Use separate answer scripts for each unit

B.Sc.(Honours) Examination, 2024

Semester-V (CBCS)
Mathematics (Honours)
Paper : CCMA 11
(Analysis-V and Differential Equation-III)

Time: Three Hours **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meaning.

Unit-I [Analysis-V (Marks-30)]

Answer *any three* questions

1. (a) Show that $\int_0^{\infty} e^{-x}x^{n-1}dx$ converges for $n > 0$. [3]

   (b) Test for convergence of the functions
   (*i*) $\int_1^2 \frac{\sqrt{x}}{\log x}$ dx (*ii*) $\int_0^1 \frac{1}{\sqrt{x^3 - x^6}}dx$ [2+2]

   (c) Evaluate $\int_0^{\infty} \frac{\log(1+4x^2)}{x^2}dx$. [3]

2. (a) Let $\{f_n\}$ be a sequence of continuous functions converges to f uniformly on $[a, b]$. Show that f is continuous on $[a, b]$. Also show that the condition of uniform convergence can not be omitted from above statement. [3+1]

   (b) Let $\{r_i\}$ be the listing of rational numbers in $[0, 1]$. Consider the sequence $\{f_n\}$,

   $$f_n(x) = \begin{cases} 1 & \text{if } x = r_1, r_2, \ldots, r_n, \\ 0 & \text{otherwise} \end{cases}$$

   Show that each f_n is Riemann-integrable but the limit function is not. Is the convergence of $\{f_n\}$ uniform? [3]

   (c) Show that the sequence of functions $\{nxe^{-nx^2}\}$ converges uniformly on $[1/2, 1]$ but not on $[0, 1]$. [3]

3. (a) State and prove Weierstass M-test for uniform convergence of a series of function. [4]

   (b) Test for uniform convergence of the series $\sum_{n=1}^{\infty} \frac{x}{n(1+nx^2)}$ in $\mathbb{R}$. [3]

   (c) State Abel test and Dirichlet test for uniform convergence of a series of functions. [3]

4. (a) Define radius of convergence of a power series. State and prove Cauchy-Hadamard Theorem for power series of real variables. [2+4]

   (b) Find the interval of convergence of the power series $\sum\limits_{n=1}^{\infty} (-1)^n \frac{(x-2)^n}{n10^n}$ [2]

   (c) If a power series $\sum\limits_{n=0}^{\infty} a_n x^n$ has radius of convergence $\rho, 0 < \rho < \infty$ and it converges at ρ then show that

   $$\lim_{x \to \rho-} \sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} a_n \rho^n$$ [2]

5. (a) When is the trigonometric series $\frac{a_0}{2} + \sum\limits_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ referred to as a Fourier series? Write the Bessel's inequality of Fourier series and prove it. [1+4]

   (b) If f is Riemann-integrable in [a,b] then show that $\lim\limits_{n \to \infty} \int\limits_a^b f(x) \cos nx dx = 0$. [4]

   (c) Find the Fourier coefficients of a constant function in $[-\pi, \pi]$ [1]

Unit-II (Differential Equations-III)
(Full Marks: 30)

Answer question number 1 and *any two* from the rest.

1. Answer *any five* questions.

(a) Form a partial differential equation (PDE) from $\phi(\frac{u}{x^3}, \frac{y}{x}) = 0$ by eliminating the arbitrary function ϕ.

(b) Find the integral surface of the Cauchy value problem $\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 3,\ u(x,0) = x^2$.

(c) Construct a PDE of the family of orthogonal surfaces, each member of which cuts each member of the family $u^2 = kxy,\ k \in \mathbb{R}$ is a parameter.

(d) Find a singular solution of the following non-linear PDE, if any:

$$u = x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{2}{3}\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}.$$

(e) What are the basic limitations of Charpit's method for solving a first order PDE $f(x, y, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y})$?

(f) Comment on the initial strip of the following PDE in connection with the Cauchy's method of characteristics:

$$\left(\frac{\partial u}{\partial x}\right)^2 + y^2\left(\frac{\partial u}{\partial y}\right)^2 = u^2,\ u(x,1) = x^2.$$

(g) Find the particular integral (PI) of the PDE $\left(\frac{\partial u}{\partial x} - 3\frac{\partial u}{\partial y} - 2u\right)^2 = 2e^{2x}\tan(3x+y)$.

