arate answer
or each unit

B.Sc.(Honours) Examination, 2022

Semester-II (CBCS)
Mathematics (Honours)
Core Course : CC-3
(Analysis-II and Algebra-II)

Time: Three Hours

Full Marks: 60

Questions are of values as indicated in the margin.
Notations and symbols have their usual meaning.

Unit-I [Analysis-II (Marks-30)]

Answer ***Question no.1*** and ***any four*** from the rest

1. Answer any five questions : [2 × 5]

(a) Find the derived sets of the sets $\mathbb{N}$, $\mathbb{R}-\mathbb{Q}$, $\mathbb{Q}-\mathbb{Z}$, and $\{\frac{1}{n} : n = 1, 2, \cdots\}$

(b) Find the closure of the set $\bigcup_{i=0}^{\infty}[(i, i+1) - \{i+\frac{1}{3}, i+\frac{2}{3}\}]$.

(c) Give examples with justification of an open set which is not an interval and a closed set which is also not an interval.

(d) Show that arbitrary union of closed sets may not be an closed set.

(e) Find envelope of the family of lines $\frac{x}{a}+\frac{y}{b}=1$, where $ab=c$.

(f) Give examples with justification of two sets which are both open and closed.

(g) Find area of the loop of the curve $y^2 = x(x-2)^2$.

2. State and prove Taylor's theorem with Lagranges's form of remainder. [5]

3. Find total length of perimeter of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$. [5]

4. Expand $\log(1+x)$ in power series with proper justtification [5]

5. State and prove Lindelöff covering theorem. [5]

6. Show that every sequence of reals has a monotone subsequence. [5]

7. If ρ_1 and ρ_2 are the radii of curvature at the ends P and D of conjugate diameters CP and CD respectively of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then show that $\rho_1^{\frac{2}{3}}+\rho_2^{\frac{2}{3}} = \frac{a^2+b^2}{ab^{2/3}}$, where C is centre of the ellipse. [5]

Unit-II(Marks:30)
(Algebra-II)

Answer ***all the*** questions.

1. Answer ***any two*** questions. $(2\times 5 = 10)$

(a) i. If a is odd, then show that $32|(a^2+3)(a^2+7)$. [2]

ii. State and proof Division Algorithm of integers. [1+2]

(b) i. Show that there are infinite number of primes. [3]

ii. If $gcd(a,b)=1$ then find $gcd(a^2+b^2, a^2b^2)$. [2]

(c) i. Find the general solution in integers of the equation $5x + 12y = 80$. [3]

ii. Examine if the equation $7x + 56y = 90$ is Diophentine equation. [2]

2. Answer ***any two*** questions. ($2 \times 10 = 20$)

(a) i. Give an example of a mapping which is neither one-one nor onto? Justify your answer. [2]

ii. Show that composition of two bijective mappings is a bijective mapping. [3]

iii. In a group G, show that $(ab)^{-1} = b^{-1}a^{-1}$. Find a condition on G such that $(ab)^{-1} = a^{-1}b^{-1}$. 3 [3+2]

(b) i. In the group $GL_2(\mathbb{R})$ find two elements other than identity one of which is finite and other is of infinite order. 1 [1+1]

ii. Can a group be expressed as a union of its two proper subgroups? Support your answer. [2]

iii. Find all subgroups of the group $U_{50} = \{[a] \in \mathbb{Z}_{50} | \ gcd(a, 50) = 1\}$. [6]

(c) i. Let G be a finite cyclic group of order n. Then show that there is an element $b \in G$ of order n. Hence show that D_4 is noncyclic. [3+2]

ii. Let G be a finite cyclic group of order n. Then show that there is a subgroup of order m in G for each $m|n$. [5]

Internal Assessment, 2022

GE - (ii): Python Programming Language

Time: 1 Hour

Full Marks: 20

1. Answer any four of the following. 4 x 1 = 4

a. What will be the output of the following Python function?

```
max (1, -5,True,min(max(False,-3,-4), 2,7))
```

b. What will be the output of the following Python program?

```
a=set('abc')
a.add('i')
a.update(set(['p' 'q']))
a
```

c. How to return multiple values from a called function? justify your answer.

d. Explain the use of the 'in' and 'is' operator in python?

e. What is the use of the self and elif keyword in python?

f. What do you mean by Built-in data type in programming language?

