**M. Sc. Examination-2024**
**Semester-I**
**Mathematics**
**Paper: MMC-11**
**(Real Analysis)**

**Time: 3 Hours** **Full Marks: 40**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer ***any four*** questions.

1. (a) What do you mean by the outer measure $m^*(A)$ of a set $A \subset \mathbb{R}$? Give example. Show that $m^*$ is countably sub-additive. [2+4]

   (b) Prove or disprove: All intervals are measurable. [4]

2. (a) If $E_1, E_2, \cdots$ are measurable subsets of $\mathbb{R}$ such that $E_1 \subseteq E_2 \subseteq \cdots$, then show that $m\left(\bigcup_{n=1}^{\infty} E_n\right) = \lim_{n\to\infty} m(E_n)$. [4]

   (b) When is a property said to hold *a.e.* on a measurable set $A$? Explain with example. If $A \subseteq \mathbb{R}$ is a measurable set and $\phi$ is an extended measurable function on $A$ and $\phi = \psi$ *a.e.* on $A$, show that $\psi$ is measurable on $A$. [2+4]

3. (a) Construct the Canor-Lebesgue function $\phi$ on $[0,1]$ Show that the function $\psi : [0,1] \longrightarrow \mathbb{R}$ defined by $\psi(x) = \phi(x) + x$ maps a measurable set onto a non-measurable set. [3+4]

   (b) Let $f : A \longrightarrow [0,\infty)$ be ameasurable function on $A \subseteq \mathbb{R}$ such that $\int_A f = 0$. Prove that $f = 0$ *a.e.* on $A$. [3]

4. (a) State and prove Fatou's Lemma. Show by an example that it does not hold good unless you take a sequence of non-negative measurable functions. [4+3]

   (b) If $\{f_n\}$ is a sequence of measurable functions on a set $A$ of finite measure such that $f_n \longrightarrow f$ pointwise *a.e.* on $A$ and $f$ is finite *a.e* on $A$, show that $f_n \longrightarrow f$ in measure on $A$. [3]

5. (a) Let $f$ be bounded function on a closed and bounded interval $[a,b]$. If $f$ is continuous *a.e.* on $[a,b]$, show that $f$ is Riemann integrable over $[a,b]$. [5]

   (b) State Monotone Convergence Theorem and use it to evaluate the Lebesgue integral of the function $f : [0,1] \longrightarrow \mathbb{R}$ defined by $f(x) = \begin{cases} 0 & \text{if } x = 0 \\ \frac{1}{x^{\frac{1}{3}}} & \text{if } 0 < x \leq 1. \end{cases}$ [1+4]

6. (a) Let $\{f_n\}$ be a sequence of bounded measurable functions on a set $E$ of finite measure. If $f_n \longrightarrow f$ uniformly on $E$, show that $\lim_{n\to\infty} \int_E f_n = \int_E f$. Does the conclusion hold if $f_n \longrightarrow f$ pointwise only on $E$? Justify your answer. [4+2]

   (b) Let a function $f : X = [1,\infty) \longrightarrow \mathbb{R}$ be given by $f(x) = \frac{1}{2^n}$ if $n \leq x < n+1$. Show that $\int_X f = 1$. [4]

# M. Sc. Examination - 2024

**Semester-I**
**Mathematics**
**Paper: MMC 12**
**(Complex Analysis)**

**Time: 3 Hours** **Full Marks: 40**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer ***any four*** questions.

1. (a) State and prove Maximum modulus theorem. [1+3]
   (b) Using Maximum modulus theorem show that if $|f(z)| > M$ on $|z| = R, f(z)$is regular for $|z| \leq R$ and $|f(\alpha)| < M, |\alpha| < R$, then $f(z)$ has at least one zero in $|z| < R$. [2]
   (c) State and prove Taylor's theorem. [1+3]

