B. Sc. Examination-2022

Semester-III
Mathematics
CC-6 (New)
(Algebra-III)

Time: Three Hours **Full Marks: ~~40~~ 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer ***all three*** questions.

1. Answer any two questions.$(10 \times 2 = 20)$

   (a) i. How many distinct $r-$cycles are there in a symmetric group S_n? [4]

   ii. Find the order of the permutation $(1\ 2\ 3\ 4)o(5\ 6)$ in the symmetric group S_6. [2]

   iii. Find a conjugate element of $(2\ 3\ 6\ 8)$ other than itself in S_8. [2]

   iv. How many elements of order 2 are there in S_4? [2]

   (b) i. Let H be a subgroup of a finite group G. Prove that $[G:H] = \frac{|G|}{|H|}$. [3]

   ii. Let $\mathbf{C}^*$ be the group of nonzero complex numbers under multiplication and $H = \{a + bi \in \mathbb{C}^* : a^2 + b^2 = 1\}$ be a subgroup of $\mathbf{C}^*$. Give a geometric description of the coset $(3 + 4i)H$. [2]

   iii. State Cauchy's Theorem on finite groups. Let H and K be two subgroups of an Abelian group. If $|H| = 12$ and $|K| = 18$ then prove that $H \cap K$ is cyclic. [1+4]

   (c) i. Write down the Caley table for the quotient group $\frac{\mathbb{Z}_{18}}{<[6]>}$. Hence find the order of $[13]+ < [6] >$ in $\frac{\mathbb{Z}_{18}}{<[6]>}$. [4+1]

   ii. Let H be a proper subgroup of a group G such that for all $x, y \in G \backslash H,\ xy \in H$. Show that H is normal in G. [2]

   iii. Show that A_4 is the only subgroup of S_4 of order 12. [3]

2. Answer any two questions.$(10 \times 2 = 20)$

   (a) i. Prove that a finite ring without zero divisor contains an unity. [4]

   ii. Give an example of a non commutative ring having exactly 81 elements. Justify your answer. [3]

   iii. Find all units (if exists)of the ring $S = \left\{ \begin{pmatrix} a & a \\ a & a \end{pmatrix} | a \in \mathbb{Q} \right\}$. [3]

   (b) i. Prove that $\mathbb{Z}_n$ is a field if and only if n is prime. [4]

   ii. Give an example of an infinite ring with characteristic 2. Justify your answer. [3]

   iii. Let $(F, +, \cdot)$ be a field of characteristic p, where p is prime. Show that $|F| = p^m$ for some $m \in \mathbb{N}$. [3]

   (c) i. Show that a commutative ring with unity is a field if and only if it has no proper ideals. [2+3]

   ii. Give an example of subring S of a ring R such that S contains an unity but R does not contains an unity. Justify your answer. [2]

   iii. Let R is a Boolean ring. Then find the value of $(a + b)ab$ for $a, b \in R$. [3]

3. Answer any two questions.$(10 \times 2 = 20)$

(a) i. Let $V = \{(x, y) : x, y \in F\}$, where F is a field. Define addition of elements of V coordinatewise, and for $c \in F$ and $(x, y) \in V$, define $c(x, y) = (x, 0)$. Is V a vector space over F with these operations? Justify your answer [3]

ii. Determine whether $u = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \end{pmatrix}$ and $v = \begin{pmatrix} 2 & 2 & 2 \\ 3 & 3 & 3 \end{pmatrix}$ are linearly dependent or not. Find a subset S of $\mathbb{R}$ which is linear dependent in the vector space $\mathbb{R}_{\mathbb{R}}$ but linear independent in the vector space $\mathbb{R}_{\mathbb{Q}}$. [2+2]

iii. Find two subspaces X and Y of $\mathbb{R}^3$ such that $\mathbb{R}^3 = X + Y$ but $\mathbb{R}^3 \neq X \oplus Y$. [3]

(b) i. Show that every finitely generated vector space has a basis. [4]

ii. Prove that $\mathbb{R}_{\mathbb{Q}}$ is not a finite dimensional vector space [3]

iii. Extend the set $\{(-2,\ 1,\ 1,\ 0), (-1,\ 2,\ 0,\ 1)\}$ to a basis of $\mathbb{R}^4$. [3]

