M.Sc. Examination - 2023

Semester-III

Mathematics

Paper MMC-31

(Discrete Mathematics)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin

Notations and symbols have their usual meaning

Answer *any four* questions

1. (a) Define well-formed formula in classical propositional logic. When is a form of argument considered valid? Construct a formal proof of validity of the argument form:

$H \to (I \to J)$
$K \to (I \to J)$
$(\sim H \wedge \sim K) \to (\sim L \vee \sim M)$
$(\sim L \to \sim N) \wedge (\sim M \to \sim O)$
$(P \to N) \wedge (Q \to O)$
$\sim (I \to J) \;/ \therefore \;\; \sim P \vee \sim Q.$ [3]

(b) Examine whether the argument is valid:

$(P \to Q) \wedge (R \to S)$
$(Q \to T) \wedge (S \to U)$
$(\sim P \to T) \wedge (\sim Q \to S)$
$\sim T \;/ \therefore \;\; \sim R \vee \sim Q.$ [3]

(c) Using the technique of assigning truth values, demonstrate the argument's invalidity:

$W \leftrightarrow (X \vee Y)$
$X \leftrightarrow (Z \to Y)$
$Y \leftrightarrow (Z \wedge \sim A)$
$Z \leftrightarrow (A \to B)$
$A \leftrightarrow (B \leftrightarrow Z)$
$B \vee \sim W \;/ \therefore \;\; W \leftrightarrow B.$ [4]

2. (a) Construct formal proof of validity for the argument form:

$(\forall x)(Q(x) \to R(x)$
$(\exists x)(Q(x) \vee R(x))$
$\therefore (\exists x)R(x).$ [3]

(b) Construct formal proof of validity for the argument form:

Bananas and grapes are fruits. Fruits and vegetables are nourishing. Therefore, bananas are nourishing. [4]

(c) Examine whether the argument is valid:

$(\forall x)(K(x) \to L(x))$
$(\exists x)(M(x) \wedge L(x))$
$(\exists x)(M(x) \wedge \sim L(x))$
$\therefore (\forall x)K(x) \to M(x).$ [3]

3. (a) Show that if G is a loop free simple graph and $\delta(G) \geq 3$, then G has a cycle of even length. [3]

(b) Show that a connected graph with n vertices has exactly one cycle if and only if it has exactly n edges. [3]

(c) Show that any two longest paths in a connected graph have a vertex in common. [4]

4. (a) Define Hamiltonian graph. Show that if G has $n \geq 3$ vertices and every vertex has degree at least $n/2$, then G is Hamiltonian. [5]

   (b) Define a planar graph. Deduce Euler's formula for planar graphs. Show that $K_{3,3}$ is not a planar graph. [5]

5. (a) Determine the number of bags with n pieces of fruit (apples, bananas, oranges and pears) such that the number of apples is even, number of bananas is a multiple of five, the number of oranges is atmost four and the number of pears is either zero or one. [2]

   (b) If we draw n ovals on the plane such that an oval intersects each of the other ovals at exactly two points, into how many regions do these ovals divide the plane. [4]

   (c) State the Inclusion-Exclusion principle and use it to find the number integers between 1 and 1000 that are not divisible by 2, 3 and 5. [4]

6. (a) State the Pigeon-hole principle. Use it to demonstrate that if we select 101 integers from the integers $1, 2, \cdots 200$ then there will be at least two integers such that one of them is divisible by the other. [5]

   (b) State Burnside's theorem on counting the number of equivalence classes. The sides of a square are to be coloured by either red or blue. How many different arrangements are there if a colouring that can be obtained from another by rotation is considered identiocal? [5]

Use separate answer script for each unit

M.Sc. Examination - 2023

Semester-III
Mathematics
Paper MMC-32
(Advanced Mathematical Statistics - I and Theory of Chaos)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin
Notations and symbols have their usual meaning

Unit - I : Marks - 20
(Advanced Mathematical Statistics - I)
Answer *any two* questions

1. Explain the procedure to construct a one-sample non-parametric sign test. IQ data of arrested drug-abusers who are aged sixteen years or older is given in the following table: [10]

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
99	100	90	94	135	108	107	111	119	104	127	109	117	105	125

Is there any evidence that the median IQ of drug-abusers in the population is greater than 107? (Test at $\alpha = 0.05$)

2. (a) Define conditional mean and conditional variance associated with two continuous random variables X and Y. If the conditional expectation of Y given $X = x$ is linear in x then show that

$$E(Y \mid X = x) = \mu_Y + \rho\frac{\sigma_Y}{\sigma_X}(x - \mu_X).$$

Let the joint pdf of X and Y be given by

$$f(x,y) = \begin{cases} e^{-y} & ; \quad 0 < x < y < \infty \\ 0 & ; \quad \text{otherwise.} \end{cases}$$

What is the conditional variance of Y given $X = x$? [1+1+3+2=7]

(b) Define regression function. Let the joint pdf of X and Y be given by

$$f(x,y) = \begin{cases} xe^{-x(1+y)} & ; \quad 0 < x, 0 < y \\ 0 & ; \quad \text{otherwise.} \end{cases}$$

What is the regression function of Y on X? [1+2=3]

3. (a) Show that the least squares estimators $\hat{\alpha}$ and $\hat{\beta}$ of a linear model

$$E(Y \mid x) = \alpha + \beta x$$

are unbiased. [2+2=4]

(b) Let each random variable $Y_{ij} \sim N(\mu_i, \sigma^2)$ where $Y_{ij} = \mu_i + \epsilon_{ij}; i = 1, .., m$ and $j = 1, .., n$. Compute the MLE of the parameters μ_i and σ^2. [4+2=6]

Unit - II : Marks - 20
(Theory of Chaos)
Answer *any two* questions

1. Explain the general theory of bifurcation of one dimensional system $\frac{dy}{dx} = f(x, \mu)$, where x is a real variable and μ is a parameter. Hence establish the conditions for Pitchfork bifurcation and transcritical bifurcation. Also draw their phase portraits and bifurcation diagrams. [2+(2+2)+(2+2)]

2. (a) State Devine's definition of chaos. [2]

   (b) Obtain the Lyapunov exponent of Bernoulli's shift map and Skinny Bekar's map. When does the value of the Lyapunov exponent becomes 0 and when becomes $-\infty$. [3]