2. (a) State Laurent's theorem. [2]
   (b) Expand $f(z) = \frac{1}{z(z-1)(z-2)}$ in Laurent series in the region $1 < |z| < 2$. [3]
   (c) State and prove Riemann's theorem on removable singularity. [3]
   (d) Show that if $f(z) = (z-\alpha)^n \phi(z)$,where $\phi(z)$ is analytic at $\alpha$ and $\phi(\alpha) \neq 0$ then $\alpha$ is a zero of $f(z)$ of order $n$. [2]

3. (a) State and prove open mapping theorem. [1+3]
   (b) If $\alpha$ be a pole of $f(z)$ of order $m$ and a pole of $g(z)$ of order $n(> m)$ then what kind of a point $\alpha$ relative to the function $f(z) - g(z)$. [2]
   (c) If $f(z)$ has a pole of multiplicity $m$ at $z = \alpha$, then show that the residue of $f(z)$ at $\alpha$ is given by
   $\frac{1}{(m-1)!} \lim_{z \to \alpha} \frac{d^{m-1}}{dz^{m-1}}[(z-\alpha)^m f(z)]$. [3]
   (d) State Riemann mapping theorem. [1]

4. (a) Determine the nature of the singularities of $cosec\frac{1}{z}$. [3]
   (b) If $f(z) = \frac{P(z)}{Q(z)}$, where $P(z)$ and $Q(z)$ are analytic at $\alpha$ and $P(\alpha) \neq 0$, while $\alpha$ is a simple zero of $Q(z)$, then prove that $\alpha$ is a simple pole of $f(z)$ with residue $\frac{P(\alpha)}{Q'(\alpha)}$. [3]
   (c) State and prove Cauchy's residue theorem. Using this theorem evaluate $\oint_{|z|=1} \frac{e^{3z}}{(6z-\pi)^2} dz$. [3+1]

5. (a) State and prove Rouche's theorem. [1+3]
   (b) If $k > 1$, show that the equation $z^n = e^{z-k}$ has $n$ roots inside the circle $|z| = 1, n$ being a positive integer. [2]
   (c) Let $f(z)$ be analytic within and on a positively oriented simple closed curve $C$ except for a finite number of poles within $C$ and let $f(z) \neq 0$ any where on $C$, then show that $\oint_C \frac{f'(z)}{f(z)} dz = i\Lambda_C arg f(z)$. [4]

6. (a) If $f(z)$ be a function having only a finite number of singularities then show that the sum of the residues of $f(z)$ in the entire z-plane including the point at infinity is zero. [3]
   (b) Calculate the residue of $\frac{1}{z^2-1}$ at the point at infinity. [2]
   (c) Evaluate any one of the following by the method of contour integration:
   (i)$\int_{-\infty}^{\infty} \frac{dx}{x^4+a^4}, (a > 0)$; $(ii)$ $\int_0^{\infty} sin x^2 dx$ . [5]

# M. Sc. Examination-2024
## Semester-I
## Mathematics
## MMC-13(Linear Algebra)

**Time: Three Hours** **Full Marks:** 40

Questions are of values as indicated in the margin.
($V$ is a finite dimensional vector space over $F$.)

Answer ***any four*** questions.

1. (a) Let $A \in M_n(F)$ and $\lambda \in F$. Show that $\lambda$ is an eigen value of $A$ if and only if $|A - \lambda I_n| = 0$. [3]
   (b) Let $\lambda$ be an eigen value of a matrix $A \in M_n(F)$. Show that the geometric multiplicity of $\lambda$ is $n - r(A - \lambda I_n)$ [3]
   (c) Let $X, Y$ be two nonzero column vectors in $\mathbb{R}^n$ and $A = XY^t$. Show that 0 is an eigen value of $A$ and every eigen vector corresponding to 0 is orthogonal to $Y$. [4]

2. (a) Let $T : V \longrightarrow V$ be a linear operator, $\beta$ be a basis of $V$ and $\lambda$ be an eigen value of $T$. Show that $v \in V$ is an eigen vector of $T$ corresponding to $\lambda$ if and only if $[v]_\beta$ is an eigen vector of $[T]_\beta$ corresponding to $\lambda$. [4]
   (b) Let $T : M_n(\mathbb{R}) \longrightarrow M_n(\mathbb{R})$ be defined by $T(A) = A^t$. Find all eigen values and eigen spaces of $T$. [3]
   (c) Let $T : V \longrightarrow V$ be a linear operator. If every nonzero vector of $V$ is an eigen vector of $T$, then show that $T = cI$ for some scalar $c$. [3]