(c) i. Show that for each subspace W of a vector space V_F there is a subspace W_1 of V_F such that $V = W \oplus W_1$. [4]

ii. Find the dimension of the quotient space $p_{10}(t)/p_5(t)$, where $p_n(t)$ denotes the vector space of all polynomials of degree less or equal to n over $\mathbb{R}$. [3]

iii. Find $\left\| \begin{pmatrix} i & 1 \\ 2+i & i \end{pmatrix} \right\|$ in the inner product space $M_{2\times2}(C)$, with the inner product defined by $< A, B >= tr(B^*A)$ for $A, B \in M_{2\times2}(C)$. [3]

B. Sc. (Honours) Examination-2022
Semester-III(CBCS)
Mathematics
Paper: CCMA-5
(Analysis-III)

Time: 3 Hours **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer *any six* questions.

1. (a) What is the sequence of partial sums of a series $\sum a_n$? Show that a series $\sum a_n$ of non-negative terms is convergent if and only if the sequence of its partial sums is a bounded sequence.
(b) Examine the convergence of the series $\sum\{(n^3+1)^{\frac{1}{3}} - n\}$ and $\sum \log(1+\frac{1}{n})$. [(1+4)+(3+2)]

2. (a) If $\sum a_n$ is a series of positive terms and $\lim\limits_{n\to\infty} n \log \frac{a_n}{a_{n+1}} > 1$, show that $\sum a_n$ must converge.
(b) Find the values of α for which the series $\sum\limits_{n=2}^{\infty} \frac{1}{(\log n)^{\alpha}}$ converges.
(c) Discuss the convergence of the series $\sum \frac{1}{3n+2}$. [4+4+2]

3. (a) What do you mean by absolutely and conditionally convergent series? Show that the series $\sum \frac{(-1)^{n-1}n}{n^2+1}$ is conditionally convergent.
(b) Let $\sum a_n$ be an absolutely convergent series and $\{b_n\}$ be a bounded sequence. Show that $\sum a_n b_n$ is absolutely convergent. [(2+4)+4]

4. (a) Using ϵ-δ definition, explain when a function f is not continuous at a point c.
(b) Show that the function $f:\mathbb{R}\longrightarrow\mathbb{R}$ defined by $f(x) = \begin{cases} 1 & \text{if } x\in\mathbb{Q} \\ 0 & \text{if } x\notin\mathbb{Q}\end{cases}$ is not continuous at any point of $\mathbb{R}$.
(c) Let $f:[a,b]\longrightarrow\mathbb{R}$ be continuous on $[a,b]$. Prove that f is bounded on $[a,b]$. [2+4+4]

5. (a) Let $f,g:\mathbb{R}\longrightarrow\mathbb{R}$ be continuous on $\mathbb{R}$. Prove that the set $S=\{x\in\mathbb{R}: f(x)=g(x)\}$ is closed in $\mathbb{R}$.
(b) Prove that the set of points of discontinuities of a monotone function on a closed and bounded interval is a countable set. [4+6]

6. (a) When is a function said to be uniformly continuous on an interval? Show that a continuous function $f:(a,b)\longrightarrow\mathbb{R}$ becomes uniformly continuous on (a,b) if and only if f admits of a continuous extension to $\mathbb{R}$.
(b) Check the continuity and uniform continuity of the function $f:\mathbb{Q}\longrightarrow\mathbb{R}$ given by $f(x) = \begin{cases} 0 & \text{if } x<\sqrt{5} \\ 1 & \text{if } x>\sqrt{5}.\end{cases}$ [(2+4)+4]

7. (a) Let $f:[a,b]\longrightarrow\mathbb{R}$ be continuous on $[a,b]$ and let f' be bounded on (a,b). Prove that f is function of bounded variation on $[a,b]$. Is the boundedness of f' necessary here? Justify your answer.
(b) Prove or disprove: A function $f:[a,b]\longrightarrow\mathbb{R}$ of bounded variation can be expressed as the difference of two monotone increasing functions on $[a,b]$. [(4+2)+4]

8. (a) For a bounded function $f:[a,b]\longrightarrow\mathbb{R}$, prove that the lower Riemann integral cannot exceed the upper Riemann integral. Explain when they become equal.
(b) Find the lower and upper Riemann integrals of the function $f(x)=e^x$ on $[a,b]$ and detemine if f is Riemann integrable on $[a,b]$. [4+6]