(h) Solve:

$$x^3y^2\frac{\partial^3 u}{\partial x^3}\frac{\partial^2 u}{\partial y^2} - x^2y^3\frac{\partial^2 u}{\partial x^2}\frac{\partial^3 u}{\partial y^3} = 0.$$

[5 × 2=10]

2. (a) Find the integral surface of the following PDE:

$$(xy^3 - 2x^4)\frac{\partial u}{\partial x} + (2y^4 - x^3y)\frac{\partial u}{\partial y} = 9u(x^3 - y^3),\ u(x,x) = x^3.$$

(b) Find the complete solution of the following PDE using Charpit's method:

$$2ux + \left(\frac{\partial u}{\partial y}\right)^2 = x\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right).$$

[5+5]

3. (a) Prove that the PDE

$$\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right)\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right) = 1 \text{ is compatible with the PDE } \frac{\partial u}{\partial x} = 2024\frac{\partial u}{\partial y},$$

and hence find the one-parameter family of solutions.

(b) Solve:

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial y} - u = \cos(x + 2y) + e^y.$$

[5+5]

4. (a) Define Cauchy-Euler PDE with one dependent and two independent variables. Hence, solve the following PDE:

$$x^2\frac{\partial^2 u}{\partial x^2} - y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = \log_e(xy).$$

(b) Find the integral surface satisfying the PDE

$$\frac{\partial^2 u}{\partial x^2} - 4\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} = 0$$

and passing through the space curves $\alpha u = y^2,\ x = 0$ and $\beta u = x^2,\ y = 0$. [5+(1+4)]

B.Sc. (Honours) Examination, 2024

Semester-V (CBCS)

Mathematics

Paper: CCMA-12

(Numerical Analysis)

Time: Three Hours **Full Marks: 60**

Questions are of values as indicated in the margin.

Notations and symbols have their usual meanings.

Answer *question no. 1* and *any five* from the rest.

1. Answer any **five** questions: [5 × 2=10]

   (a) Round off the numbers 2.47235, 0.0035008, 42.3085, −72.0506 to three decimal places.

   (b) Show that the average operator μ and shift operator E are commutative.

   (c) Prove or disprove: If the n-th forward difference of a polynomial is constant, then it is of degree n.

   (d) How big should the spacing (h) be so that the error in computing $\int_0^1 e^x dx$, using the composite Trapezoidal rule, is less than or equal to 0.5×10^{-5}?

   (e) State one advantage and one disadvantage of Newton-Raphson method.

   (f) State the principle of numerical differentiation.

   (g) Prove or disprove: If A is a strictly diagonally dominant matrix of order n, then A^T is strictly diagonally dominant.

   (h) Compute $y(0.1)$, from the initial value problem $y' = x^2 - y$, $y(0) = 1$, taking $h = 0.1$, by Modified Euler's method, correct to three decimal places.

2. (a) What do you mean by memory overflow and underflow? Explain. [3]

   (b) Find the decimal equivalent to the following floating point machine number:
   0 10000001010 1001001100000000000000000000000000000000000000000000. [4]

   (c) Find absolute, relative and percentage error in $f(x) = 4\cos x - 6x - 9$ for $x = 0$ when the error in x is 0.005. [3]

3. (a) Show that $1 - e^{-hD} \equiv -\frac{\delta^2}{2} + \delta\sqrt{1 + \frac{\delta^2}{4}}$. [4]

   (b) Find a formula for $\Delta\left[\frac{f(x)}{g(x)}\right]$ and hence find $\Delta\left[\frac{2^x}{(x+1)!}\right]$, taking $h = 1$. [3]

   (c) Show that $f(E)a^x = a^x f(a)$ where $f(E)$ is a polynomial of degree n in E taking unity as the interval of spacing. [3]

4. (a) Considering the limit of the three point Lagrange's interpolating polynomial for the values $x_0, x_0 + \varepsilon$ and x_1 as $\varepsilon \to 0$, obtain the following formula

$$f(x) = \frac{(x_1 - x)(x + x_1 - 2x_0)}{(x_1 - x_0)^2} f(x_0) + \frac{(x - x_0)(x_1 - x)}{(x_1 - x_0)} f'(x_0) + \frac{(x - x_0)^2}{(x_1 - x_0)^2} f(x_1) + E(x),$$

   where $E(x) = \frac{1}{6}(x - x_0)^2(x - x_1)f'''(\xi), \xi \in (x_0, x_1)$. [5]

   (b) The following table gives pressure (P) of a steam plant at a given temperature (T).

