B.Sc Sem-III 2023

Use separate answer script for each group

B. A./ B. Sc. Examination-2023

Semester-III (CBCS)
Mathematics
Paper: GEC 3
(Differential Equations and its Applications)

Time: Three Hours **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Group A (Ordinary Differential Equations)

(Full Marks: 40)
Answer *any four* questions.

1. (a) For which value of m, the function f defined by $f(x) = (2x^2+2e^x+3)e^{mx}$ is a solution of the differential equation
$$\frac{dy}{dx} = 6e^x + 4xe^{-2x}.$$
(b) Obtain the differential equation of all the circles each of which touches the x-axis at the origin.
(c) When a differential equation of the form $M(x,y)\,dx+N(x,y)\,dy = 0$ is called exact? Examine whether the equation $(1-2xy-y^2)\,dx-(x+y)^2\,dy = 0$ is exact or not.

[2+5+3]

2. (a) Solve: $\cos y\frac{dy}{dx} + \frac{1}{x}\sin y = 1$.
(b) Solve: $(x^3+3xy^2)\,dx + (y^3+3x^2y)\,dy = 0$.
(c) Show that $x(x^2-y^2)^{-1}$ is an integrating factor of the equation $(x^2+y^2)dx - 2xy\,dy = 0$ and hence solve the equation.

[2+4+4]

3. (a) Find the orthogonal trajectories of the family of curves $x = \frac{y^4}{4} + \frac{c}{y^2}$, c being a variable parameter.
(b) Solve: $x\frac{dy}{dx} + y = y^2\ln x$, $y(x=1) = \frac{1}{2}$.

[5+5]

4. (a) Write down the geometrical significance of the singular solution in Clairaut's equation.
(b) Solve: $\alpha yp^2 + (2x-\beta)p - y = 0$, $p = \frac{dy}{dx}$, α and β are two parameters.
(c) Solve the equation $y = px + \sqrt{a^2p^2+b^2}$ and show that the singular solution lies on a parabola, a and b are two parameters.

[2+3+5]

5. (a) Show that the functions $\{e^{2x},\ e^{2x}\cos 4x,\ e^{2x}\sin 4x\}$ are linearly independent.
(b) Solve: $\frac{d^2y}{dx^2} - 4y = xe^{2x}$.
(c) Solve: $(D^2+4)(D^2+1)y = \cos 2x$, $D^n \equiv \frac{d^n}{dx^n}$.

[2+3+5]

6. (a) Solve the system of equations $\frac{dx}{dt} = -\omega y$ and $\frac{dy}{dt} = \omega x$. Hence show that the solution lies on a circle.
(b) Solve by using the method of variation of parameters $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = xe^{2x}$.
(c) Solve: $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = \cos(\ln x) + x\sin(\ln x)$.

[3+3+4]

Group B (Partial Differential Equations)

(Full Marks: 20)

Answer *any two* questions.

1. (a) Form the partial differential equation by eliminating arbitrary constants 'a' and 'b' from

$$\ln(az-1) = x^2 + ax^3y^3 + b.$$

What kind of partial differential equation is this?

(b) Find the order and degree of the following partial differential equations:

(i) $\left(\frac{\partial^3 z}{\partial x^3} + \frac{\partial z}{\partial x}\right)^{\frac{1}{4}} = xy^2z^{\frac{1}{2}} + \cos\left(\frac{\partial z}{\partial x}\right)$

(ii) $\left(\frac{\partial^5 z}{\partial y^5}\right)^{\frac{1}{9}} + xz\frac{\partial^2 z}{\partial x^2} = (x^2+y^2)e^x z^{\frac{1}{3}}$.

(c) Solve the partial differential equation $\frac{(b-c)}{a}yzp + \frac{(c-a)}{b}zxq + \frac{(a-b)}{c}xy = 0$, $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$.

[4+2+4]

2. (a) Form the partial differential equation by eliminating the arbitrary function ϕ from

$$\sin z = xy\,\phi\left(\frac{x^2}{y} - \frac{z}{x}\right).$$

(b) Classify the following partial differential equations:

(i) $(\ln x - xy^2)p + (y-z)q = ze^x$,

(ii) $xyp + e^y q = \sin(x+y+z)$.