9. (a) If $f:[a,b]\longrightarrow\mathbb{R}$ is bounded on $[a,b]$, prove that f is Riemann integrable on $[a,b]$ if and only if for each $\epsilon>0$, there exists a $\delta>0$ such that $U(P,f)-L(P,f)<\epsilon$ for every partiton P of $[a,b]$ with $||P||<\delta$.
(b) Show by an example that a bounded function which is continuous on $[a,b]$ execpt for an infinite set $S\subset[a,b]$ having infinite number of limit points may be Riemann integrable on $[a,b]$. [5+5]

B.Sc. (Honours) Examination, 2022

Mathematics

Semester - III

Paper/Course SECMA - 1

Boolean Algebra and Circuit Design

Time: Two Hours **Full Marks: 25**

Questions are of value as indicated in the margin.
Notations and symbols have their usual meaning.

Answer *any five* questions

1. Define Boolean algebra. Give an example of your choice. Explain some unusuality in its definition. Would it be possible to have a three element Boolean algebra? Define a Boolean expression. [1+1+1+1+1=5]

2. Show from definition that elements of a Boolean algebra obey association law, absorption law and De-Morgan's law. [2+2+1=5]

3. Define equivalence of two Boolean expressions. Define complete conjunctive normal form of a Boolean expression. Show that there are only a finite number of distinct Boolean expressions in n Boolean variables. [2+2+1=5]

4. Define a Boolean function. Define a Karnaugh map. State how Boolean functions are represented in a Karnaugh map. Use Karnaugh map to represent the Boolean function $NAND(x, y, z)$. [1+1+3=5]

5. Minimize the function $f(x, y, z, w) = \Sigma(0, 2, 8, 9, 10, 12, 13, 14)$ and implement the same using two-input NAND gates. [2+3=5]

6. Design a full adder logic circuit. [5]

7. State the function of flip-flop in the design of digital circuits. Draw the block diagram, state table, state equation, external state diagram, steering table and the $NAND$ implementation of a JK-ff. [1+4=5]

8. State the functions of an SR-ff. Design an SR-ff from $NAND$ gates. Determine the logic levels at each output of a gate. [1+3+1=5]

B. Sc.(H) Examination-2022

Semester-III

Mathematics

Core Course: CC-7

(Differential Equations-I)

Time: Three Hours **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer Question No. 1 and any *four* from the rest.

1. Answer any *ten* questions from the following: [$10 \times 2 = 20$]

(a) Solve: $(D^2 + 2D + 5)^2(D^2 + 5D + 4)^3(D + 1)^2 y = 0$. [2]

(b) Find a third order linear homogeneous differential equation whose linearly independent solutions are e^x, e^{-2x}, and e^{3x}. [2]

(c) Find the orthogonal trajectories of the family of parabolas $y^5 = cx^7$, c being the parameter. [2]

(d) Find three different solutions of the differential equation $\frac{dy}{dx} = xy^{1/2}$, $y(0) = 0$. [2]

(e) Find an integrating factor of the differential equation $(xy^2 - e^{\frac{1}{x^3}})dx - (x^2 y)dy = 0$. [2]

(f) Find the differential equation of the circles $(x - \alpha)^2 + (y - \beta)^2 = r^2$, where α and β are arbitrary constants, and r is a fixed constant. [2]

(g) Solve the differential equation $x^2(\frac{dy}{dx})^2 + 5xy\frac{dy}{dx} + 4y^2 = 0$. [2]

(h) Solve: $\frac{dy}{dx} = \frac{y}{x} + cot(\frac{y}{x})$. [2]

(i) Consider the differential equation $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = 0$. If $W(1) = 9$, then find the value of $W(4) - W(2)$. [2]

(j) Find an integrating factor of the second order ordinary differential equation $2x^2(x + 1)\frac{d^2y}{dx^2} + x(7x + 3)\frac{dy}{dx} - 3y = x^2$. [2]

(k) Let $x(t)$ be the solution of the differential equation $\frac{dx}{dt} = (x - 5)(x - 7)$ satisfying the initial condition $x(0) = 6$. Find the value of $x(t)$ when $t \to -\infty$. [2]

(l) Solve: $\frac{d^2y}{dx^2} = f(y)$. [2]

(m) Find the condition under which the general solution of the differential equation $\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$ tends to zero as $x \to \infty$. [2]

(n) Solve the differential equation $\frac{dy}{dx} = 5x\left(1 - \frac{x}{2}\right)$, $y(0) = 1$, and find $y(x)$ when $x \to \infty$. [2]

