = l$, then show that $\lim_{z\to\alpha} \Re\{f(z)\} = \Re\{l\}$. [3]
   (b) Prove that the limit of a function of complex variable when exists is unique. [4]
   (c) If $\lim_{z\to\alpha} f(z) = l \neq 0$,then show that there exists a deleted neighbourhood of α in which $f(z) \neq 0$. [3]

2. (a) Show that if the functions $f(z)$ and $g(z)$ are continuous at $z = \alpha$, then $f(z).g(z)$ is also continuous there. [4]
   (b) If $f(z)$ is continuous in a bounded closed region D, then show that it is bounded in D. [6]

3. (a) If $f(z)$ is differentiable at α, then show that it is continuous at α. Show by an example converse is not always true. [3]
   (b) Show that for the following function C-R equations are satisfied at the origin but $f'(0)$ does not exist:
   $f(z) = \frac{xy^2(x+iy)}{x^2+y^4}$,for $z \neq 0$
   $= 0$, for $z = 0$. [4]
   (c) If the functions $f(z) = u + iv$ and $\overline{f(z)} = u - iv$ are both analytic in a region R, then show that $f(z)$ is constant in R. [3]

4. (a) Define a harmonic function. Find a harmonic conjugate $v(x, y)$ when $u(x, y) = sinhx\ siny$. [4]
   (b) Show that if $lim_{n\to\infty} z_n = z$ then $lim_{n\to\infty}|z_n| = |z|$. [3]
   (c) Prove that any absolutely convergent series is convergent. [3]

5. (a) Prove that cross ratio remains invariant under any Bilinear Transformation. [2]
   (b) Show that the equation $|\frac{z-\alpha}{z-\beta}| = k(k \neq 1)$ represents a circle with respect to which α and β are inverse points. [4]
   (c) Find the stereographic projection of a circle in the complex plane whose equation is given as $x^2 + y^2 + 2gx + 2fy + c = 0$. [4]

Unit-II (Full Marks: 30)
(Metric Spaces)

Answer ***any three*** questions.

1. (a) Define a metric space. Give an example of it. [4]
   (b) For any four points x, y, z, w in a metric space (X, ρ), prove that $|\rho(x, y) - \rho(z, w)| \leq \rho(x, z) + \rho(y, w)$. [3]
   (c) If $\rho(x, y)$ is a metric for a set X, then show that $3\rho(x, y)$ is also a metric for X. [3]

2. (a) Let (X, ρ) be a metric space. Then show that each open sphere in X is an open set. [4]
   (b) Show that arbitrary union of open sets is open in a metric space. [3]
   (c) Is the arbitrary intersection of open sets open ? Give reason. [3]

3. (a) Define a Cauchy sequence in a metric space. [2]
   (b) Show that if a sequence $\{x_n\}$ converges, then it is a Cauchy sequence. [4]
   (c) Give an example of a Cauchy sequence in a metric space. [4]

4. (a) Define a complete metric space. [2]
   (b) Prove that $C[a, b]$ is a complete metric space. [6]
   (c) Give an example of an incomplete metric space. [2]

5. (a) Let X be a complete metric space and let Y be a subset of X. Then show that Y, considered as a metric space is complete if and only if it is closed. [6]
   (b) Let $X = (0, \frac{1}{4})$ be a metric space with the metric of R. Let $T : X \longrightarrow X$ be given by $Tx = x^3$. Show that T is a contraction mapping. [4]

B. A./B. Sc. Examination-2020

Semester-VI

Mathematics

CCMA 14 (Algebra IV)

Time: Three Hours **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Group - A (Group Theory)
Answer ***any four*** questions.

1. (a) Prove that every subgroup of index two is normal. [2]
   (b) Give an example of an infinite group G and a subgroup H such that $[G:H]=2$. [1]
   (c) Show that the quotient group $(\mathbb{Q}/\mathbb{Z},+)$ is an infinite group such that each of its elements is of finite order. [2]

2. Let $f: G \longrightarrow G'$ be a group homomorphism.
   (a) If K is a normal subgroup of G', then prove that $f^{-1}(K)$ is a normal subgroup of G. [2]
   (b) Give an example of a normal subgroup H of G such that $f(H)$ is not normal in G'. [2]
   (c) Show that for each $a \in G$, $o(f(a)) \mid o(a)$. [1]