(c) Solve the partial differential equation $qxy + pq + px - xz = 0$, using Charpit's method.

[3+2+5]

3. (a) Find the general integral of the partial differential equation

$$(x^3 + 4xy^2)p - (4x^2y + y^3)q + (y^2 - x^2)z = 0.$$

Hence find the particular integral which passes through the straight line $z = 1$, $x = y$.

(b) Solve the partial differential equation $(p^2+q^2)y = qz$, using Charpit's method.

[5+5]

Sem - III

B. Sc. (Honours) Examination-2023

Semester-III (CBCS)

Mathematics

Paper: CCMA-5

(Analysis-III)

Time: 3 Hours

Full Marks: 60

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer *any six* questions.

1. (a) When does a series $\sum a_n$ converge? Show that $\lim_{n\to\infty} a_n = 0$ is a necessary condition for the convergence of the series $\sum a_n$. Is this condition sufficient? Justify your answer.
(b) Prove that $\sum_{n=0}^{\infty} \frac{1}{3^n}$ is convergent and find the sum. [(1+2+3)+(2+2)]

2. (a) If $\sum a_n$ is a convergent series of positive terms, prove that the series $\sum a_n^2$ is also convergent.
(b) Examine the convergence of the series $\sin\frac{\pi}{2} + \sin\frac{\pi}{4} + \sin\frac{\pi}{6} + \cdots$.
(c) Show that the series $1 + \frac{1}{1!} + \frac{2^2}{2!} + \frac{3^3}{3!} + \cdots$ cannot converge. [4+3+3]

3. (a) What is Cauchy's root test for positive series? Use it to test the convergence of the series $\sum n^3 e^{-n}$.
(b) State Gauss' test for positive series. Hence discuss the convergence of the series $\left(\frac{1}{2}\right)^4 + \left(\frac{1\cdot 3}{2\cdot 4}\right)^4 + \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^4 \cdots$. [(2+3)+(2+3)]

4. (a) Define the continuinty of a function at a point using ϵ-δ notations. Show that $f : [a, b] \longrightarrow \mathbb{R}$ is continuous at $c \in [a, b] \subset \mathbb{R}$ if for every sequence $\{x_n\}$ in $[a, b]$ converging to c, the sequence $\{f(x_n)\}$ converges to $f(c)$.
(b) Show that the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ defined by $f(x) = \begin{cases} \sin\frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is not continuous at 0.
(c) Prove or disprove: Every continuous function on $\mathbb{R}$ is bounded. [(2+3)+3+2]

5. (a) If $f : [a, b] \longrightarrow \mathbb{R}$ is continuous on $[a, b]$, prove that $|f|$ is continuous on $[a, b]$. Is the converse true? Justify your answer.
(b) Classify the discontinuities of a function defined on $[a, b]$. Find the points of discontinuities of the function $f(x) = [x] + [-x]$. [(3+2)+(2+3)]

6. (a) Let $f, g : \mathbb{R} \longrightarrow \mathbb{R}$ be continuous on $\mathbb{R}$. Show that the set $A = \{x \in \mathbb{R} : f(x) < g(x)\}$ is an open set in $\mathbb{R}$.
(b) Let $f : (a, b) \longrightarrow \mathbb{R}$ be uniformly continuous on (a, b). If $\{x_n\}$ is a Cauchy seqeunce in (a, b), show that $\{f(x_n)\}$ is a Cauchy sequence in $\mathbb{R}$. Hence examine if the function $f(x) = \sin\frac{\pi}{x}$ is uniformly continuous on $(0, 1)$. [3+(3+4)]

7. (a) Show that $f(x) = \sin x + \cos x$ is a function of bounded variation on $[0, \frac{\pi}{2}]$. Find the variation function V of f on $[0, \frac{\pi}{2}]$.
(b) Prove or disprove: A bounded function $f : [a, b] \longrightarrow \mathbb{R}$ is necessarily a function of bounded variation on $[a, b]$. [(3+3)+4]

8. (a) Prove that a monotone function $f : [a, b] \longrightarrow \mathbb{R}$ is Riemann integrable on $[a, b]$.
(b) Check if $f(x) = \operatorname{cosec} x$ is Riemann integrable on $[0, \frac{\pi}{2}]$.