3. (a) Let $T : V \longrightarrow V$ be a linear operator. Prove that the eigen vectors belonging to the distinct eigen values are linearly independent. [4]
   (b) Let $T : V \longrightarrow V$ be a linear operator and $V$ be a $T$-cyclic space. If a linear operator $S : V \longrightarrow V$ commutes with $T$, then show that $S = g(T)$ for some polynomial $g(t)$. [3]
   (c) Let $T : V \longrightarrow V$ be a linear operator, $v \in V$ and $W = < v >_T$. If $\dim W = k$, then show that $\{v, T(v), \cdots, T^{k-1}(v)\}$ is a basis of $W$. [3]

4. (a) Let $A \in M_n(F)$. If $F^n$ has a basis of eigen vectors of $A$, then show that $A$ is diagonalizable. [3]
   (b) State and prove the spectral theorem for the diagonalizable linear operators. [4]
   (c) Show that two reflections $R_1$ and $R_2$ on a vector space are similar if and only if $\dim E_1(R_1) = \dim E_1(R_2)$. [3]

5. (a) Let $T : V \longrightarrow V$ be a linear operator and $f(t) \in F[t]$ be such that $f(t) = g(t)h(t)$ for some $g(t), h(t) \in F[t]$. If $\gcd(g(t), h(t)) = 1$, then show that $\ker f(T) = \ker g(T) \oplus \ker h(T)$. [4]
   (b) Give an example with justification of a non-diagonalizable matrix $A \in M_3(\mathbb{R})$ which satisfies $A^k = I_3$ for some $k \in \mathbb{N}$. [3]
   (c) Let $X, Y \in \mathbb{R}^n$ be two nonzero column vectors. If $X \perp Y$, then show that the matrix $A = XY^t$ is not diagonalizable. [3]

6. (a) Let $T : V \longrightarrow V$ and $\beta = \{v_1, v_2, \cdots, v_n\}$ be a basis of $V$. Then prove that $[T]_\beta$ is upper triangular if and only if $T(vj) \in span(\{v_1, v_2, \cdots, v_j\})$ for all $1 \leqslant j \leqslant n$. [4]
   (b) Let $T : V \longrightarrow V$ and $\chi_T(t) = (t - \lambda_1)^{n_1}(t - \lambda_2)^{n_2} \cdots (t - \lambda_k)^{n_k}$ where $\lambda_1, \lambda_2, \cdots, \lambda_k$ are distinct eigen values of $T$. Show that $\dim(T - \lambda_i I)^{n_i} = n_i$ for all $1 \leqslant i \leqslant k$. [3]
   (c) Find all possible Jordan canonical forms of a linear operator $T$ such that $\chi_T(t) = (t-2)^5$ and $m(t) = (t-2)^2$. In each case find $\dim E_2$. [3]

Use separate answer script for each unit

# M.Sc. Examination-2024

**Semester-I**
**Mathematics**
**MMC 14**
**(Ordinary Differential Equations)**

**Time: Three Hours** **Full Marks: 40**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer ***any four*** questions.

1. (a) With a suitable example show that Lipchitz Criteria is a weaker concept than Picard's condition for uniqueness of solutions of a first order IVP. [2]

   (b) What is meant by Interval of Definition for the solution of an IVP. Find the Interval of Definition for the solution of the following IVP.
   $\frac{d^3x}{dt^3} + \frac{1}{t^2-4}x = cos(t), \qquad x(1) = 3, x'(1) = 0, x''(1) = 0.$ [1+2]

   (c) Show that the initial value problem $\frac{dy}{dx} = (y-1)/x, y(0) = 1$ has infinite solutions. [2]