   (c) Explain Sharkovskii's order and state Sharkovskii's theorem. [2]

   (d) Show that the map $B[0,1] \to [0,1]$ defined by B(x) = { $2\,x$, $0 \leq x \leq 0.5$ $2x - 1$, $0.5 \leq x \leq 1$ is chaotic. [3]

3. (a) Construct a Koch curve. Show that the length of the Koch curve tends to ∞ as the steps are repeated. [1+1]

   (b) Define Mendelbort set, and Julia set. Write some properties of the Julia set. [2+2]

   (c) What is a triadic number? Convert the decimals 17 into a triadic number. [1]

   (d) What do you mean by box dimension of any object? Find the box dimension of Shirpinsky triangle and Von Koch curve. [1+2]

M.Sc. Examination - 2023

Semester-III

Mathematics

Paper: MMC-33

(Fluid Mechanics)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin

Notations and symbols have their usual meaning

Answer *any four* questions

1. (a) Derive the equation of mass conservation equation in Lagrangian form. Hence find the corresponding equation in Eulerian form. [3+3]

   (b) The velocity components of a flow in cylindrical polar coordinates are $(trz\sin\theta, trz\cos\theta, -\frac{1}{2}z^2t\sin\theta)$. Determine the components of the acceleration of a fluid particle. [4]

2. (a) Derive Euler's equation of motion and hence deduce Bernoulli's equation in its most general form. [3+2]

   (b) The particle velocity $\vec{q}$ for a fluid motion referred to a rectangular Cartesian axes with unit vectors $\hat{i}$, $\hat{j}$, $\hat{k}$ is given by $\vec{q} = v_0\left(\hat{i}\cos\frac{\pi x}{2a}\cos\frac{\pi z}{2a} + \hat{k}\sin\frac{\pi x}{2a}\sin\frac{\pi z}{2a}\right)$, where, v_0 is a constant. Show that the pressure p associated with this velocity field in an incompressible fluid under no body force is given by

   $p = \frac{\rho v_0^2}{4}\left(\cos\frac{\pi z}{a} - \cos\frac{\pi x}{a}\right) + \text{constant}.$ [5]

3. (a) Derive the curvature at any point of a stream line. [2]

   (b) Derive the Cauchy-Riemann equations in polar form. [2]

   (c) Show that the velocity potential

   $$\phi = \frac{1}{2}\log\frac{(x+a)^2+y^2}{(x-a)^2+y^2}$$

   gives a possible motion. Determine the equation of the stream line and show also that the curves of equal speed are ovals of Cassini given by $rr' =$ constant where r, r' are the distances of the point $P(x,y)$ from the points $(a,0)$ and $(-a,0)$. [2+2+2]

4. (a) State and prove the Blasius Theorem [2+4]

   (b) In a region bounded by a fixed quadrant arc and its radii, deduce the motion due to a source and an equal sink situated at the ends of one of the bounding radii. Show that the streamline leaving either end at an angle $\pi/6$ with radius is $r^2\sin(\pi/6+\theta) = a^2\sin(\pi/6-\theta)$, where a is radius of the quadrant. [4]

5. (a) State and prove the Kelvin's Theorem on constancy of circulation. Hence show that under certain conditions which will be stated by you, the motion of an inviscid fluid, if once irrotational, remains irrotational even afterwards. [2+3+2]

   (b) A velocity field is given by $\vec{q} = (-\hat{i}y + \hat{j}x)/(x^2+y^2)$. Determine whether the flow is irrotational. Calculate the circulation round a unit circle with center at the origin. [3]

6. (a) Derive the Navier-Stokes equation of motion for an incompressible viscous fluid assuming the constitutive equations for Newtonian fluid. [5]

(b) Find the velocity distribution in the plane Poiseuille flow. Find maximum and average velocities. Also find the coefficient of friction at the upper wall in terms of Reynolds number, $Re = hu_a/\nu$ where h is the gap between two parallel plates, u_a is the average velocity of the fluid and ν is the kinematic viscosity of the fluid. [5]

Use separate answer script for each unit

M.Sc. Examination, 2023

Semester-III
Mathematics
Paper: MMC-34
(Calculus of Variations and Special Functions)

Time: Three Hours **Full Marks: 40**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I (Full Marks: 20)
(Calculus of Variations)
Answer *any two* questions.

1. (a) If $f(x) \in C[a,b]$, $g(x) \in C^1[a,b]$, and if $\int_a^b [f(x)h(x) + g(x)h'(x)]dx$ vanishes for every $h(x) \in C^1[a,b]$ such that $h(a) = h(b) = 0$, then show that $g'(x) = f(x)$ for all $x \in [a,b]$. [3]

   (b) Find the extremal of the functional $\int_0^{\frac{\pi}{2}} (y'^2 - y^2 + 4y\sin^2 x)dx$ that satisfies the boundary conditions $y(0) = y\left(\frac{\pi}{2}\right) = \frac{1}{3}$. [4]

   (c) If the integrand of the functional $J[x(t), y(t)] = \int_{t_1}^{t_2} \phi(t, x(t), y(t), \dot{x}(t), \dot{y}(t))dt$ does not contain t explicitly and is a homogeneous function of first degree in $\dot{x}$ and $\dot{y}$, then show that the functional $J[x(t), y(t)]$ depends only on the type of the curve $x = x(t), y = y(t)$, and not its parametric representation. [3]

2. (a) Find the extremal of the isoperimetric problem $I[y(x)] = \int_0^1 (x^2 - y'^2)dx$ with $y(0) = 0, y(1) = 0$ subject to the condition $\int_0^1 y^2 dx = 2$. [4]

   (b) What is geodesics? Find the geodesics on a right circular cone of semi-vertical angle α. [1+5]

3. (a) Find the shortest distance between the point $(1,1)$ and the curve $x^2 + y^2 + xy = 1$. [6]

   (b) Using Legendre condition, test for an extremum of the functional $J[y(x)] = \int_0^1 (x + 2y - \frac{1}{2}y'^2)\, dx$, subject to the boundary conditions which can be determined from the relation $y(x) = x - x^2$. [4]

Unit-II (Full Marks: 20)
(Special Functions)
Answer *any two* questions.