2. Answer any four of the following. (4x4=16)

a. (2+2)

I. Write down the difference between a programming and scripting language.

II. What are the differences between list, tuple and set?

b. (2+2)

i. Explain the membership Operators and comparison Operators available in Python with examples.

ii. Explain Polymorphism in python.

(1+3)

i. What will be the output of the following Python code snippet?

```
d = {"john":40, "peter":45}
d["john"]
```

ii. Write a program to implement binary search in python.

d. (1+3)

i. What is the usefulness of __init__() method in python?

ii. Write a program in python to sort a list using a recursive function.

(2+2)

i. What is inheritance and explain its types in python?

ii. Write a Python program to find the second smallest number in a list.

(4)

Write a Python class named Student with two instances student1, student2 and assign given values to the said instances attributes. Print all the attributes of student1, student2 instances.

1

Undergraduate Examination, 2022

Semester-II

Computer Science

Generic Elective Course: GEC-2

(Programming in Python)

Time: 3 Hours

Full marks: 40

Questions are of value as indicated in the margin.

Answer **Question No. 1** and any **four** from the rest.

1. a) Differentiate between compilation and interpretation.

b) Write down the structure of defining a function?

c) What is the use of __*init*__() method?

d) What will be the output of the following code?

```
i = 2
while True:
    if i%5 == 0:
        break
    print(i)
    i += 1
```

2×4=8

2. a) Write down the differences between scripting and programming languages?

b) Write a program to implement binary search.

3+5=8

3. a) What is a list? Explain the following list methods: *list.append()* , *list.insert()*, *list.pop()*, and *list.remove()*.

b) Write a program to print Fibonacci series upto *n* terms.

(1+4)+3=8

4. a) Briefly explain the recursive function?

b) Write a program to sort a list using a recursive function.

3+5=8

5. a) Explain the role of *break* and *continue* statements with examples.

b) Write a program to implement a queue using two stacks.

4+4=8

6. a) What do you mean by class and object? What are the key features of an object-oriented programming language?

b) Write a program to create a class by name *students*, and initialize attributes like *name, age,* and *grade* while creating an object.

(2+3)+3=8

7. Write short notes on **any two:** 4×2=8

a) Reserved/key words vs. unreserved words

b) Branching statements

c) Loops

d) Polymorphism

Use separate answer script for each unit

B.Sc. (Honours) Examination, 2022

Semester-II (CBCS)
Mathematics
Course: CCMA-4
(Geometry & Vector Calculus)

Time: Three Hours

Full Marks: 60

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I (Full Marks: 30)
(Geometry)

Answer **Question no. 1** and *any four* from the rest.

1. Answer any **five** questions: [5 × 2=10]

(a) Determine the eccentricity of the conic $xy = c^2$, where the rectangular axes are rotated through the angle $\frac{\pi}{4}$.

(b) Find the condition that the lines joining the origin to the point of intersection of the line $lx + my + n = 0$ and the curve $y^2 = 4a(x + a)$ to be at right angle.

(c) Find the length of the tangent from $(2, 2)$ to the circle $x^2 + y^2 - 2x - 2y + 1 = 0$.

(d) Find the value of a for which the equation $ax^2 + xy - 12y^2 + 2x - 31y - 20 = 0$ represents a pair of straight lines.

(e) Find the pole of the straight line $x+y+3 = 0$ with respect to the circle $x^2+y^2-2x+5 = 0$.

(f) Examine whether the numbers $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$ are the direction cosines of any directed line or not. Find the direction cosines of the line joining the points $(1, 2, 3)$ and $(2, 3, 4)$.

(g) Determine the values of a for which the plane $x + y + z = a\sqrt{3}$ touches the sphere $x^2 + y^2 + z^2 - 2x - 2y - 2z - 6 = 0$.