   (d) Find the third approximation of the solution of the equation $\frac{dy}{dx} = z, \frac{dz}{dx} = x^3(y+z)$ by the Picard's Method where $y = 1, z = \frac{1}{2}$ where $x = 0$. [3]

2. (a) What is a Wronskians ? State and prove Abel's Theorem for a 2nd order ODE. Hence show that the Wronskians is either zero everywhere or zero nowhere. [1+3+1]

   (b) When a set of solutions is said to be Fundamental? What do you mean by a Natural Fundamental Sets of Solutions? Find the Natural Fundamental set of solutions of the following Differential Equations $y'' - y' - 2y = 0$, with the initial time 0. [1+1+3]

3. (a) Find the nature and stability of the fixed points of
   $\dot{x} = -ax + y \ , \ \dot{y} = -x - ay$
   For the different values of the parameter $a$. [3]

   (b) Draw the phase diagrams for the linear harmonic undamped oscillators represented by $\ddot{x} + \omega^2 x = 0$. [2]

   (c) Find the solution of the non-homogeneous differential equation $\dot{\underset{\sim}{x}} = A\underset{\sim}{x} + F(t)$, where $A = \begin{bmatrix} 6 & -3 \\ 2 & 1 \end{bmatrix}$ and $F(t) = \begin{bmatrix} e^{5t} \\ 4 \end{bmatrix}$ [5]

4. (a) State and prove Sturm Separation Theorem. [4]

   (b) Let $f$ and $g$ be any two solutions of $\frac{d}{dt}\left[P(t)\frac{dx}{dt}\right] + Q(t)x = 0$ on the interval $a \le t \le b$. Then show that for all $t$ on $a \le t \le b$, $P(t)[f(t)g'(t) - f'(t)g(t)] = k$, where $k$ is a constant. [3]

   (c) If $A = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then prove that $e^{At} = e^{aIt}[I\cos(bt) + J\sin(bt)]$, where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. [3]

5. (a) Solve the boundary-value problem
   $x^2y'' - 3xy' + 3y = 24x^5, \quad y(1) = 0, y(2) = 0.$
   by using Green's function. [6]

   (b) Solve the initial-value problem
   $y'' + 4y = x, \quad y(0) = 0, y'(0) = 0.$
   by using Green's function . [4]

6. (a) State and prove Gronwall's inequality. [1+4]

   (b) Find the eigenvalues and eigen-functions of the following Strum-Liouville's system
   $\frac{d^2y}{dx^2} + \lambda y = 0, \ y(0) = y'(0), \ y(\pi) = y'(\pi).$ [5]

M. Sc. Examination-2024
Semester-I
Mathematics
Paper: MMC 15
(Partial Differential Equations)

**Time: 3 Hours** **Full Marks: 40**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer ***any four*** questions.

1. (a) When is a partial differential equation (PDE) said to be well-posed in the sense of Hadamard? [2]

   (b) Solve the the following Cauchy problem and find the value of $u\left(1/\sqrt{2}, 1/\sqrt{2}\right)$.

$$x\frac{\partial u}{\partial y} - y\frac{\partial u}{\partial x} = u,$$
$$u(x,0) = \sin\left(\frac{\pi x}{4}\right).$$

   Also, characterize the nature of the integral surface so obtained. [3+1]

   (c) Obtain the singular integral of the following PDE and give its geometrical interpretation.

$$\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} - u\right)^2 = 1 + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2.$$

[4]

2. (a) Find by the Cauchy's method of characteristics the integral surface of the following PDE that passes through the curve $u = y$, $x = 0$.