1. (a) Find the indicial equations of Bessel's equation and Laguerre equation by using Frobenius method. [4]

   (b) For Legendre polynomial, prove that $G(x,t) = (1 - 2xt + t^2)^{-1/2}$ is a generating function. Hence establish, $(1 + \frac{1}{2}P_1 \cos\theta + \frac{1}{3}P_2 \cos\theta + \ldots) = \log \frac{1 + \sin\frac{\theta}{2}}{\sin\frac{\theta}{2}}$. [3+3]

2. (a) Show that Laguerre polynomials are orthogonal over $(0, \infty)$ with respect to the weight function e^{-x}. [4]

   (b) Show that $H_n'(x) = 2nH_{n-1}(x)$ and $H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$ where $H_n(x)$ is the Hermite polynomial of order n. [2+2]

   (c) Prove that $J_n(x) = 0$ has no repeated root except at $x = 0$. [2]

3. (a) Define Hypergeometric function. Derive the integral representation for hypergeometric function. [1+2]

   (b) Express x^3 as a series in Legendre's ploynomial. [2]

   (c) Prove the following recurrence relations for Bessel's function $(J_n(x))$
   $(i) \frac{d}{dx}(x^n J_n(x)) = x^n J_{n-1}(x)$,
   $(ii) \frac{d}{dx}(x^{-n} J_n(x)) = -x^{-n} J_{n+1}(x)$,
   $(iii) J_n'(x) = J_{n-1}(x) - (n/x)J_n$. [5]

Use separate answer script for each unit

M. Sc. Examination-2023

Semester-III
Mathematics
Elective Course: MMC-35
(Galois Theory -I and Multivariable Analysis)

Time: Three Hours **Full Marks: 40**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Unit I: Galois Theory I (Full Marks: 20)

Answer *any four* questions.

1. (a) Prove that $\mathbb{Q}[\sqrt{2}] = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$ is a field.
(b) Let F be a field. Find all elements $a \in F$ such that $a = a^{-1}$.
(c) Let F be a field. Prove that there does not exist a non-zero proper ideal I in F. [3+1+1]

2. (a) Prove that $\mathbb{Q}(\sqrt{2}, \sqrt{5}) = \mathbb{Q}(\sqrt{2} + \sqrt{5})$.
(b) Find the degree of α over $\mathbb{Q}$ where α is a root of $X^3 - 12X^2 + 44X - 52$. [3+2]

3. (a) Prove that the fields $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(\sqrt{3})$ are not $\mathbb{Q}$-isomorphic.
(b) Prove that -1 cannot be expressed as a sum of finitely many square elements in the field $\mathbb{Q}(2^{\frac{1}{7}} e^{\frac{2\pi i}{7}})$. [3+2]

4. (a) Prove that every finite extension of fields is algebraic.
(b) Let F and K be fields. Prove that $[K : F] = 1$ if and only if $K = F$.
(c) Is $\mathbb{R}$ a finite extension of $\mathbb{Q}$? Justify your answer. [2+2+1]

5. (a) Let K be a field extension of F. Prove that the set of elements of K that are algebraic over F forms a field.
(b) Let α and β be two complex numbers such that both $\alpha + \beta$ and $\alpha\beta$ are algebraic over $\mathbb{Q}$. Prove that both α and β are algebraic over $\mathbb{Q}$.
(c) Let K be a field extension of F with $[K : F]$ odd. Prove that $F(\alpha) = F(\alpha^2)$ for any $\alpha \in K \setminus F$. [2+2+1]

6. (a) For a prime number p and a positive integer r, prove that there exists a field of order p^r.
(b) Prove that the product of the non-zero elements of a finite field is -1.
(c) Prove that a finite field cannot be algebraically closed. [3+1+1]

Unit-II : Multivariable Analysis (Full Marks: 20)

Answer any two questions

1. (a) Suppose $f : \mathbb{R}^n \to \mathbb{R}^m$ be such that for some matrix T, $\lim\limits_{n\to\infty} \frac{||f(x)-f(a)-T(x-a)||}{||x-a||} = 0$ then show that T is the matrix of partial derivatives at a. [4]
(b) Suppose $f : A \to \mathbb{R}^m$ is differentiable at $x_0 \in A$ where $A \subset \mathbb{R}^n$ be open set. Show that there exists M and δ such that $||f(x) - f(x_0)|| \leq M||x - x_0||$ for every $x \in A$ satisfying $||x - x_0|| < \delta$. Hence show that f is continuous at x_0 [3+1]
(c) Compute $f'(0)$ for the function $f(x, y) = (\sin x, \cos y, \sin y, \cos x)$ [2]

2. (a) Suppose $f : A \to \mathbb{R}^m$ has $(k + 1)$th order continuous partial derivatives at $x_0 \in A$ where $A \subset \mathbb{R}^n$ be open set and $R_k(x)$ is kth degree remainder of f at x_0. Then show that

$$\lim_{x\to x_0} \frac{|R_k(x)|}{||x - x_0||^k} = 0$$

[3]

(b) Define directional derivative $D_v f(a)$ of the function $f : \mathbb{R}^n \to \mathbb{R}^m$. Show that $\{D_v f(a) : v \in \mathbb{R}^n\}$ is a linear subspace of $\mathbb{R}^m$ [4]
(c) Suppose $f : A \to \mathbb{R}^m$ has $(k + 1)$th order continuous partial derivatives at $x_0 \in A$ where $A \subset \mathbb{R}^n$ be open set. If Q is a polynomial of degree k such that $\lim\limits_{x\to x_0} \frac{f(x)-Q(x-x_0)}{||x-x_0||^k} = 0$ then show that Q is the kth degree Taylor's polynomial of f at x_0. [3]

3. (a) Define the quadratic form $q(h)$ of a function $f : \mathbb{R}^n \to \mathbb{R}$ at a critical point a. Show that f has a local minimum at the point a if $q(h)$ is positive definite. [3]
(b) Suppose $f : \mathbb{R}^N \to \mathbb{R}^N$ is C^1 and $det(f(a)) \neq 0$. Show that f is an open map. [4]
(c) Find critical point for the function $f(x, y) = x^2 \sin y - 4x$ and the quadratic form at that point. Show that the critical point is neither a local maximum nor a local minimum. [3]

M. Sc. Examination 2023

Semester-III

Mathematics

Paper: MMO-31 (A08) (New Syllabus)

(Magnetohydrodynamics-I)

Time: Three Hours **Full Marks: 40**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer *any four* questions.

