Use separate answer script for each unit

B. Sc. Examination - 2021

Semester-VI
Mathematics
Paper: CCMA-13
(Analysis VI)

Time: 3 Hours **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I (Complex Analysis)(Marks: 30)

Answer ***any three*** questions.

1. (a) Examine the continuity and analyticity of the function $f(z) = z^{\frac{1}{2}}$ in $\mathbb{C}$. [4]
   (b) Explain the difference between the differentiability and analyticity of a complex function. Justify your answer with examples. [4]
   (c) Prove or disprove: If a function $f : \mathbb{C} \longrightarrow \mathbb{C}$ is continuous, then $f'(z)$ exists at least at some $z \in \mathbb{C}$. [2]
2. (a) Let $f : \mathbb{C} \longrightarrow \mathbb{R}$ be differentiable. Prove that f is constant. [4]
   (b) If $f(z)$ is continuous at a point z_0, prove that $\overline{f(z)}$ is also continuous at z_0. [3]
   (c) Determine the convergence of the sequence $\left\{\sin \dfrac{n\pi}{4}\right\}$. [3]
3. (a) Prove or disprove: If $\{|z_n|\}$ converges, then $\{z_n\}$ also converges. [2]
   (b) Show that the Cauchy-Riemann equations are satisfied for the function
   $$f(z) = \begin{cases} \frac{xy}{x^2+y^2} & \text{for} \quad z \neq 0 \\ 0 & \text{for} \quad z = 0 \end{cases}$$
   at $z = 0$ but still $f'(0)$ does not exist. [4]
   (c) Find all the analytic functions whose imaginary part is $e^x \cos y$. [4]
4. (a) Determine all the bilinear transformations of the half-plane $Re(z) \geq 0$ into the unit disc $|w| \leq 1$. [6]
   (b) Prove or disprove: $|\sin z| \leq 1$, for all $z \in \mathbb{C}$. [4]
5. (a) Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{x \sin 3x}{x^2+9} dx$ using the method of contour integration. [6]
   (b) Determine the image of the exterior of the circle $|z+2| = 2$ under the mapping $f(z) = \dfrac{z}{2z+8}$. [4]

Unit-II (Metric Spaces)(Marks: 30)

Answer ***any three*** questions.

6. (a) Let X be a non empty set and ρ be a real valued function of ordered pairs of elements of X which satisfies the following two conditions:
   $\rho(x, y) = 0$ if and only if $x = y$ and $\rho(x, y) \leq \rho(x, z) + \rho(y, z)$. Show that $\rho(x, y)$ is a metric on X. [3]
   (b) Show that the space of all convergent sequences of real numbers with a suitable metric defined by you form a metric space. [4]
   (c) Let (X, ρ) be a metric space and $V \subset X$. Then V is an open set if and only if it is the union of open spheres. [3]
7. (a) Show by an example the union of an infinite number of closed sets need not be closed. [2]
   (b) Prove that a Cauchy sequence which has a convergent subsequence is itself convergent. [3]
   (c) Show that the Euclidean space R^n is a complete metric space. [5]
8. (a) State and prove Cantor's intersection theorem. [4]
   (b) Define a set of second category. Give an example of it. [2]
   (c) State and prove Baire's category theorem. [4]
9. (a) Define a contraction mapping. Give an example of it. [3]

(b) Show that any contraction mapping is uniformly continuous. [2]

(c) State and prove Banach's contraction principle theorem. [5]

10. (a) A function $f : X \to Y$ is continuous if and only if $f^{-1}(B)$ is an open subset of X whenever B is an open subset of Y-prove it. [4]

(b) Define a connected metric space.
Prove that a metric space (X, ρ) is connected if and only if X and ϕ are the only sets which are both open and closed in (X, ρ). [6]

B. A./B. Sc. (Hons) Examination-2021

Semester-VI

Mathematics

CCMA 14 (Algebra IV)

Time: Three Hours **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Group - A (Group Theory)
Answer ***any four*** questions.

1. (a) Let $f : G \longrightarrow G'$ be a homomorphism. Show that for every $a \in G$, $f^{-1}(f(a)) = aKer\ f$. [2]
   (b) If $|Ker\ f| = n$, then show that f is an $n - to - 1$ map. [2]
   (c) Consider the group homomorphism $f : \mathbb{R} \longrightarrow S^1$ defined by $f(x) = e^{i\pi x}$. Find the kernel of f. [1]
2. (a) Consider the subgroup $G = \{\begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix} \mid n \in \mathbb{Z}\}$ of the group $GL_2(\mathbb{R})$. Show that $G \simeq \mathbb{Z}$. [2]
   (b) Show that every finite cyclic group of order n is isomorphic to $\mathbb{Z}_n$. [3]
3. (a) State and prove the first isomorphism theorem for groups. [3]
   (b) Define a suitable homomorphism $f : \mathbb{R} \longrightarrow \{-1, 1\}$ so that $\mathbb{R}/Ker\ f \simeq \{-1, 1\}$. [2]
4. (a) Find all homomorphisms from $\mathbb{Z}_8$ into $\mathbb{Z}_6$. [2]
   (b) Show that there are only two groups of order 4 up to isomorphism. [3]
5. (a) Show that $Inn(G)$ is a normal subgroup of $Aut(G)$. Also show that $G/Z(G) \simeq Inn(G)$. [2+2]
   (b) Show that $|Aut(\mathbb{Z})| = 2$. [1]
6. (a) Find $Aut(S_3)$. [2]
   (b) Prove that $Aut(\mathbb{Z}_n) \simeq U_n$. [3]