3. State and prove the Cayley's Theorem for the representation of groups. [5]

4. (a) Let G be a group. Prove that the mapping $f: G \longrightarrow G$ defined by $f(a)=a^{-1}$ is a homomorphism if and only if G is abelian. [2]
   (b) Find all group homomorphisms $f: \mathbb{Z} \longrightarrow \mathbb{Z}_6$. [3]

5. (a) State and prove the second isomorphism theorem. [3]
   (b) Show that $\mathbb{Z}_7$ can not be a homomorphic image of $\mathbb{Z}_{12}$. [2]

6. (a) State the result regarding the characterization of the subgroups of the quotient group $\mathbb{Z}/24\mathbb{Z}$. Find all subgroups of this quotient group. [3]
   (b) Prove that $GL(2,\mathbb{R})/SL(2,\mathbb{R}) \simeq \mathbb{R}^*$. [2]

Group - B (Ring Theory)
Answer ***any four*** questions.

1. (a) Show that the sum $12\mathbb{Z}+8\mathbb{Z}$ of the ideals $12\mathbb{Z}$ and $8\mathbb{Z}$ is the ideal $4\mathbb{Z}$. [2]
   (b) Let $I=\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M(2,\mathbb{Z}) \mid a,b,c,d$ are even integers$\}$. Find the number of elements of the quotient ring $M(2,\mathbb{Z})/I$. [3]

2. (a) Find all ring homomorphisms from $\mathbb{Z}$ into $\mathbb{Z}$. [3]
   (b) Show that the ring $2\mathbb{Z}$ is not isomorphic to $4\mathbb{Z}$. [2]

3. (a) Prove that $R[x]/<x> \simeq R$. [2]
   (b) Let I and J be two ideals of a ring R. Prove that $R/(I\cap J)$ is isomorphic to a subring of $R/I \times R/J$. [3]

4. (a) Let R be a commutative ring with unity. If $a_0+a_1x+\cdots+a_nx^n$ is a unit in $R[x]$, then show that a_0 is a unit and $a_1,a_2,\cdots,a_n$ are nilpotents in R. [3]
   (b) In $\mathbb{Z}_8[x]$ prove that $[4]x^2+[2]x+[4]$ is a zero divisor and $[2]x$ is a nilpotent element. [2]

5. State and prove the Division Algorithm for polynomials. [5]

6. (a) Let F be a field and $f(x), g(x) \in F[x]$ be two nonzero polynomials in $F[x]$. Prove that the greatest common divisor of $f(x)$ and $g(x)$ exists uniquely in $F[x]$. [3]
   (b) Show that there is no irreducible polynomial over $\mathbb{R}$ of degree greater than 2. [2]

Group - C (Linear Algebra)
Answer ***any four*** questions.

1. (a) A linear transformation $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ is defined by $T(x, y, z) = (x + y + z, x - y + 2z, 3x + y + 4z)$. Find a basis and dimension of both the $\ker T$ and $Im\ T$. [2+2]

   (b) Does there exist a linear transformation $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ such that $\ker T = Im\ T$? [1]

2. (a) Let $\{u_1, u_2, \cdots, u_n\}$ be a basis of U and $v_1, v_2, \cdots, v_n$ be n vectors in V. Then prove that there is a unique linear transformation $T : U \longrightarrow V$ such that $T(u_i) = v_i$ for all $i = 1, 2, \cdots n$. [3]

   (b) Find the linear transformation $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ that represents the counter clock-wise rotation about the origin by 60^o. [2]

3. (a) If $T : V \longrightarrow V$ is a linear transformation such that $T^2 = T$, then prove that $V = \ker T \oplus Im\ T$. [3]

   (b) Give an example with justification to show that the converse of the above does not hold. [2]

4. (a) Let $T : P_3(\mathbb{R}) \longrightarrow P_3(\mathbb{R})$ be the linear operator defined by, $T(p(t)) = \frac{dp(t)}{dt}$. Find the matrix of T in the basis $\{1, 1 + x, x + x^2, x^2 + x^3\}$. [3]

   (b) Find the determinant and the trace of the above linear transformation. [2]