1. (a) What do "Quasineutrality" and "Collective behaviors" mean concerning plasmas? [2]

   (b) Explain the phenomenon of Debye shielding in a medium of charged particles. Hence define the Debye length and the Debye sphere. Show that in a plasma medium with inertialess thermal electrons and stationary positive ions, the electrostatic potential drops exponentially with the distance (relative to the Debye length) from the center of the Debye sphere. [3+2+3]

2. (a) Discuss the motion of a charged particle in presence of uniform electric (**E**) and magnetic (**B**) fields. Hence define the **E** × **B**-drift velocity. [4+1]

   (b) Show that the sum of potential and kinetic energies of a charged particle under uniform electromagnetic fields is constant. Does it hold for time-dependent fields? [4+1]

3. (a) Discuss the basic concepts of phase velocity and group velocity of a plane propagating wave in a conducting medium. [2+2]

   (b) Obtain the dispersion relation for Langmuir waves in an unmagnetized plasma with stationary positive ions and find the expressions for the wave's phase velocity and group velocity. [4+2]

4. Starting from a set of hydrodynamic and Maxwell's equations, obtain the dispersion relations for the ordinary (O-mode) and extraordinary (X-mode) modes in a plasma under a uniform magnetic field. Discuss their characteristics. [8+2]

5. Write short notes on (i) Lower-hybrid frequency (ii) Upper-hybrid frequency of electrostatic waves propagating in a magnetoplasma. [5+5]

6. Show that the dispersion relation for electrostatic ion-cyclotron waves in a cold electron-ion quasineutral magnetoplasma can be written in the form: $\omega^2 = \omega_{ci}^2 + c_s^2 k^2$. Discuss how it modifies the dispersion relation for electrostatic ion-acoustic waves in a cold quasineutral plasma. [8+2]

M.Sc Examination-2023

Mathematics

(MMO-31 (P5): Algebraic Coding Theory-I)

Time: Three Hours **Full Marks: 40**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.
Answer ***any four*** questions.

1. (a) Let $C = \{00000, 00111, 11111\}$ be a binary code. Find distance of the code C. [3]
   (b) Find all the cyclotomic cosets of 2 modulo 15. [3]
   (c) Let α be a root of $2 + x + x^2 \in F_3[x]$. Find the minimal polynomial of α^5 [4]

2. (a) Obtain the factorization of $x^{13} - 1$ over F_3 into monic irreducible polynomials. [4]
   (b) Let V be a n-dimensional vector space over a finite field F_q, where q is a prime. Show that V has $\frac{1}{k!}\sum_{i=0}^{n-1}(q^m - q^i)$ different bases. [4]
   (c) Let C be a linear code with the parity check matrix $H = \begin{pmatrix} 110100 \\ 101010 \\ 011001 \end{pmatrix}$. Decode the codeword 110110. [2]

3. (a) Let C be a linear code of length n over a field F_q, where q is a prime. Show that $C^{\perp}$ is a linear code and $dim(C) + dim(C^{\perp}) = n$. [3+3]
   (b) Show that the dimension of a self-orthogonal code of length n must be less than $n/2$, and the dimension of a self-dual code of length n is $n/2$. [2+2]

4. (a) Let C be the binary [5, 3]-linear code with the generator matrix $G = \begin{pmatrix} 10110 \\ 01011 \\ 00101 \end{pmatrix}$.
   What is the encoded form of the message word $u = 101$. [3]
   (b) Let q be a prime. Define $Tr_{F_{q^m}/F_q}(\alpha) = \alpha + \alpha^q + \ldots + \alpha^{q^m-1}$ for any αF_{q^m}. Show that $Tr_{F_{q^m}/F_q}$ is an element of F_q for all αF_{q^m} [7]

5. (a) For an integer $q > 1$ and integers n, d such that $1 \leq d \leq n$. show that

$$\sum_{i=0}^{\lfloor\frac{d-1}{2}\rfloor} \binom{n}{i} (q-1)^i.$$

   [4]
   (b) Show that the dimension of $Ham(r, 2)$ is $2^r - 1 - r$. [3]
   (c) Show that the weight of every codeword in G_{24} is a multiple of 4. [3]

6. (a) For $m > 1$, show that the Reed-Muller code $R(1, m)$ is a binary $[2^m, m+1, 2^{m-1}]$-linear code, in which every codeword except 0 and 1 has weight 2^{m-1}. [6]
   (b) If G_m is a generator matrix for $R(1, m)$, then show that the generator matrix for $R(1, m+1)$ is $G_{m+1} = \begin{pmatrix} G_m & G_m \\ 0\ldots0 & 1\ldots1 \end{pmatrix}$ [4]

M. Sc. Examination - 2023
Semester-III
Mathematics
Paper: MMO 31 (A5) (New)
(Dynamics of Ecological System-I)

Time: 3 Hours **Full Marks: 40**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer *any four* questions.

1. Analyze a logistic model for a single species non-age structured population. Show that the population growth curve possesses a point of inflexion when half the final population size is attained. Find also the time at which the point of inflexion occurs. [(2+4)+4]

2. Consider the following competing species model:

$$\begin{aligned}\frac{dx}{dt} &= x(7-3y-2x),\\ \frac{dy}{dt} &= y(19-7x-5y),\\ x(t=0) &\geq 0,\ \ y(t=0)\geq 0.\end{aligned}$$

(a) Determine and classify the co-existence equilibrium point.
(b) Sketch the phase portrait using the method of nullclines.
(c) What happens to the competing populations as time increases? [3+4+3]

3. Consider the following Lotka-Volterra two species competition model:

$$\begin{aligned}\frac{du_1}{dt} &= r_1u_1\left(1-\frac{u_1}{K_1}\right)-\gamma_1u_1u_2,\\ \frac{du_2}{dt} &= r_2u_2\left(1-\frac{u_2}{K_2}\right)-\gamma_2u_1u_2,\\ u_1(t=0) &\geq 0,\ \ u_2(t=0)\geq 0,\end{aligned}$$

where all the system parameters are positive. Construct a suitable Lyapunov function for which the co-existence equilibrium point is globally asymptotically stable (GAS). [10]