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{\partial u}{\partial x}\frac{\partial u}{\partial y}.$$

[5]

   (b) Show that a linear PDE of the form $\sum_{r,s} C_{rs}x^r y^s \frac{\partial^{r+s}u}{\partial x^r \partial y^s} = f(x,y)$ can be reduced to one with constant coefficients by the substitutions: $\xi = \log_e x$ and $\eta = \log_e y$. Hence, find the complete integral of the following PDE.

$$x^2\frac{\partial^2 u}{\partial x^2} - 3xy\frac{\partial^2 u}{\partial x \partial y} + 2y^2\frac{\partial^2 u}{\partial y^2} + 5y\frac{\partial u}{\partial y} - 2u = \cos\left[\log_e(xy)\right].$$

[(1+4)]

3. (a) Solve the following PDE by Monge's method.

$$xy\left(\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2}\right) - (x^2 - y^2)\frac{\partial^2 u}{\partial x \partial y} + y\frac{\partial u}{\partial x} - x\frac{\partial u}{\partial y} = 0.$$

[5]

   (b) Find the two families of characteristics of the PDE, given by,

$$\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = e^{2x+3y}$$

   and hence convert the equation to a canonical form. [(2+3)]

4. (a) Using Laplace integral transform method, solve the following initial boundary value problem (IBVP):

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} - \cos(2t),\ 0 \le x < \infty, 0 \le t < \infty,$$

BC's: $u(0,t) = 0$, $u(x,t)$ is bounded as $x \to \infty$,
IC's: $u(x,0) = 0$, $\frac{\partial u}{\partial t}|_{(x,0)} = 0$. [5]

(b) Solve the following PDE by using the finite Fourier sine transform method:

$$\begin{aligned} \frac{\partial u}{\partial t} &= 4\frac{\partial^2 u}{\partial x^2},\ 0 < x < 2,\ t > 0, \\ u(0,t) &= 0 = u(2,t),\ t > 0, \\ u(x,0) &= \begin{cases} x, & 0 \le x \le 1 \\ 2 - x, & 1 \le x \le 2. \end{cases} \end{aligned}$$

[5]

5. (a) Find the value of $u(2,3)$ for the following PDE.

$$\frac{\partial^2 u}{\partial x \partial y} + x^3 y^2 u = 0,$$
$$u(x,0) = e^{(x^4/4)}.$$

[4]

(b) If $u(x,t) = Ae^{-t}\sin x$ is a solution of the following IBVP, then find the value of $A$.

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2},\ 0 < x < \pi,\ t > 0, \\ u(0,t) &= 0 = u(\pi,t),\ t > 0, \\ u(x,0) &= \begin{cases} 60, & 0 \le x \le \frac{\pi}{2}, \\ 40, & \frac{\pi}{2} \le x \le \pi. \end{cases} \end{aligned}$$

[3]

(c) Let $u(x,t)$ be a function that satisfies the PDE:

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} = e^x + 6t,\ x \in \mathbb{R},\ t > 0,$$

with $u, \frac{\partial u}{\partial x} \to 0$ as $x \to \pm\infty$ and the initial conditions: $u(x,0) = \sin x$, $\frac{\partial u}{\partial t}|_{(x,0)} = 0$ for every $x \in \mathbb{R}$. Find the value of $u(\frac{\pi}{2}, \frac{\pi}{2})$. [3]

6. (a) State maximum-minimum principle for a function $u(x,y)$, which is continuous in a closed region: $\overline{\mathbb{R}}$ $(= \mathbb{R} \cup \partial\mathbb{R})$ and which satisfies the Laplace equation: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in the interior of $\mathbb{R}$. [2]

(b) Prove that, if the Dirichlet problem has a solution $u(x,y)$, which is continuous in a bounded region: $\overline{\mathbb{R}}$ and which satisfies the Laplace equation: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in the interior of $\mathbb{R}$, then it is unique. [2]

(c) Solve the following Neumann problem for a rectangle.

PDE: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,\ 0 \le x \le a,\ 0 \le y \le b,$
BC's: $\frac{\partial u}{\partial x}|_{(0,y)} = 0 = \frac{\partial u}{\partial x}|_{(a,y)}$, $\frac{\partial u}{\partial y}|_{(x,0)} = 0$, $\frac{\partial u}{\partial y}|_{(x,b)} = f(x)$. [6]

# M. Sc. Examination-2024

**Semester-I**

**Mathematics**

**Paper: MMC-16**

**(Integral Transforms and Integral Equations)**

**Time: Three Hours** **Full Marks: 40**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer Question No. 1 and any *three* from the rest.