Group - B (Ring Theory)
Answer ***any four*** questions.

1. (a) State and prove the second isomorphism theorem for rings. [3]
   (b) Let R be a ring with 1. If the characteristic of R is 0, then show that R has a subring isomorphic to $\mathbb{Z}$. [2]
2. (a) Find all ring homomorphisms $f : \mathbb{Z}_n \longrightarrow \mathbb{Z}_n$. [3]
   (b) Show that the quotient field of $\mathbb{Z}[\sqrt{2}]$ is $\mathbb{Q}[\sqrt{2}]$. [2]
3. (a) If D is an integral domain and $f(x)$ is a nonzero polynomial in $D[x]$ of degree n, then show that $f(x)$ has at most n roots in D counted according to multiplicity. Show that the result fails if D is not an integral domain. [3]
   (b) Find the number of elements in the quotient ring $\mathbb{Z}_5[x]/ < x^3 + x + 1 >$. [2]
4. (a) If F is a field, then show that every ideal in $F[x]$ is a principal ideal. [3]
   (b) Find all ideals of the quotient ring $\mathbb{R}[x]/ < (x - 1)(x - 2) >$. [2]
5. (a) Show that gcd of any two nonzero polynomials over a field exists. [3]
   (b) Suppose that over a field F, $f(x) \mid g(x)h(x)$ and $\gcd(f(x), g(x)) = 1$. Does $f(x) \mid h(x)$? Justify. [2]
6. (a) Define irreducible polynomial over field. State and prove a necessary and sufficient condition for a polynomial over a field to be irreducible. [3]
   (b) Find the total number of monic irreducible cubic polynomials over the field $\mathbb{Z}_5$. [2]

Group - C (Linear Algebra)
Answer ***any four*** questions.

1. (a) A linear transformation $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ is defined by $T(x, y, z) = (x - y + z, x + 2y - z, 3x + z)$. Find the rank and nullity of T. [2]

   (b) Is there any linear transformation $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ such that $\ker T = \{(x, y, z) \in \mathbb{R}^3 \mid x = y = z\}$ and $Im\ T = \{(x, y, z) \in \mathbb{R}^3 \mid x + y + z = 0\}$? If so, find one such linear transformation. [3]

2. (a) Let $T : V \longrightarrow V$ be a linear transformation. Then show that $ST = TS$ for every $S \in \mathcal{L}(V)$ if and only if $T = cI_V$ for some scalar c. [3]

   (b) A linear transformation $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is given by $T(x, y) = (x, x + y)$. Find the image of the line $x + y = 1$ under T. [2]

3. (a) Let $T : U \longrightarrow V$ be a linear transformation. For every subspace W of V, show that $T(T^{-1}(W)) = Im\ T \cap W$. [3]

   (b) A linear transformation $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is given by $T(x, y) = (ax + by, cx + dy)$. If T is an isomorphism then show that $ad - bc \neq 0$. [2]

4. (a) Let $T : V \longrightarrow V$ is a linear transformation and β an γ be two bases of V. Show that $[T]_\beta$ and $[T]_\gamma$ are similar. [2]

   (b) Let U, V and W be three finite dimensional vector spaces and $T : U \longrightarrow V$ and $S : V \longrightarrow W$ be two linear transformations. Then show that $rank(ST) = rank(T) - \dim(Im\ T \cap \ker S)$. [3]

5. (a) Let $f, g \in V^*$ be two linear functionals such that $f(v) = 0$ implies that $g(v) = 0$ for every $v \in V$. Prove that $g = cf$ for some $c \in F$. [2]

   (b) Define transpose T^t of a linear transformation $T : U \longrightarrow V$. Show that $\ker T^t = (Im\ T)^o$. [3]

6. (a) Consider the linear transformation $T(a + bt + ct^2) = -(a + 2b + c) + (2a + 3b)t - 2(a + b)t^2$. Show that 1 is an eigen value of T. Find E_1 and a basis of E_1. Is T diagonalizable? [3]