4. (a) What do you mean by prey refuge in an interacting species system?
(b) Comment on the uniformly boundedness of the system and influence of the refuge parameter r about the co-existence equilibrium point of the following Gause-type predator-prey model:

$$\begin{aligned}\frac{du}{dt} &= au\left(1-\frac{u}{K}\right)-\frac{b(1-r)uv}{1+c(1-r)u},\\ \frac{dv}{dt} &= \frac{eb(1-r)uv}{1+c(1-r)u}-dv,\ \ 0<e<1,\ \ 0\leq r<1,\\ u(t=0) &\geq 0,\ \ v(t=0)\geq 0,\end{aligned}$$

where all the system parameters are positive. [2+(4+4)]

5. (a) Explain Kermack-McKendrick SIR model for the spread of infectious disease. Define basic reproductive number R_0 with its biological significance.
(b) Consider an infectious disease for which there is no recovery, such as untreated tuberculosis (Mycobacterium Tuberculosis) or many plant diseases. A simple transmission model for such a disease which includes vital rates and disease caused mortality is given by

$$\begin{aligned}\frac{dS}{dt} &= N\mu-\mu S-\beta SI,\\ \frac{dI}{dt} &= \beta SI-\mu I-\delta I,\\ S(t=0) &\geq 0,\ \ I(t=0)\geq 0.\end{aligned}$$

Locate the equilibria and create their stability in respect of basic reproductive number R_0. [(2+2)+6]

6. Consider the following SIS model:

$$\begin{aligned}\frac{dS}{dt} &= rSe^{-aS} - \beta IS - \mu S,\\ \frac{dI}{dt} &= \beta IS - (\alpha + \mu)I,\\ S(t=0) &> 0,\ I(t=0) > 0,\end{aligned}$$

where S is the number of susceptibles, I is the number of infected, β is the transmission factor, α is the disease-induced death rate, r and a are positive constants associated with declining with population size per capita birth rate re^{-aS}, and μ is the natural death rate.

(a) Determine the basic reproduction number and equilibria of the system.

(b) Calculate the Jacobian of each equilibrium and determine the stability.

(c) Use the Dulac criterion to rule out periodic solutions. [3+4+3]

M. Sc. Examination-2023

Semester-III

Mathematics

Paper: MM0-31(A7/P8)(New)

(Lie Theory of Ordinary and Partial Differential Equations)

Time: Three Hours **Full Marks: 40**

Questions are of values as indicated in the margin.

Notations and symbols have their usual meanings.

Answer ***any four*** questions.

1. Define Lie group, Lie group of transformations, and Lie group of transformations in infinitesimal form. Introduce an infinitesimal generator for a Lie group of transformations. Examine whether the set of transformations $(x, y) \to (x^*, y^*) = \mathcal{T}_\epsilon \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cosh\epsilon & \sinh\epsilon \\ \sinh\epsilon & \cosh\epsilon \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$ is a Lie group of transformations in the geometric space $\mathbb{R}^2$. Find the infinitesimal generator if yes. [1+1+1+1+5+1]

2. Derive the formula for getting an infinitesimal generator $\hat{Y}(\equiv \sum_{i=1}^{n} \eta^i(y^1, \cdots, y^n)\frac{\partial}{\partial y^i})$ of a one-parameter Lie group transformations in the geometric space $\mathbb{R}^n (n \geq 2)$ in one set of variables whenever the infinitesimal generator $\hat{X}(\equiv \sum_{k=1}^{n} \xi^k(x^1, \cdots, x^n)\frac{\partial}{\partial x^k})$ in another set of variables describing the points of the same geometric space is given. Define canonical variables in the geometric space for a Lie group of point transformations. State the principle for obtaining canonical variables for a given Lie group of transformations. Find canonical variables for the Lie group of transformations $(x, y) \to (x^*, y^*) = (\cos\epsilon\ x - \sin\epsilon\ y,\ \sin\epsilon\ x + \cos\epsilon\ y)$. [4+1+2+3]

3. Define Lie algebra. When is a Lie algebra said to be solvable? Show that the set infinitesimal generators for the group of transformations $x \to x^* = \frac{a_1}{1+a_3x+a_4y}$, $y \to y^* = \frac{a_2}{1+a_3x+a_4y}$ is a Lie algebra. Is it solvable? Provide a solvable Lie subalgebra of this Lie algebra. [2+2+5+1]

4. Find equations for the coefficient $\phi(x)$ so that the infinitesimal generator $\phi(x)\frac{\partial}{\partial u}$ generates a Lie group of symmetry transformations admitted by the second-order linear nonhomogeneous ordinary differential equation $u''(x) + p(x)u'(x) + q(x)u(x) = f(x)$. Reduce the order of the differential equation by using the infinitesimal generator mentioned above. [4+6]

5. Derive the system of determining equations for the coefficients in the infinitesimal generator $\hat{X} = \xi(x, y, u)\partial_x + \eta(x, y, u)\partial_y + \phi(x, y, u)\partial_u$ of the Lie group of symmetry transformations admitted by the equation $u_y = u_{xx} + u_x$. [10]

6. Find the group invariant solution of the heat equation $u_t = u_{xx}$, $(x, t) \in \mathbb{R} \times \mathbb{R}^+$ satisfying the initial condition $u(x, 0) = \delta(x)$. Here $\delta(x)$ being the Dirac delta function is defined by $\int_{\mathbb{R}} g(x)\delta(x - a) = g(a),\ \forall\ g \in C_0^\infty(\mathbb{R})$. [10]

M. Sc. Examination-2023

Semester-III

Mathematics

Paper: MMO-31(A04)

Differential Equations in Ecology

Time: Three Hours

Full Marks: 40

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer ***any ten*** questions.