5. (a) Let $T : U \longrightarrow V$ and $S : V \longrightarrow W$ be two linear transformations. Then prove that for any three ordered bases α, β and γ of U, V and W, respectively, $[ST]^\gamma_\alpha = [S]^\gamma_\beta [T]^\beta_\alpha$. [3]

   (b) Let $T : U \longrightarrow V$ be a one-to-one linear transformation. Show that T maps every linearly subset of U into a linearly independent subset of V. [2]

6. (a) Let $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{pmatrix}$ be a linear transformation $T : \mathbb{R}^3 \longrightarrow P_2(\mathbb{R})$ in the basis $\beta = \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$ and $\beta' = \{1 + t, t + t^2, 1 + t^2\}$. Find T. [3]

   (b) Let $T : V \longrightarrow V$ be a linear transformation and $T_0 : V \longrightarrow V$ be the zero transformation. Prove that $T^2 = T_0$ if and only if $Im\ T \subseteq \ker T$. [2]

B. Sc. (Honours) Examination-2020

Semester-VI

Mathematics

Course: DSE-3

(Computer Fundamentals (Theory))

Time: Two Hours **Full Marks: 20**

Questions are of values as indicated in the margin.

Notations and symbols have their usual meanings.

Answer ***any four*** questions.

1. (a) Define algorithm. State four important features of an algorithm. [1+2]

   (b) Write an algorithm to calculate and print the value of a function F given by

$$F(x) = \begin{cases} x^2 + 9 & x < 0 \\ x^4 + 3x^2 + 7 & 0 \leq x < 20 \\ 12x + 7 & x \geq 20 \end{cases}$$

[2]

2. (a) What do you mean by priority of computation? [3]

   (b) (i) Given the values $I = 3,\ J = 4,\ K = 2,\ A = 3.0,\ B = 4.0$. Find the value of the expression $B + J * K * *J/B/I$.

   (ii) If $A = 3.0,\ B = 2.0,\ C = 4.0,\ D = 6.0,\ E = 5.0$, then find the order of evaluation and the value of the expression $((A + B) * C * *1.5 - D - E - A)/C + B$. [1+1]

3. (a) What are the differences between 'STOP' statement and 'END' statement. [2]

   (b) Assuming that I and J are integer variables having the values -6 and 9 respectively and that A, B and C are real variables with values -4.23, 11.2 and 31.2 respectively. Find the value of the following expressions:

   (i) $NINT(C).EQ.(6 * I + 3)$

   (ii) $.NOT.((I.GT.J).OR.(A.LT.C)).EQV.((I.LE.J).AND.(A.GE.C))$

   (iii) $.NOT.((I.GT.J).AND.(A.LT.C)).NEQV.((I.LE.J).OR.(A.GE.C))$ [3]

4. (a) Describe logical 'IF' statement. [2]

   (b) Write a complete program to compute and print K where $K = \sum_{I=1}^{m} \sum_{J=1}^{n} I * J$ using the logical 'IF' statement. [3]

5. Describe execution of a 'DO' statement. [5]

6. Describe unconditional 'GOTO' and computed 'GOTO' statements. [5]

B. Sc. (Honours) Examination-2020

Semester-VI

Mathematics

Course: DSE-3B

(Computer Laboratory (Practical))

Time: Three Hours **Full Marks: 40**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer ***any eight*** questions.

1. Write a Fortran program to calculate the monthly telephone bill at the following rates:
First 300 calls : no charges
Calls between 301 and 501: at the rate of 50p per call
Calls between 501 and 1000: at the rate of 80p per call
Calls more than 1000: at the rate of Rs. 1.5 per call
and rental charges of Rs. 75 per month.
[5]

2. A Fibonnaci sequence is defined as follows:
0,1,1,2,3,5, ... ($T_{n+1} = T_n + T_{n-1}, n \geq 1$)
Write a Fortran program to find all numbers of this sequence which are$\leq$ 200. [5]

3. Draw a flow chart to find the inner product $x_1y_1 + x_2y_2 + ... + x_ny_n$ of two vectors X and Y, each containing n elements. [5]

4. Write a Fortran program to compute $\int_a^b f(x)dx = \frac{h}{2}\left[f(a) + 2f(a+h) + ... + 2f(b-h) + f(b)\right]$
$h = (b-a)/n, f(x) = \frac{x+sinx}{x+e^x}$. [5]