(c) Let $f : [a, b] \longrightarrow \mathbb{R}$ Riemann integrable on $[a, b]$ and $\int_a^b f^2(x)dx = 0$. Show that $f(x) = 0$ at each point $x \in [a, b]$ at which f is continuous.

[3+2+5]

9. (a) Let $f : [a, b] \longrightarrow \mathbb{R}$ be Riemann integrable on $[a, b]$ and define $g(x) = \int_a^x f(t)dt$, $x \in [a, b]$. Prove that g is uniformly continuous on $[a, b]$.

(b) Let $f : [0, 1] \longrightarrow \mathbb{R}$ be defined by $f(x) = \begin{cases} x^2 + x^3 & \text{if } x \text{ is rational} \\ x + x^2 & \text{if } x \text{ is irrational} \end{cases}$. Evaluate $\underline{\int}_0^1 f$ and $\overline{\int}_0^1 f$ and examine if f is Riemann integrable on $[0, 1]$.

[4+(2+2+2)]

B.A/B. Sc. Examination-2023

Semester-III
Mathematics
CC-6 (New)
(Algebra-III)

Time: Three Hours **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer ***all three*** questions.

1. Answer any two questions.$(10 \times 2 = 20)$
   (a) i. For $n > 1$, show that A_n has order $n!/2$. Give a reason why the set of odd permutations in S_n is not a subgroup. Justify your answer [2+2]
   ii. Find all odd permutations of order 5 in S_5. [4]
   iii. Let $\beta = (123)(145)$. Write β^{99} in disjoint cycle form. [2]
   (b) i. Let H be a subgroup of R^*, the group of nonzero real numbers under multiplication. If $R^+ \subseteq H \subseteq R^*$, prove that either $H = R^+$ or $H = R^*$. [3]
   ii. Suppose that K is a proper subgroup of H and H is a proper subgroup of G. If $|K| = 42$ and $|G| = 420$, what are the possible orders of H? [3]
   iii. Prove that a group of order 63 must have an element of order 3. [2]
   iv. Give an example to show that the converse of Lagrange's Theorem does not hold in general. [2]
   (c) i. Prove that a quotient group of a cyclic group is cyclic. [2]
   ii. What is the order of the element $[14] + < 8 >$ in the quotient group $\mathbb{Z}_{24}/ < 8 >$? [2]
   iii. Consider the subgroup $H = \{(1), (12)\}$ of S_3. Does there exist a quotient group of S_3 modulo H? Justify your answer [3]
   iv. Give an example with proper justification of a non commutative group of which every proper subgroup is normal. Find all normal subgroups (if exists) in the group $\mathbb{Z}_{47}$? [2+1]

2. Answer any two questions.$(10 \times 2 = 20)$
   (a) i. Show that a ring that is cyclic under addition is commutative. [4]
   ii. Explain why every subgroup of $\mathbb{Z}_n$ under addition is also a subring of $\mathbb{Z}_n$. [3]
   iii. Prove that an element $A \in M_2(\mathbb{Z})$ is a unit if and only if $\det A = \pm 1$. [2+1]
   (b) i. Prove that characteristic of a finite ring is a divisor of the additive order of that ring. Hence show that characteristic of a finite ring is always finite. [3+1]
   ii. Show that the set $S = \left\{ \begin{pmatrix} a & a \\ a & a \end{pmatrix} : a \in \mathbb{R} \right\}$ is a field with respect to usual matrix addition and multiplication. [4]
   iii. Give an example of a commutative ring without zero-divisors that is not an integral domain. justify your answer [2]
   (c) i. Show that in a commutative ring with unity every maximal ideal is a prime ideal. Give an example of a ring in which there exists maximal ideal that is not prime. [3+2]
   ii. Check whether the ideal $< 2 + 2i >$ of the ring $\mathbb{Z}[i]$ is prime or not? [3]
   iii. If an ideal I of a ring R contains a unit, show that $I = R$. [2]