1. Consider the differential equation $\frac{dx}{dt} = ax - bx^2 + hx$. Sketch the bifurcation diagram by considering h as the bifurcation parameter. [4]

2. What is functional response? Draw the curves of Holling type-I, type-II, type-III, and type-IV functional responses. [4]

3. What are the advantages of a dimensionless model? Using suitable transformation, transform the system

$$\begin{aligned}\frac{dx}{dT} &= ax - \frac{bxy}{(1+Ax)(1+By)},\\ \frac{dy}{dT} &= -cy + \frac{mxy}{(1+Ax)(1+By)}\end{aligned}$$

into the system

$$\begin{aligned}\frac{du}{dt} &= u - \frac{uv}{(1+\alpha u)(1+\beta v)},\\ \frac{dv}{dt} &= -\gamma v + \frac{uv}{(1+\alpha u)(1+\beta v)}.\end{aligned}$$

[4]

4. By constructing a Lyapunov function $v(x,y) = ax^2 + by^2$ with suitable a and b, show that the system

$$\begin{aligned}\frac{dx}{dt} &= -(x-2y)(1-x^2-3y^2),\\ \frac{dy}{dt} &= -(y+x)(1-x^2-3y^2)\end{aligned}$$

is globally asymptotically stable. [4]

5. Prove that the system

$$\begin{aligned}\frac{dx}{dt} &= rx - bx^2 - \frac{px^2y}{q+x^2},\\ \frac{dy}{dt} &= \frac{px^2y}{q+x^2} - \frac{ryz}{1+\alpha y} - d_2y,\\ \frac{dz}{dt} &= \frac{ryz}{1+\alpha y} - d_3z\end{aligned}$$

is bounded. [4]

6. Consider the "rabbits vs sheep" problem

$$\frac{dx}{dt} = x(8-2x-2y),$$
$$\frac{dy}{dt} = y(6-2x-y),$$

where $x, y \geq 0$. Draw the nullclines, find the equilibrium points and investigate their stability. [4]

7. Find the Z-controller $U_{pred}(t)$ of the system

$$\frac{dx}{dt} = ax - bx^2 - \frac{pxy}{q+x^2},$$
$$\frac{dy}{dt} = \frac{pxy}{q+x^2} - my - yU_{pred}(t),$$

so that the prey population $x(t)$ achieves a desired state $x_d(t)$. [4]

$\frac{(q+x^2)(p\dot{x}y + px\dot{y}) - pxy \cdot 2x\dot{x}}{(q+x^2)^2}$

8. What is Hopf bifurcation? Prove that the system

$$\frac{dx}{dt} = y,$$
$$\frac{dy}{dt} = -x + (\mu - x^2)y$$

undergoes Hopf bifurcation by considering μ as the bifurcation parameter. [4]

9. Using center manifold theorem, determine the stability of the origin of the system

$$\frac{dx}{dt} = -x^3 + y^2,$$
$$\frac{dy}{dt} = -3y + x^2.$$

[4]

10. What is bistability? Show that the system

$$\frac{dx}{dt} = x(x-a)(b-x)$$

exhibits bistability behavior. [4]

11. Find the interior equilibrium point and Jacobian matrix of the delay model

$$\frac{dx}{dt} = ax(t) - m_1x(t) - bx(t)y(t),$$
$$\frac{dy}{dt} = bx(t-\tau)y(t-\tau) - cy(t)z(t) - m_2y(t),$$
$$\frac{dz}{dt} = cy(t-\tau)z(t-\tau) - m_3z(t),$$

where τ is the delay parameter. [4]

12. Write a MATLAB code to draw the phase portrait and time series solutions in a single figure for the system

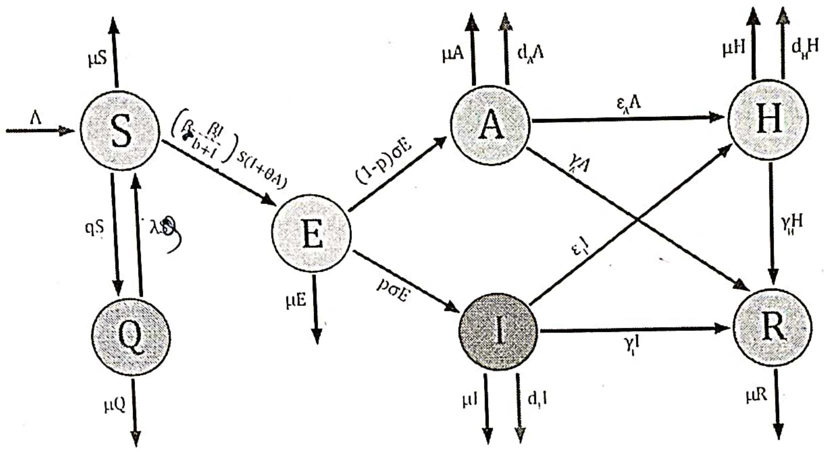
$$\frac{dx}{dt} = ax\left(1 - \frac{x}{k}\right) - \frac{pxy}{q + rx + wx^2},$$
$$\frac{dy}{dt} = \frac{pxy}{q + rx + wx^2} - my.$$

[4]

13. Consider a linear birth-death process in a stochastic model. Derive a stochastic differential equation for p_n, where p_n is the probability of n individuals in the system at any time t. [4]

14. What is the Allee effect? Discuss strong and weak Allee effects with examples. [4]

15. Write down the system of differential equations from the following schematic diagram:



[4]

M.Sc. Examination-2023

Semester-III
Mathematics
MMO-31
(Nonlinear Differential Equations)

Time: Three Hours   Full Marks: 40

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer *any four* questions.

1. (a) Using Direct method solve the Zakharov-Kuznetsov (ZK) equation. $u_t + 4uu_x + u_{xxx} + \frac{\partial}{\partial y} X_{xx}$ [5]

   (b) Solve the following nonlinear differential equation

$$xy''' + 3y'' = xe^{-y'}.$$

[5]

2. Establish the relation among the amplitude, velocity and width for existence of *Sech* solution profile of the modified-KdV equation. [10]

3. Find approximate solution of the Damped and Forced KdV equation. [10]

4. Use *tanh* method to solve the KdV-Burgers' equation. $u_t + 4uu_x + u_{xx} = 0$ [10]

5. Find the solution of Burgers' equation using *tanh* − *coth* method. [10]

6. Find the soliton solution of the KdV equation using *sine* − *cosine* method. [10]

$1 - 2\tanh^2 + \tanh^4$

M. Sc. Examination, 2023

Semester-III

Mathematics

Paper : MMO-31 (A9) (New)

(Mathematical Pharmacology-I)

Time: Three Hours **Full Marks: 40**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer ***any four*** questions.