$T(^\circ C)$:	140	150	160	170	180
$P(\text{dyn/cm}^2)$:	3.685	4.854	6.302	8.076	10.225

   Using suitable interpolation formula, compute the pressure for a temperature of 172°C correct to three decimal places. [5]

5. (a) Deduce numerical differentiation formula for the function $y = f(x)$ from Newton's backward interpolation formula at an interpolating point. Also derive the error involved in this formula. [2]

   (b) The solid of revolution obtained by rotating the region under the curve $y = f(x), a \leq x \leq b$, about the x-axis has surface area given by

$$A = 2\pi \int_a^b y\sqrt{1+y'^2}dx.$$

   Find the surface area for the function $f(x) = \cos x, 0 \leq x \leq \frac{\pi}{4}$, using Weddle's rule by taking 13-ordinates. [5]

6. (a) Discuss the convergence/divergence of the fixed point iteration method geometrically in solving one real root of the equation $f(x) = 0$ or $x = \phi(x)$, for the cases $0 < \phi'(x) < 1$, $-1 < \phi'(x) < 0$, $\phi'(x) > 1$ and $\phi'(x) < -1$. [5]

   (b) The iteration scheme

$$x_{n+1} = x_n - \frac{3\log_e x_n - e^{-x_n}}{p}$$

   is used to find the real root of the equation $e^{-x} - 3\log_e x = 0$. Determine the value of p for which the iteration scheme ensures the fastest possible convergence. [5]

7. (a) Show that the n-th order forward difference of a function $f(x)$, which is continuously differentiable sufficient number of times, is related with its n-th order derivative by the following relation

$$\Delta^n f(x) = h^n f^{(n)}(x) + O(h^n),$$

   where, h is the step length and $O(h^n)$ is a function of the form $h^n R_n(x,h)$ such that $R_n(x,h) \longrightarrow 0$ as $h \longrightarrow 0$. [5]

   (b) Find the root of the equation $\log_{10}(x) + 8 = 2x^2$ by Regula-Falsi method, correct to 6-significant figures. [5]

8. (a) Carry out the total operational count required for solving a system of n linear equations in n unknowns by Gauss-Jordan elimination method. [5]

   (b) Solve the following system of equations

$$\begin{aligned} x_1 + x_2 + 4x_3 &= 9 \\ 8x_1 - 3x_2 + 2x_3 &= 20 \\ 4x_1 + 11x_2 - x_3 &= 33 \end{aligned}$$

   using the Gauss-Seidel iteration method in error format correct to four decimal places. [5]

9. (a) If $f(x,y)$ has continuous partial derivatives upto sufficient number of times for $(x,y) \in D$ where, $D = \{(x,y) : a \leq x \leq b, -\infty < y < \infty\}$ then show that the grid error E for a single-step method in solving the initial value problem $\frac{dy}{dx} = f(x,y)$, $y(a) = y_0$ is given by

$$E = \frac{Kh^p}{M}\left(e^{(b-a)M} - 1\right),$$

   where, K, M are positive constants, p is the order of the single-step method. [5]

   (b) Compute $y(0.2)$ from the initial value problem $\frac{dy}{dt} = t + t^2y$, $y(0) = 1$ by Picard's method, correct to five decimal places. [5]

se separate answer script for each unit

B.Sc. (Honours) Examination-2024

Semester-V (CBCS)
Mathematics
Course: DSEMA-1
(Mechanics-II)

Time: Three Hours **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I (Full Marks: 40)

(Dynamics of a rigid body)

Answer ***any four*** questions.

1. (a) Given the Moments and Products of Inertia of a rigid body with respect to a set of coordinate axes, find the M.I. of the body about a line having d.cs. (l, m, n) passing through the origin. [5]

   (b) Find the M.I. of a truncated cone of mass M about its axis, the radii of its ends being a, b. [5]

2. (a) What is meant by momental ellipsoid? [3]

   (b) If α, β, γ and h be the distances of the vertices and the centre of inertia of a uniform triangular lamina of mass m from any straight line, then prove that the M.I. about that line is

$$\frac{1}{12}m\left(\alpha^2+\beta^2+\gamma^2+9h^2\right).$$

[7]

3. (a) State D'Alembert's principle. Obtain the general equations of motion of a rigid body from D'Alembert's principle. [1+4]

   (b) A thin rod of length $2a$ revolves with uniform angular velocity ω about a vertical axis through a small joint at one extremity of the rod so that it describes a cone of semi-vertical angle α. Prove that

$$\omega^2 = \frac{3g}{4a\cos\alpha}.$$

[5]

4. (a) Prove that the rate of change of angular momentum of a body about the axis of rotation is equal to the sum of the moments of all forces acting on the body about the same axis. [5]

   (b) A heavy wheel and axle is free to turn about its axis which is horizontal and fixed. A mass m is suspended at the end of a string which is coiled round the axle. Find the angular acceleration of the wheel and tension of the string. [5]