3. Answer any two questions.($10 \times 2 = 20$)

(a) i. Find conditions on $a, b, c \in \mathbb{R}$ so that $v = (a, b, c) \in \mathbb{R}^3$ belongs to $W = span((1, 2, 0), (-1, 1, 2), (3, 0, -4))$. [2]

ii. Let U and W be two subspaces of a vector space V. Show that $U + W$ is the smallest subspace containing U and W. Hence show that $U + W = L(U \cup W)$. [3+2]

iii. Determine whether the vectors $(1, 2, -3, 1), (3, 7, 1, -2), (1, 3, 7, -4)$ in $\mathbb{R}^4$ are linearly dependent or independent. [3]

(b) i. Show that every vector space has a basis. [4]

ii. The set of all 3×3 matrices having trace equal to zero is a subspace W of $M_{n \times n}(\mathbb{R})$. Find the dimension of W? [2]

iii. Let V be a finite dimensional vector space, and let S be a subset of V. Prove that there is a subset of S that is a basis for $L(S)$. [4]

(c) i. Let V be an inner product space over $\mathbb{R}$. Then show that for all $x, y \in V$, $< x, y > \leq ||x|| \cdot ||y||$. [3]

ii. Show that every orthogonal set of an inner product space is linear independent. [3]

iii. Find the orthogonal projection of the vector $4 + 3t - 2t^2$ on the given subspace $P_1(R)$ of the inner product space $P(R)$ with the inner product $< f, g >=$

$$\int_0^1 f(t)g(t)\, dt.$$

[4]

B. Sc.(H) Examination-2023

Semester-III

Mathematics

Paper: CC-7

(Differential Equations-I)

Time: Three Hours **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer Question No. 1 and any *four* from the rest.

1. Answer any *ten* questions from the following: [10 × 2 = 20]
   (a) Find three different solutions of the differential equation $\frac{dy}{dx} = xy^{1/2}$, $y(0) = 0$. [2]
   (b) Solve the differential equation $(\frac{dy}{dx})^2 + 3\frac{dy}{dx} + 2 = 0$. [2]
   (c) Solve: $2x(y+1)dx - ydy = 0$. [2]
   (d) Find the value of α for which the differential equation $(1 + x^2y^3 + \alpha x^2y^2)dx + (2 + x^3y^2 + x^3y)dy = 0$ becomes exact. [2]
   (e) Find a third order linear homogeneous differential equation whose linearly independent solutions are e^x, e^{2x}, and e^{3x}. [2]
   (f) Find the orthogonal trajectories of the family of circles $x^2 + y^2 = c^2$. [2]
   (g) Consider the differential equation $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = 0$. If $W(1) = 1$, then find the value of $W(7) - W(3)$. [2]
   (h) Solve: $(D^2+1)^3(D+2)^2(D^2-1)^3y = 0$. [2]
   (i) Find an integrating factor of the differential equation $y(ydx - xdy) + x^2(2ydx + 2xdy) = 0$. [2]
   (j) Find the differential equation of all circles passing through the origin and having their centers on the axis of x. [2]
   (k) Let $x(t)$ be the solution of the differential equation $\frac{dx}{dt} = x(x-4)(x-11)$ satisfying the initial condition $x(0) = 6$. Find the value of $x(t)$ when $t \to \infty$. [2]
   (l) Explain why it is always possible to express any homogeneous differential equation $M(x,y)dx + N(x,y)dy = 0$ in the form $\frac{dy}{dx} = F(\frac{x}{y})$. [2]
   (m) If an integral curve of the differential equation $(y-x)\frac{dy}{dx} = 1$ passes through the points $(0,0)$ and $(\alpha, 1)$ then find the value of α. [2]
   (n) Solve: $\frac{d}{dx}(x\frac{dy}{dx}) = x; y(1) = 0, y'(1) = 0$. [2]