   (b) Let $T : V \longrightarrow V$ be a linear operator, β be a basis of V and λ be an eigen value of T. Then show that $v \in V$ is an eigen vector of T corresponding to λ if and only if $[v]_\beta$ is an eigen vector of $[T]_\beta$ corresponding to λ. [2]

B.Sc. (Honours) Examination, 2021

Semester-VI (CBCS)
Mathematics
Course: DSEMA-3A
(Computer Fundamentals)

Time: Two Hours **Full Marks: 20**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

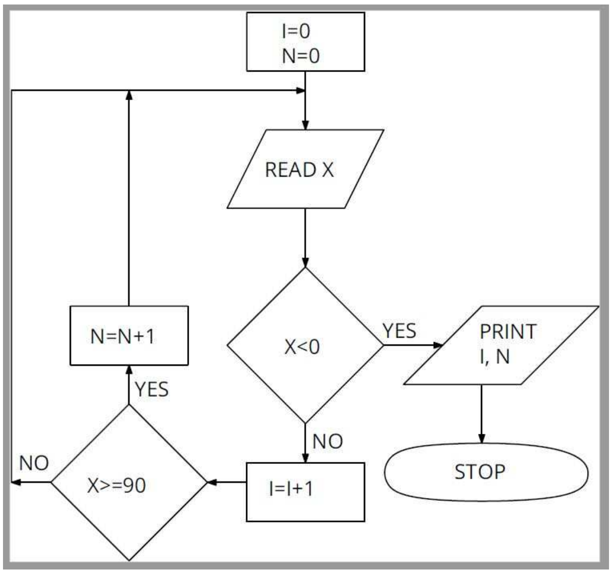
Answer ***any four*** questions.

1. (a) What do you mean by machine level language? How does they differ from assembly level language? [1+1]
   (b) Draw a flowchart to find out whether a given year is a leap year or not. [3]

2. (a) Write down the FORTRAN expression of the following mathematical expression:
   $\sqrt{|\sin(a-|b|)|} + \log_{10}(a-b)^2 + \dfrac{\tanh 3x}{1+x^{-\frac{2}{9}}}$ [2]
   (b) Using the statement function defined by $\mathrm{DVD(X,Y)} = \dfrac{\mathrm{X-Y}}{\mathrm{X+Y}}$, write assign statements to evaluate [3]
   $$\alpha = \frac{\sqrt{p}+5}{\sqrt{p}-5}, \quad \beta = \left[\frac{(a+b)^2+(c+d)^2}{(a+b)^2-(c+d)^2}\right]^{\frac{3}{2}}, \quad \gamma = 3\log_{10}\left|\frac{1+r}{1-r}\right|^{\frac{1}{2}}.$$

3. (a) Write a FORTRAN 77 program to compute the Euclidean norm of a given real 10×10 matrix $M = (m_{ij})$, where the norm is defined by $EN = \left(\sum_{i=1}^{10}\sum_{j=1}^{10} m_{ij}^2\right)^{\frac{1}{2}}$. [3]
   (b) Describe the output if the following is executed: [2]
   WRITE(6, 20) (A(J, K), J= 2, 8, 3), K=1, 4)
   20 FORMAT(1X, 5(3X, F10.2))

4. (a) Translate the following flowchart to equivalent FORTRAN 77 program. [3]



   (b) State (with examples) two important differences between ARITHMETIC IF statement and LOGICAL IF statement. [2]

5. (a) Find errors if any, in each of the following FORTRAN 77 segments.
   (i) DO 400 K=4, 2∗M, 3
   (ii) WRITE(6,40) (A(L, K+3), K=1, 7), L=1, 5)
   [1+1]
   (b) If P=2.0, Q=3.0, R=4.0, S=5.0, T=6.0, then find the order of the evaluation and value of the expression ((P+Q)∗ R ∗∗ 1.5−S−T+P)/Q +R [3]

6. (a) Find the label of the statement to which control is transferred for each of the following FORTRAN 77 segments.

   (i)
   ```
   I=2
   NEXT=4−I
   GO TO(21, 31, 41, 51), NEXT
   ```

   (ii)
   ```
   INTEGER TYPE
   K=3
   TYPE=2+K
   GO TO(50, 60, 70, 80), TYPE
   ```

   [1+1]

   (b) What will be the final values of the variables in the following FORTRAN 77 program after all the statements have been executed? You must mention all the necessary steps involved to evaluate the variables.

   ```
       P=5
       I=1
       ASSIGN 55 TO K
   55  GO TO(35, 35, 45, 25), I
   25  ASSIGN 65 TO K
   35  P=P+2
   45  I=I+1
       GO TO K, (55, 65)
   65  P=P-1
       Q=P+I
   ```

   [3]

B.Sc. (Honours) Examination, 2021

Semester-VI (CBCS)
Mathematics
Course: DSEMA-3B
(Computer Laboratory)

Time: Three Hours **Full Marks: 40**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer ***four*** questions in TOTAL taking ***any two*** from Question No. 1 to Question No. 3 and ***any two*** from Question No. 4 to Question No. 6.