2. (a) Solve the second order linear differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = sin(2x) + e^{2x}$. [4]

(b) A certain radioctive material is known to decay at a rate proportional to the amount present. If initially there is 50 miligrams of the material present and after two hours it is observed that the material has lost 10% of its original mass, then find the mass of the material after four hours. [2]

(c) Construct a direction field and sketch a few solution curves of the differential equation $\frac{dy}{dx} = 2y - x$. [2]

(d) Solve: $\frac{dy}{dx} = y^{\frac{1}{3}}(sin2x)$, $y(0) = 0$. [2]

3. (a) Find the singular solution and extraneous loci of the differential equation $x^3p^2 + x^2yp + 27 = 0$. [4]

(b) Using the method of undetermined coefficients, solve the equation $(D^2 + D - 6)y = -18e^{3x} - 6x - 11$. [3]

(c) Find the orthogonal trajectories of the family of curves $r^n cos(n\theta) = c^n$, c being the parameter. [3]

4. (a) Using the method of variation of parameters, solve the equation $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 log(x)$, given that $y = x^3$, and $y = \frac{1}{x}$ are two linearly independent solutions of the corresponding homogeneous equation. [3]

(b) If u and v are any two solutions of the equation $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$ on an interval $[a, b]$, then prove that their Wronskian $W(u, v)$ is either identically zero or never zero on $[a, b]$. [3]

(c) Solve: $y = 2x\frac{dy}{dx} + x^4(\frac{dy}{dx})^8$. [3]

(d) Solve the boundary value problem $\frac{d^2y}{dx^2} + y = 0, y(0) = 0, y(\frac{\pi}{2}) = 1$. [1]

5. (a) Solve the Cauchy-Euler equation $x^2y'' - 4xy' + 6y = log(x^2)$, $x > 0$. [3]

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(b) Prove that the transformation $v = y^{1-n} (n \neq 0 \text{ or } 1)$ reduces the Bernoulli equation $\frac{dy}{dx} = P(x)y + Q(x)y^n$ to a linear equation in v. Hence solve the equation $x\frac{dy}{dx} + y = x^2y^2$. [4]

(c) If $e^{\int \phi(\frac{x}{y}) d(\frac{x}{y})}$ is an integrating factor of the differential equation $M(x,y)dx + N(x,y)dy = 0$, then find the expression of $\phi(\frac{x}{y})$. [3]

6. (a) Solve the differential equation $\frac{dy}{dx} + y = f(x)$ with initial condition $y(0) = 6$, where $f(x) = 5$, when $0 \leq x < 10$ and $f(x) = 1$, when $x \geq 10$. Hence find the value of $y(12)$. [3]

(b) Reduce the equation $xy(y - px) = x + py$ to Clairaut's form by the substitution $x^2 = u$ and $y^2 = v$. Hence solve the equaion. [3]

(c) Show that in a first order and first degree homogeneous differential equation $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$, the variables can be separated by the substitution $y = vx$. Hence solve the initial value problem $(y + \sqrt{x^2 + y^2})dx - xdy = 0,\ y(2) = 0$. [4]

7. (a) Let $y(x)$ be the solution of the differential equation $(xy + y + e^{-x})dx + (x + e^{-x})dy = 0$ satisfying the condition $y(0) = 1$. Then find the value of $y(-1)$. [3]

(b) Prove that the second order linear differential equation $p_0(x)\frac{d^2y}{dx^2} + p_1(x)\frac{dy}{dx} + p_2(x)y = 0$ is exact iff $p_0''(x) - p_1'(x) + p_2(x) = 0$. [4]

(c) Solve: $\frac{d^2y}{dx^2} + y = x^2$. [2]

(d) Convert the differential equation $y = xf(p) + \phi(p)$ to a linear equation in x. [1]

x

Use separate answer script for each unit

B. A./B. Sc. Examination-2022

Semester-III
Mathematics
Elective Course: GEC-3
(Differential Equations and Its Applications)

Time: Three Hours | Full Marks: 60

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I (Ordinary Differential Equation)

Answer *any four* questions.