5. Write a Fortran program to read the values of the n nodal points and the corresponding function values and to calculate the value of the nth degree Lagrange interpolating polynomial for a given value of x which is also to be read. [5]

6. (a) Write a Fortran program to read values of the sides a,b,c of a triangle and to calculate and print the perimeter and area of the triangle. [3]
   (b) Given a series in geometric progression, with first term a and common ratio r. Write a Fortran program to find the sum of the first n terms of this series. [2]

7. Write a complete Fortran program to evaluate
$cosx = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ...$
by direct summation of successive terms, neglecting the first term whose absolute value becomes lesser than EPS or adding M terms, which ever is earlier, for $x = 0.1$ to 0.5 in steps of 0.05(x in radians) using DO statement. [5]

8. Write a Fortran program to arrange a sequence of n real numbers ($n \leq 100$) in ascending order. [5]

9. Write a Fortran program to find the smallest positive root of an algebraic or transcendental equation using fixed point iteration method. [5]

10. Write a Fortran program to find the smallest positive root of an algebraic or transcendental equation using regula falsi method. [5]

11. Write a Fortran program to find the function value of y for a given value of x using Newton's forward interpolation formula. [5]

12. What will be the final values of the variables in the following sections of program after all the statements have been executed in each case? Also show the steps of evaluation of variables.
   (a)
```
INTEGER A,B

A = -4

B = -2
```

```
50    IF (A-B) 20, 30, 30
20    A = A+5
      GO TO 50
30    IF (A.LT.0) A = A+3
      B = A + B
      IF (B .LE. 0) GO TO 50
      B = A * B
```

[3]

(b)

```
      A = 4
      K = 1
      ASSIGN 50 TO I
50    GO TO (30,30,40,20), K
20    ASSIGN 60 TO I
30    A = A + 2
40    K = K + 1
      GO TO I, (50,60)
60    A = A-1
      B = A + K.
```

[2]

B. Sc. (Honours) Examination-2020
Semester-VI
Mathematics
Course: DSE-4
(Mathematical Modelling)

Time: Three Hours **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer question no. 1 and any five form the rest (taking at least two from each group)

1. State the basic features of mathematical modelling. Mention the classification of mathematical models and explain the basic differences among them with appropriate illustrations. [3+3+4]

Group A (Physical System)

2. Construct a mathematical model for the motion of a simple pendulum in presence of a frictional force. Discuss the case of either under damped or over damped oscillations. [6+4]

3. Consider the one-dimensional linear heat diffusion in a uniform rod of length l and construct the model for the heat flow along a fixed direction. [10]

4. (a) Starting from the Maxwell's equations, obtain the one-dimensional wave equation for the electromagnetic (EM) fields. Hence show that the phase velocity of EM waves is the same as that of light in vacuum. [3+2]

   (b) Consider an elastic string fixed at its extremities and model the dynamics of the vibrating string with necessary assumptions. [5]

5. (a) Define autonomous and non-autonomous dynamical systems and give examples. [2+2]

   (b) Discuss the notions of center, node, saddle point and spiral associated with isolated critical points of a dynamical system. [6]

6. (a) Find the equilibrium points and analyze the local stability of the dynamical systems: (i) $\dot{x} = x + x^3$ and (ii) $\dot{x} = -x - x^3$. [3+3]

   (b) Find the nature and stability of the fixed points of the dynamical system $\dot{x} = -ax + y$, $\dot{y} = -x - ay$ for different values of a. [4]

Group-B (Biological System)

7. Develop a deterministic model for a single species non-age structured population. Carry out a mathematical analysis to show that there is a point of inflexion in the population growth curve when half the final population size is attained. Find also the critical time at which the point of inflexion occurs. [3+5+2]

8. Frame a stochastic model for a single species population based on the consideration that each individual is capable of reproducing with a certain mortality rate. Find the mean and the variance of the distribution and state what happens when birth and death rates are equal. [4+6]

9. Write down the basic equations governing the interactions between prey and predator species population. Carry out an analysis and explain the trajectories of the model in order to understand the dynamics of their interactions. Comment on the stability of the model. [3+5+2]

10. Discuss the Fick's first and second laws of diffusion. Write down the prey-predator and two competitive species models with one dimensional diffusion. Outline the steps to estimate the influence of diffusion on stability of these models. [2+5+3]