2. If, by a rotation of rectangular axes about the origin, the expression $(ax^2 + 2hxy + by^2)$ changes to $(a'x'^2+2h'x'y'+b'y'^2)$, then prove that (i) $a+b = a'+b'$ and (ii) $ab-h^2 = a'b'-h'^2$. [5]

3. Find the equation of the line joining the feet of the perpendiculars from the point $(r, 0)$ on the lines $ax^2 + 2hxy + by^2 = 0$. [5]

4. Tangents are drawn to the parabola $y^2 = 4ax$, at the points whose abscissae, are in the ratio $p : 1$. Show that the locus of their point of intersection is a parabola. [5]

5. The normal to the rectangular hyperbola $xy = c^2$ at a point P on it meets the curve again at Q and touches the conjugate hyperbola. Show that $(PQ)^4 = 512c^4$. [5]

6. A sphere of constant radius k passes through the origin O and cuts the axes in A, B, C respectively. Prove that the locus of the foot of the perpendicular from O to the plane ABC is $(x^2 + y^2 + z^2)^2(x^{-2} + y^{-2} + z^{-2}) = 4k^2$. [5]

7. The section of a cone whose guiding curve is $\frac{x^2}{9} + \frac{y^2}{4} = 1$, $z = 0$ by the plane $x = 0$ is a rectangular hyperbola. Show that the locus of the vertex is $\frac{x^2}{9} + \frac{y^2 + z^2}{4} = 1$. [5]

Unit-II (Full Marks: 30)

(Vector Calculus)

Answer **Question no. 1** and *any four* from the rest.

1. Answer any **five** questions: [5 × 2=10]

   (a) Find the area of the parallelogram having diagonals $3\vec{i}+\vec{j}-2\vec{k}$ and $\vec{i}-3\vec{j}+4\vec{k}$.

   (b) If $\vec{a}$ and $\vec{c}$ be any two given vectors and k, a given scalar. Determine all vectors $\vec{r}$ such that $\vec{a}\times\vec{r}=\vec{c}$ and $\vec{a}\cdot\vec{r}=k$.

   (c) Define scalar triple product of three vectors $\vec{a}, \vec{b}, \vec{c}$ and give its geometric interpretation.

   (d) Find the work done by the forces $2\vec{i}-3\vec{j}+2\vec{k}$ and $\vec{i}+2\vec{k}$ which together work on a particle and displace it from the point $(1,1,2)$ to the point $(0,1,-1)$.

   (e) Find the maximum value of the directional derivative at $(1,1,-1)$ of $\phi=x^2-2y^2+4z^2$.

   (f) For what value of λ, the vector $\vec{v}=(x+3y)\vec{i}+(y-2z)\vec{j}+(x+\lambda z)\vec{k}$ is solenoidal.

   (g) Evaluate $\oint_C \{(3x+4y)dx+(2x-3y)dy\}$ by Green's theorem where C is the circle in the xy-plane with centre at the origin and radius 2 units.

2. (a) Use vector method to show that $\cos(A-B)=\cos A\cos B+\sin A\sin B$. [3]

   (b) Prove the identity $\vec{\alpha}\times[\vec{\alpha}\times\{\vec{\alpha}\times(\vec{\alpha}\times\vec{\beta})\}]=|\vec{\alpha}|^4\vec{\beta}$, where $\vec{\alpha}\cdot\vec{\beta}=0$. [2]

3. (a) State Serret-Frenet formulae. [2]

   (b) Show that for the curve $\vec{r}=\vec{r}(s)$, $\frac{d\vec{r}}{ds}\cdot\frac{d^2\vec{r}}{ds^2}\times\frac{d^3\vec{r}}{ds^3}=\kappa^2\tau$, where κ is the curvature, τ the torsion and s the arc length of the curve. [3]

4. State and prove a necessary and sufficient condition for a vector function $\vec{r}=\vec{F}(t)$ to have a constant direction. [5]

5. Show that the vector $\vec{f}=(2x-yz)\vec{i}+(2y-zx)\vec{j}+(2z-xy)\vec{k}$ is irrotational. For $\vec{f}$, find a scalar function ϕ such that $\vec{f}=\vec{\nabla}\phi$. [2+3]

6. Prove the identity $\vec{\nabla}\times(\vec{\nabla}\times\vec{F})=-\nabla^2\vec{F}+\vec{\nabla}(\vec{\nabla}\cdot\vec{F})$. [5]

7. (a) If the vectors $\vec{F}$ and $\vec{G}$ be irrotational, then show that the vector $\vec{F}\times\vec{G}$ is solenoidal. [3]

   (b) If $\vec{R}$ be a unit vector in the direction of $\vec{r}$, then prove that $\vec{R}\times\frac{d\vec{R}}{dt}=\frac{1}{r^2}\vec{r}\times\frac{d\vec{r}}{dt}$, where $r=|\vec{r}|$. [2]