1. (a) Show that the equation $\frac{dy}{dx} = 3xy^{\frac{1}{3}}$, $y(0) = 0$ does not have a unique solution. [3]

   (b) Form the differential equation of the one parameter family of curves
$$\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1,$$
λ being the parameter of the family. [3]

   (c) Solve: $(x^3 + xy^2 + m^2y)\,dx + (y^3 + yx^2 + m^2x)\,dy = 0$. [4]

2. (a) Describe the following equations by giving their order, degree and stating whether they are linear or non-linear:

   (i) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 9\,sin y = 0$,

   (ii) $\left(\frac{d^2y}{dx^2}\right)^7 = \left(\frac{d^2y}{dx^2}\right)^3$. [1+1]

   (b) Find the orthogonal trajectories of the family of curves
$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = r^{\frac{2}{3}},$$
r being the parameter of the family. [4]

   (c) Solve: $xy\,dx + (2x^2 + 3y^2 - 20)\,dy = 0$. [4]

3. (a) Reduce the equation
$$sin\,y\,\frac{dy}{dx} = cos\,x\,(2\,cos\,y - sin^2 x)$$
to a linear equation and hence solve it. [5]

   (b) Solve: $xy\left\{\left(\frac{dy}{dx}\right)^2 - 1\right\} = (x^2 - y^2)\frac{dy}{dx}$. [5]

4. (a) Solve: $(D-1)^3\,(D^2-4)\,(D+2)\,y = 0$. [2]

   (b) Compute the Wronskian $W(y_1, y_2)$ of the functions
$$y_1 = e^{2x}\,sin(2x), \text{ and } y_2 = e^{-2x}\,cos(2x).$$
[1]

   (c) Solve: $\frac{d^3y}{dx^3} + \frac{1}{x}\frac{d^2y}{dx^2} + 2\left(\frac{d^2y}{dx^2}\right)^2 = 0$. [4]

   (d) Solve the differential equation
$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x sin\,x.$$
[3]

5. (a) Solve: $\frac{d^2y}{dx^2} = sec^2y\ tan\ y$. [3]

(b) Solve the equations

$$\frac{dx}{mz - ny} = \frac{dy}{nx - lz} = \frac{dz}{ly - mx}.$$

[2]

(c) By the method of variation of parameters, solve the differential equation

$$\frac{d^2y}{dx^2} + 4y = 4\ tan(2x).$$

[5]

6. (a) Show that the differential equation

$$(2x^2 + 3x)\frac{d^2y}{dx^2} + (6x + 3)\frac{dy}{dx} + 2y = (x + 1)e^x$$

is exact and hence solve it. [1+4]

(b) Solve the simultaneous equations

$$\frac{dx}{dt} + 5x - 2y = e^t,$$
$$\frac{dy}{dt} - x + 6y = e^{2t}.$$

[5]

Unit-II (Partial Differential Equations)

Answer *any two* questions.

1. (a) Form partial differential equation by eliminating arbitrary function ϕ from

$$xz^2 = e^{(2x+3y)}\phi\left(\frac{x}{y} + \frac{y}{z} - \frac{z}{x}\right).$$

What kind of partial differential equation is this? [3+1]

(b) Find the order and degree of the following partial differential equations:

(i) $\left(\frac{\partial^2 z}{\partial x^2} + \left(\frac{\partial z}{\partial x}\right)^{\frac{1}{3}}\right)^{\frac{1}{5}} = z^{\frac{1}{15}} + xz^2 + y^3.$

(ii) $\left(\frac{\partial^3 z}{\partial y^3}\right)^{\frac{1}{4}} + xz\frac{\partial^2 z}{\partial x^2} + sin\left(\frac{\partial z}{\partial x}\right) = x^2e^z + y.$

[1+1]

(c) Solve the partial differential equation

$$x^3(y - z)p + y^3(z - x)q + z^3(y - x) = 0.$$

[4]

2. (a) Form partial differential equation by eliminating arbitrary constants from the equation

$$z^2(1 + a^2) = (ax + y + b)^2.$$

[3]

(b) Solve the partial differential equation

$$(z^2 - 2yz - y^2)p + x(y + z)q = x(y - z).$$

[3]

(c) Solve the partial differential equation $xq = 2xy + log(p)$ using Charpit's method. [4]

3. (a) Find the equation of the integral surface given by the partial differential equation

$$(x - y)y^2p + (y - x)x^2q = (x^2 + y^2)z,$$

which passes through the curve $y = 0$, $xz = a^3$. [5]

(b) Solve the partial differential equation $xzq^2 - p = 0$. [5]