5. (a) Define the centre of suspension of a compound pendulum. Show that the length of a simple equivalent pendulum of a circular disc of radius a, oscillating about a horizontal axis tangent to it, is $\frac{5}{4}a$. [2+3]

   (b) A uniform elliptic board swings about a horizontal axis at right angle to the board and passing through one focus. If the centre of oscillation be at the other focus, prove that the eccentricity of the ellipse is $\sqrt{\frac{2}{5}}$. [5]

6. (a) Prove that the total moment of momentum of a rigid body about the origin is $Mvp + Mk^2\dot{\theta}$. [5]

   (b) A uniform solid cylinder is placed with its axis horizontal on a plane, whose inclination to the horizon is α. Show that the least coefficient of friction between it and the plane is $\frac{1}{3}\tan\alpha$, when it may roll but not slide. [5]

Unit-II (Full Marks: 20)

(Hydrostatics)

Answer ***any two*** questions.

1. (a) Define pressure of a liquid at a point. Show that an increase of pressure at any point of a liquid at rest under given external forces is transmitted equally to every part of the liquid. [1+5]

   (b) Equal volumes of two liquids of densities σ and 3σ, which do not mix, together just fill a cone which is held with its axis vertical and its vertex uppermost. Show that the pressure at any point of the base is $3 - \sqrt[3]{4}$ times the pressure at the same point when the cone is filled with the lighter liquid. [4]

2. (a) Define lines of force. Show that lines of force cut the surfaces of equi-pressure orthogonally. [1+4]

   (b) The lighter of two liquids of density σ rests on the heavier of density ρ to a depth h. A square of side l $(l > h)$ is immersed in a vertical position with one side in the surface of the upper liquid. If the thrust on the two portions of the square in contact with the two liquids be equal, prove that $\sigma h(3h - 2l) = \rho(l - h)^2$. [5]

3. (a) The forces per unit mass at (x, y, z) parallel to the co-ordinate axes are $y(a-z)$, $x(a-z)$, xy respectively. Find the surfaces of equal pressure and the curves of equal pressure and density. Also state their geometric nature. [4+1]

   (b) An area is bounded by two concentric semi-circles with common bounding diameter in the surface. Find the depth of its centre of pressure. [5]

se separate answer
script for each unit

(1+5)

B.Sc Examination-2024

Semester-V (CBCS)

Mathematics

Paper: DSE-2

(Linear Programming Problem, Game Theory and Mathematical Statistics)

Time: Three Hours **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I (Full Marks:30)

(Linear Programming Problem and Game Theory)

Answer question no 1 and ***any four*** from the rest.

1. Answer any five questions $(5 \times 2 = 10)$
   - (a) Verify if $S = \{(x, y) : x^2 + y^2 = 25\}$, a subset of E^2, is a convex set.
   - (b) What is the nature of the feasible solution of an LPP having the set of constraints
   $$-x + y \geq 3,\ x - y \geq 2 \text{ where } x, y \geq 0.$$
   - (c) What is the relation between the optimal values of primal problem and dual problem (assume that both exist)?
   - (d) If the k–th constraint of a primal problem be an equation, then what can you can you say about the nature of k–th dual variable?
   - (e) How can you establish geometrically that a set of vectors a_{ij} associated with the variables x_{ij} in a T.P are linearly independent?
   - (f) In a T.P with 3 origins and 4 destinations, map the variables $x_{11}, x_{22}, x_{33}, x_{34}, x_{24}, x_{23}$ be considered as basic variables? Give reasons.
   - (g) Is the optimal solution of an assignment problem unique? Justify your answer.
   - (h) How can you solve a restricted assignment problem?

2. (a) What is the relation between the optimum value of maximization and minimization problem? What do you mean by an optimal solution of an LPP? [1+1]

   (b) A dietitian wants to create a diet plan for their clients by combining two types of food items: Food A and Food B. The diet plan must meet the following daily nutritional requirements:

   - At least 30 grams of protein
   - At least 20 grams of fat
   - At least 40 grams of carbohydrates

   The nutritional content and cost per unit of each food item are given in the following table:

Food Item	Protein (grams)	Fat (grams)	Carbohydrates (grams)	Cost (per unit)
A	3	2	4	Rs. 400
B	4	3	5	Rs. 560

Formulate a linear programming problem to minimize the total cost of the diet pl[an] while meeting the nutritional requirements.