1. Define the efficacy of a drug. Using occupancy theory, prove that the equilibrium response $\gamma = \phi' \frac{KA}{1+KA}$, K is the equilibrium association constant, A is the concentration of drug applied and ϕ' is a constant. [3+7]

2. Formulate the governing equation for receptor/ligand complex $(c(t))$ for simple monovalent cell surface binding model in which the ligand depletion from the medium is significant. Hence obtain the time-dependent solution of the complex $(c(t))$. [3+7]

3. Write down the model equations for complexes in which two non-interconverting receptor sub-classes interact with ligands to form two complex species. Solve the equations and also obtain the steady-state solutions. [3+5+2]

4. Formulate a single endosome model in which monovalent ligands bind to monovalent receptors. Write down the descriptions of variables and parameters involved. Nondimensionalise the equations. [4+2+4]

5. Define receptor cross-linking. Neglecting endocytic trafficking, obtain the steady-state solutions in case a bivalent ligand binds with monovalent receptor on the cell surface. [3+7]

6. Write short notes on
   (i) Endocytic downregulation and sorting downregulation.
   (ii) Diffusive flux and convective flux. [5+5]

M.Sc. Examination, 2023

Semester-III

Mathematics

Paper: MMO-31 (A14)

(Critical Point Theory in Fluid Topology-I)

Time: Three Hours **Full Marks: 40**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer ***question no. 1*** and ***any three*** from the rest.

1. Answer any **five** questions: [5 × 2=10]

   (a) What do you mean by phase plane and phase portrait of a system?

   (b) The velocity fields in a diffuser are given by $u = u_0 e^{-\frac{2x}{L}}$ and $\rho = \rho_0 e^{-\frac{x}{L}}$, where u_0 and ρ_0 are constants. Find the rate of change of density at $x = L$.

   (c) What is path line? Write down the differential equation of a path line.

   (d) Describe geometric interpretation of stream function.

   (e) Distinguish between streamlines and skin-friction lines.

   (f) How vortex lines are connected to skin-friction lines?

   (g) What do you mean by separation and attachment?

   (h) What is node-saddle point of attachment? How does it form in the flow field?

2. (a) Show that (0, 0) is always an unstable critical point of the following linear system [5]

$$\dot{x} = \mu x + y$$
$$\dot{y} = -x + y,$$

   where μ is a real constant and $\mu \neq 1$. When is $(0,0)$ an unstable saddle point? When is $(0,0)$ an unstable spiral point?

   (b) Classify (if possible) the critical points of the plane autonomous system corresponding to the non-linear second order differential equation $\ddot{x} + x - x^3 = 0$. [3]

   (c) Sketch phase portrait of degenerate unstable node and stable spiral point. [2]

3. (a) State and prove Reynolds transport theorem. [5]

   (b) Establish the relation: $\frac{d}{dx}[U_\infty^2 \delta^{**}] + \delta^* U_\infty \frac{dU_\infty}{dx} = \frac{\tau_w}{\rho}$, where symbols have their usual meanings. [5]

4. Show that the behaviour of the skin-friction lines in the vicinity of the critical points at $P_0(x_0, z_0)$ is governed by the nature of the solutions of a system of linear algebraic equations of the form $F\Psi = O$ where,

$$F = \begin{pmatrix} \tau_{xx}|_{P_0} - S & \tau_{xz}|_{P_0} \\ \tau_{zx}|_{P_0} & \tau_{zz}|_{P_0} - S \end{pmatrix}, \ \Psi = \begin{pmatrix} \lambda \\ \mu \end{pmatrix} \text{ and } O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

   Also, classify different critical points of the skin-friction line patterns. Describe their topological significance in fluid dynamics. [4+3+3]

5. (a) Discuss flow topology associated with a separation with one saddle point and two foci. [6]

(b) What are the key differences between separation surface of detachment type and separation surface of attachment type? [4]

6. (a) What do you mean by vortical structures in the flow field? Describe vortical structures over a delta wing. [7]

(b) State Poincaré-Bendixson formula when flow domain with surface complexity one contains node, focus, saddle point, half-saddle and half-node as critical points. [3]

M. Sc. Examination-2023

Semester-III

Mathematics

Course: MMO-31 (P-02)

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Time: Three Hours **Full Marks: 40**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer any <u>four</u> questions

1. (a) If K and C are compact and closed subsets respectively in a topological vector space (tvs) X such that $K \cap C = \phi$, show that there is a neighborhood (nbd) V of θ in X such that $(K+V) \cap (C+V) = \phi$. Hence deduce that evey tvs is regular. [5+2]

   (b) If X is a locally convex tvs, then prove that there exists a nbd base of θ consisting of convex and balanced nbds. [3]

2. (a) Let T be a non-zero linear functional define on a tvs X. Show that the following statements are equivalent:

   (i) T is continuous

   (ii) Null space $N(T)$ is closed

   (iii) $N(T)$ is non-dense in X

   (iv) T is bounded in some nbd of θ in X. [8]

   (b) If X is locally bounded tvs with Heine-Borel property, then prove that X is finite dimensional. [2]

3. (a) Let p be a seminorm on a vector space X. Then for $x, y \in X$, prove the following:

   (i) $p(\theta) = 0$.

   (ii) $|p(x) - p(y)| \leq p(x-y) \;\; \forall x, y \in X$.

   (iii) $p(x) \geq 0 \;\; \forall x \in X$.

   (iv) $\{x \in X : p(x) = 0\}$ is a subspace of X.

   (v) If $B = \{x \in X : p(x) < 1\}$, then B is convex, balanced and absorbing and $\mu_B = p$ where μ_B is the Minkowski functional on X for the set B. [6]

   (b) When a tvs X is said to be normable? Prove that X is normable iff its origin has a bounded convex nbd. [4]

4. (a) State and prove closed graph theorem. [6]

   (b) Let X be a tvs and Y be a Hausdorff topological space. If $f : X \to Y$ is continuous, then prove that the graph G of f is closed. [2]

   (c) Let X be a tvs. Then for $A, B \subset X$ prove that $\overline{A} + \overline{B} \subset \overline{A+B}$. [2]

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5. (a) When a tvs is said to be (i) locally convex (ii) an F-space? [2]

(b) Let X be a tvs and Y be a subspace of X. If $dim Y = n$ (finite), then prove that any isomorphism from C^n onto Y is a homeomorphism. [5]

(c) If $\tau_1 \subset \tau_2$ are topoloies on a set X and if τ_1 is equipped with Hausdorff topology, τ_2 is compact, then show that $\tau_1 = \tau_2$. [3]

6. (a) State and prove Krein-Milman theorem. [8]

(b) Let X be a tvs and $A \subset X$. Show that

$$\overline{A} = \cap\{A + V : V \text{ is any nbd of } \theta\}.$$

[2]