1. Answer any *five* questions from the following: [5 × 2 = 10]

(a) Find the value of the integral $\int_0^\infty te^{-5t}\cos 3t dt$. [2]

(b) Find the inverse Fourier transform of $F(s)$, where $F(s) = \frac{1}{7+8is-s^2}$. [2]

(c) Find the value of $\lambda$ for which the integral equation $y(x) = \lambda\int_0^1 x^3t^3y(t)dt$ has infinitely many solutions. [2]

(d) Solve the Fredholm integro-differential equation
$u'(x) = 9x^2 + \int_0^1 xu(t)dt,\ u(0) = 1.$ [2]

(e) Find the Laplace transform of $f(t)$, where $f(t) = e^{2t}$ when $0 < t < 5$ and $f(t) = t - 2$ when $t > 5$. [2]

(f) Using modified Adomian decomposition method, solve the integral equation
$y(x) = \cos x - (1 - e^{\sin x})x - x\int_0^x e^{\sin t}y(t)\,dt$. [2]

(g) Find the value of $u(2) + u(5)$, where $u(x)$ satisfies the equation
$xe^{-x} = \int_0^x e^{t-x}u(t)\,dt$. [2]

2. (a) Find the resolvent kernel of the integral equation
$y(x) = (1 + x^2)^7 + 7\int_0^x \frac{1+x^2}{1+t^2}y(t)\,dt$. [4]

(b) Convert the integral equation $y(x) = \int_0^x(x - t)y(t)\,dt - x\int_0^1(1 - t)y(t)\,dt$ into an equivalent differential equation. [2]

(c) Solve Volterra integral equation of first kind
$\int_0^x(1 - x^2 + t^2)y(t)\,dt = \frac{x^2}{2}$. [2]

(d) Solve the Volterra integro-differential equation $\frac{dy}{dx} = x + \int_0^x y(x - t)cost\,dt, y(0) = 4$. [2]

3. (a) State Fredholm alternative theorem. Show that the integral equation

$$y(x) = f(x) + \frac{1}{\pi}\int_0^{2\pi}\sin(x + t)y(t)\,dt$$

possesses no solution for $f(x) = x$, but it possesses infinitely many solutions when $f(x) = 1$. [1+4]

(b) Using Laplace transform, solve the initial value problem
$\frac{d^2y}{dt^2} + y = e^{-t}; y(0) = 5, y'(0) = 7$. [2]

(c) If $f$ and $g$ are piecewise continuous on $[0, \infty)$ and of exponential order then prove that $L\{f * g\} = L\{f\}.L\{g\}$.
Using the above result, find the inverse Laplace transform of the function $\frac{1}{s^2(s-1)}$. [2+1]

4. (a) Transform the differential equation $\frac{d^2y}{dx^2} + xy = 5, y(0) = 0, y(1) = 1$ into an equivalent integral equation. [4]

(b) Using Hilbert-Schmidt theorem, solve the integral equation $y(x) = x + \frac{1}{\pi} \int_0^1 (1 - 3xt)y(t)\, dt$. [4]

(c) Prove that $L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}[F(s)]$, where $L\{f(t)\} = F(s)$. [2]

5. Solve the Volterra integral equation $y(x) = x^2 - \int_0^x (x - t)y(t)dt$ by

(a) Laplace transform method. [2]

(b) series solution method. [2]

(c) Adomian decomposition method. [2]

(d) successive approximations method. [2]

(e) successive substitutions method. [2]

6. (a) Find the inverse Fourier transform of $F(s) = e^{-s|y|}$. [3]

(b) Find the Fourier sine transform of $f(x) = e^{-ax}$.
Hence using Persaval's identity, evaluate $\int_0^\infty \frac{x^2}{(a^2+x^2)(b^2+x^2)} dx$. [1.5+1.5]

(c) Find the Fourier cosine transform of $f(x)$, where $f(x) = \cos 2x$ when $0 \leq x \leq 3$, and $f(x) = 0$ when $x > 3$. [2]

(d) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$. [2]