3. (a) Show that a hyperplane is a convex set.
   (b) State Fundamental Theorem of Linear Programming Problem.
   (c) How many solution are there of the following system of equations:

$$4x_1 + 2x_2 + 3x_3 - 8x_4 = 6$$
$$3x_1 + 5x_2 + 4x_3 - 6x_4 = 8.$$

Does there exist a basic feasible solution with x_2 and x_3 simultaneously as nonbasic variables? [1+2]

4. Consider the LPP

$$\text{Max}Z = 2x_1 + 5x_2$$

Subject to:

$$2x_1 + 5x_2 \leq 40$$
$$x_1 + 2x_2 \leq 11$$
$$x_2 \geq 4,$$
$$x_1 \geq 0, x_2 \geq 0.$$

Solve it geometrically. What is the type of convex set of feasible solution of the LPP and what is impact of that on the optimality of the objective function? [3+1+1]

5. Solve the LPP (if possible exists) by Big M-method,

$$\text{Max}Z = 3x_1 - x_2$$

Subject to:

$$-x_1 + x_2 \geq 2$$
$$5x_1 - 2x_2 \geq 2$$
$$x_1 \geq 0, x_2 \geq 0.$$

[5]

6. Solve the LPP (if possible exists) by two phase method,

$$\text{Min}Z = 4x_1 + x_2$$

Subject to:

$$x_1 + 2x_2 \leq 3$$
$$4x_1 + 3x_2 \geq 6$$
$$3x_1 + x_2 = 3$$
$$x_1 \geq 0, x_2 \geq 0.$$

[5]

7. Find the ... lem:

7. Find the optimal solution and the corresponding cost of the following transportation problem:

Origin/Destination	D_1	D_2	D_3	D_4	Supply
O_1	2	5	4	7	4
O_2	6	1	2	5	6
O_3	4	5	2	4	8
Demand	3	7	6	2	18

[5]

8. In a factory there are five operators A,B,C,D,E and the five machines I,II,III, IV, V the operating cost is given if i–th operator opeartes j–th machine $[i, j = 1, 2, \cdots 5]$. But there is a restriction that C can not be allowed to operate the third machine and similarly B can not be allowed to operate the fifth machine. The cost matrix is given below. Find the optimal assignment and the optimal assignment cost. [5]

Operators/Machines	*I*	*II*	*III*	*IV*	V
A	24	29	18	32	19
B	17	26	34	22	21
C	27	16	28	17	25
D	22	18	28	30	24
E	28	16	31	24	27

Unit-II (Full Marks: 30)

(Mathematical Statistics)

Answer ***any three*** questions.

1. (a) Define population of a random variable X and a random sample of the population. Show that if the size of a random sample is large, the distribution of the sample is the statistical image of the distribution of the population. [2+3]

   (b) For a continuous population explain what are meant by *class intervals*, *class limits*, and *grouping of data*. Describe how to obtain a graphical representation of *grouped data*. [3+2]

2. Define an estimate of a population parameter. When is an estimate said to be consistent and unbiased? For a normal (m, σ) population, show that the sample variance S^2 is a consistent but biased estimate of σ^2. Obtain an estimate of σ^2 that is both consistent and unbiased. [2+2+(4+2)]

3. (a) Define a *statistic* and sampling distribution of a *statistic*. Find the sampling distribution of the mean for the gamma (l) population. [2+3]

   (b) Prove that the maximum likelihood estimate of the parameter α of a population having density function $f(x) = 2(\alpha - x)/\alpha^2$ $(0 < x < \alpha)$ for a sample of unit size is $2x$, x being the sample value, and examine whether the estimate is biased or unbiased. [4+1]

4. (a) The population of scores of 10-year old children in a psychological performance test is known to have a standard deviation 5.2. If a random sample of size 20 shows a mean of 16.9, find 95% confidence limits for the mean score of the population, assuming that the population is normal. Given $P(U > 1.96) = 0.025$, where U is a standard normal variate.

(b) Define (i) Simple hypothesis, (ii) Composite hypothesis, and (iii) Null hypothesis, and give examples of each of them. [2+2+2]

5. (a) Define the critical region of testing a statistical hypothesis. If $x \geq 1$ is the critical region for testing $H_0 : \theta = 2$ against the alternative $H_1 : \theta = 1$ on the basis of the single observation from the population with density $f(x, \theta) = \theta e^{-\theta x}$, $0 \leq x < \infty$, obtain the values of two types of errors. [2+4]

(b) Using the Neyman-Pearson theorem construct a test of the null hypothesis $H_0 : m = m_0$ against an alternative $H_1 : m = m_1$ for a normal (m, σ) population when m_0, m_1, and σ are known and $m_0 < m_1$. [4]