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M.Sc. Examination - 2023
Semester-III
Mathematics
Paper: MMO-31 (A03)
(Computational Fluid Dynamics - 1)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin
Notations and symbols have their usual meaning

Answer *any four* questions

1. (a) Find the second order accurate finite difference analog of third order partial derivative using the values of the dependent variable at the $(i \pm 2, j)$ and $(i \pm 1, j)$ points. [5]
   (b) Reduce the equation $u_{xx} + x^2 u_{yy} = 0$ to a canonical form. [5]
2. (a) Find the Peaceman-Rachford ADI method for the two dimensional heat flow equation in the domain $R = [0 \leq x, y \leq 1] \times [0, T]$ subject to some initial and boundary conditions. Also find the stability criteria of the method. [2+3]
   (b) Consider the general heat conduction equation with complex coefficient $u_t = (a + ib)u_{xx}$. Show that if $a \geq 0$, the Crank-Nicolson method is unconditionally stable. [5]
3. Solve the one dimensional heat conduction equation $u_t = u_{xx}$ subject to the initial and boundary conditions

$$u(x, 0) = \sin(\pi x), 0 \leq x \leq 1$$
$$u(0, t) = u(1, t) = 0$$

using (i) the Schmidt method, (ii) the Laasonen method, (iii) the Crank-Nicolson method and (iv) the Dufort-Frankel method for $h = 1/3$ and $k = 1/36$. Integrate upto two time levels[2+2+4+2]

4. Show that the partial differential equation $u_{tt} = u_{xx}$ with $u = f(x)$ and $u_t = g(x)$ at $t = 0$, may be replaced by the difference

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scheme

$$u_m^{n+1} = -u_m^{n-1} + 2(1-r^2)u_m^n + r^2(u_{m+1}^n + u_{m-1}^n)$$
$$u_m^0 = f_m$$
$$u_m^1 = kg_m + (1-r^2)f_m + \frac{1}{2}r^2(f_{m+1} + f_{m-1})$$

where $r = k/h$. Find an expression for the local truncation error of each of the finite difference expressions and find the order of the method. Using

$$\begin{aligned} f(x) &= x, & 0 \le x \le 1/2 \\ &= 1-x, & 1/2 \le x \le 1 \\ g(x) &= 0, & 0 \le x \le 1 \end{aligned}$$

and $u(0,t) = 0 = u(1,t)$, find the solution upto two time steps with $h = 0.2, r = 1/2$. [5+5]

5. * (a) Consider the two dimensional hyperbolic differential equation

$$u_{tt} = u_{xx} + u_{yy}$$

subject to initial conditions

$$u(x,y,0) = f(x,y) \quad 0 \le x, y \le 1$$
$$u_t(x,y,0) = g(x,y) \quad 0 \le x, y \le 1$$

and the boundary condition prescribed on the boundary of the unit square. Using the central difference for time and space variables, derive the finite difference analogue of the above differntial equation. Find the truncation error and stability criteria of the derived finite difference equation[1+3+4]

(b) Derive higher order finite difference method for $u_{tt} = u_{xx} + u_{yy}$ using
*

$$k^2\frac{\partial^2}{\partial t^2} = \frac{\delta_t^2}{1+\frac{1}{12}\delta_t^2} + O(k^6)$$
$$h^2\frac{\partial^2}{\partial x^2} = \frac{\delta_x^2}{1+\frac{1}{12}\delta_x^2} + O(h^6) \qquad \delta_x^2 u_{lm}^n =$$
$$h^2\frac{\partial^2}{\partial y^2} = \frac{\delta_y^2}{1+\frac{1}{12}\delta_y^2} + O(h^6).$$

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[2]

6. For the boundary value problem

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 4, |x| \leq 1, |y| \leq 1$$
$$u = 0, \text{ on the boundary}$$

write a five point and a nine point difference formulation with mesh spacing h in both the directions. Solve the difference equations exactly for $h = 1/2$. [10]

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M. Sc. Examination 2023
Semester-III
Mathematics
Paper: MMO-31(P-12)
(Value Distribution Theory-I)

Time: 3 Hours | Full Marks: 40

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer ***any four*** questions.

1. (a) Define a simply connected set in $\mathbb{C}$. Let G be a domain and let $H(G)$ denote the collection of all analytic functions on G. If G is simply connected, prove that for each $f \in H(G)$, there is a sequence of plolynomials $\{P_n\}$ which converges to f in $H(G)$.
(b) Let $G \subset \mathbb{C}$ be an open set and E be a subset of $\mathbb{C}_\infty \setminus G$ which meets every component of $\mathbb{C}_\infty \setminus G$. Let $R(G, E)$ be the collection of all rational functions on G with poles in E. Show that $R(G, E)$ is a dense subset of $H(G)$. [(1+5)+4]

2. (a) What do you mean by harmonic functions? Explain with examples. Prove that every harmonic function on a domain possesses the mean value property. Is the converse true? Justify your answer.
(b) Establish the maximum and minimum principle for harmonic functions. [(2+3+1)+4]

3. (a) Find, if possible, an entire function $f(z) = u + iv$ such that $u(x, y) = x^4 + y^4 - 6x^2y^2 - 4xy$.
(b) Describe Dirichlet problem for a disc. Prove that it has (assuming the existence) a unique solution. [5+(1+4)]

4. (a) For a periodic function f, discuss the difference between its periods and primitive periods. Prove or disprove: The period points of a non-constant periodic function cannot accumulate in a finite region.
(b) Define an elliptic function f on a domain. Describe, in detail, the period parallelograms asociated with such an f. [(1+4)+(1+4)]

5. (a) Let f be an elliptic function. What do you mean by a cell for f? Show that the sum of the residues at the poles of f in a cell is zero.
(b) What is Weierstrass' elliptic function $\wp(z)$? Prove or disprove: The function $\wp(z)$ is an even function but $\wp'(z)$ is an odd function. [(1+4)+(1+4)]

6. (a) Let f be a meromorphic function on $\mathbb{C}$. Describe the functions $m(r, f)$, $N(r, f)$ associated with f and define Nevanlinna's characteristic function $T(r, f)$ of f. Hence evaluate these functions for $f(z) = e^z$
(b) State and prove Nevanlinna's first fundamental theorem. [(3+3)+4]