1. (a) Write an algorithm to calculate the monthly telephone bill at the following rates: [5]
First 300 calls: no charges
Call between 301 and 501: at the rate Rs. 3.00 per call
Call between 501 and 1000: at the rate Rs. 3.50 per call
Call more than 1000: at the rate Rs. 4.50 per call
and rental charges of Rs. 450 per month.

   (b) Write a FORTRAN 77 program to find the sum of the first N terms of series in Geometric Progression (G.P), with first term A and common ratio R. [5]

2. (a) Draw a flowchart to convert a positive decimal integer to its binary equivalent. [5]

   (b) Find the output of the following program. [5]
(*You must mention all the necessary steps to find the output.*)

```
      INTEGER X(6)
      DO 100 K=1, 6, 2
      X(K)=3*K
      X(K+1)=K+2
100   CONTINUE
      DO 200 J=1, 6, 3
      X(J)=X(J)+X(J+1)
200   CONTINUE
      IF(X(2) .LT. 7) X(2)=X(3)
      WRITE(6, 20) X
 20   FORMAT(1X, 6I8)
```

3. (a) A palindrome is a number, a word, a phrase or a sentence that reads the same backwards and forwards (e.g., 12321, LEVEL, WOW). Draw a flowchart to decide if a string of characters is a palindrome or not. [6]

   (b) Write a FORTRAN 77 program to evaluate $\sum_{I=1}^{5} I^K + \sum_{I=1}^{5} IK$, where K= 1, 2, 3, 4, 5. [4]

4. Write a FORTRAN 77 program to find the root of the equation $3(2-px)^2 + x\cos x(2+px) - 2 = 0$ using Regula-Falsi Method correct upto five decimal places, where $p = 1.5 + \frac{R}{10}$, R is the last digit of your registration number. (*For example if someone's registration number is VB-123765, then* $R = 5$) [10]

5. Write a FORTRAN 77 program to solve the following system of equations by Gauss-Seidal iterative method correct up to four decimal places. [10]

$$\begin{pmatrix} 8.77 & 2.99 & 1.16 & 2.23 \\ 0.69 & 9.22 & 2.48 & 1.12 \\ 3.10 & 0.89 & 9.11 & 2.38 \\ 1.87 & 1.36 & 1.94 & 7.89 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2.12 \\ 4.15 \\ 6.29 \\ 3.16 \end{pmatrix}$$

6. Write a FORTRAN 77 program to evaluate the following integral by Weddle's rule correct up to four decimal places and taking 18 sub-intervals. [10]

$$\int_{10^\circ}^{40^\circ} \frac{\sin(1+x+e^{-x})}{\sqrt{1+x}}dx$$

B. Sc. (Honours) Examination-2021
Semester-VI
Mathematics
Course: DSE-4
(Mathematical Modelling)

Time: Three Hours **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer question no. 1 and any five form the rest (taking at least two from each group)

1. Why mathematical model is necessary? What are its uses? What are the different types of mathematical models?
Is there any mathematical modeling to solve the current Covid-19 crisis? [3+3+3+1]

Group A (Physical System)

2. Construct a mathematical model for an electrical LCR circuit. Discuss the notions of Reactance and Impedance. [6+(2+2)]

3. Derive the wave equation of a vibrating string with one space dimension. [10]

4. Discuss the motion of a simple pendulum in presence of an external periodic driving force. [10]

5. (a) Define a dynamical system. What are autonomous and non-autonomous dynamical systems? Give examples. [1+2+2]
(b) How do you reduce a non-autonomous dynamical system to an autonomous one? Give an example. [3+2]

6. (a) Define stable and unstable equilibrium points. Discuss the stability of the motion of a simple pendulum. [3+4]
(b) Find the fixed point and analyze the local stability of the system: $\dot{x} = x - x^3$. [3]

Group-B (Biological System)

7. Distinguish between deterministic and stochastic models. How do you construct a logistic growth model? Discuss the nature of its solution. [4+2+4]

8. Construct a stochastic model for a single species population assuming that each individual can give birth to new individuals and that each individual acts independently of all others. Find the expected value and variance of the number of individuals at time t. [4+(3+3)]

9. What is meant by mutualism? Develop a mathematical model for mutualistic interactions of two species where each species grows logistically in the absence of the other. Discuss the case of zero growth and the stability of the model. [1+5+(2+2)]

10. What is Fick's law of diffusion? Use it to derive one- and three-dimensional heat equations. Write down the prey-predator and two competitive species models with one dimensional diffusion. [2+5+3]