2. (a) Prove that the transformation $v = y^{1-n} (n \neq 0 \text{ or } 1)$ reduces the Bernoulli equation $\frac{dy}{dx} = P(x)y + Q(x)y^n$ to a linear equation in v. Hence solve the equation $\frac{dy}{dx} + y = xy^3$. [3+2]
   (b) If $e^{\int \phi(y)dy}$ is an integrating factor of the differential equation $M(x,y)dx + N(x,y)dy = 0$, then find the expression of $\phi(y)$. Hence solve the differential equation $y(2x^2y + e^x)dx - (e^x + y^3)dy = 0$. [3+2]

3. (a) Solve the nonlinear differential equation $y + x\frac{dy}{dx} - x^4\left(\frac{dy}{dx}\right)^2 = 0$. [3]
   (b) Solve the equation $x^2\frac{d^2y}{dx^2} - 2x(1+x)\frac{dy}{dx} + 2(1+x)y = x^3$, where $y = x$ and $y = xe^{2x}$ are two linearly independent solutions of the corresponding homogeneous equation. [4]
   (c) If u and v are any two solutions of the equation $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$ on an interval $[a,b]$, then prove that their Wronskian $W(u,v)$ is either identically zero or never zero on $[a,b]$. [3]

4. (a) Solve the homogeneous differential equation $(3xy + y^2)dx - 3x^2dy = 0$. [4]
   (b) Solve: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x cosx$. [4]
   (c) Use the method of isoclines to sketch some of the solution curves of the differential equation $\frac{dy}{dx} = x^2 + y^2$. [2]

5. (a) Using the method of undetermined coefficients, solve the equation $(D^2 + D - 6)y = e^x + x^2 + 1$. [5]
   (b) Find the singular solution and extraneous loci of the differential equation $4p^2x = (3x-1)^2$. [5]

6. (a) Reduce the equation $(px - y)(x - py) = 2p$ to Clairaut's form by the substitution $x^2 = u$ and $y^2 = v$. Hence solve the equaion. [4+1]

(b) Solve the differential equation $\frac{dy}{dx} + y = f(x)$ with initial condition $y(0) = 0$, where $f(x) = 2$, when $0 \leq x < 1$ and $f(x) = 0$, when $x \geq 1$. Hence find the value of $y(5)$. [4+1]

7. (a) Find the orthogonal trajectories of the family of curves $r = a sin\theta$. [2]

(b) Solve the equation $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4$. [4]

(c) Find the equation of the family of curves which cut the members of the family of straight lines $y = cx$ at angle of 45^o. [4]

B.Sc. (Honours) Examination, 2023
Mathematics
Semester-III
Paper SECMA - I
Boolean Algebra and Circuit Design

Time: Two Hours

Full Marks: 25

Questions are of value as indicated in the margin
Notations and symbols have their usual meaning

Answer *any five* questions

1. Define Boolean algebra. Show that the complement of an element of the underlying set B is unique. Deduce from definition that [5]

$$a + a = a \text{ for all } a \in B.$$

2. Using definition, show that for all $a, b, c \in B$ [2+1+2]

   i) $ac + b\bar{c} = ac + b\bar{c} + ab$
   ii) $a + ab = a$
   iii) $\overline{(a + c)} = \bar{a}.\bar{c}$

3. Define a Boolean expression. Show that every Boolean expression in n Boolean variables has a unique normal form. [5]

4. Define a Karnaugh map for a Boolean function in $n(\leq 4)$ Boolean variables. Represent the Boolean function

$$f(x, y, z, w) = \sum(3, 7, 11, 12, 13, 14, 15)$$

in a Karnaugh map and write down the minimal form. Design a two-level AND-to-OR gate network using as few gates as possible for the function [5]

$$g(x, y, z, w) = \sum(0, 1, 2, 3, 4, 6, 7, 8, 12, 13).$$

5. Define NOR gate. Using NOR gate implement OR, AND and NOT functions. Design a single gate network for the modulo-2 sum of two binary digits. [5]

6. Design a full subtractor logic circuit. [5]

7. State the functions of a D-flipflop. Write down the state table. State how a $JK - ff$ and $SR - ff$ can be used as a $D - ff$. [5]

8. Design a single bit comparator circuit. [5]