Use separate answer script for each unit

B.Sc. (Honours) Examination-2022

Semester-V (CBCS)
Mathematics
Course: DSE-2
(Linear Programming Problem, Game Theory and Mathematical Statistics)

Time: Three Hours **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I (Full Marks: 30)
(Linear Programming Problem, Game Theory)

Answer *any three* questions.

1. (a) Define the convex combination of a set of vectors. Prove that set of all convex combinations of a finite number of linearly independent vectors $\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_m$ is a convex set. [1+4]

   (b) Use a graphical method for obtaining an optimal solution of the LPP

   Maximize $Z = x_1 + x_2$

   subject to

$$\begin{aligned} x_1 + x_2 &\leq 1, \\ -3x_1 + x_2 &\geq 3, \\ x_1, x_2 &\geq 0, \end{aligned}$$

   if exists. [3]

   (c) Define the vertex of a convex set. Give an example of a convex set in which all boundary points are vertices. [1+1]

2. (a) Establish that whenever an LPP has a basic feasible solution, if we drop one of the basis vectors and append a suitable non-basis vector in the basis set, the new solution is also a basic feasible solution. [6]

   (b) Examine whether $x_1 = 2,\ x_2 = 1$ is a basic feasible solution to

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 4 \\ 2x_1 + x_2 + 5x_3 &= 5. \end{aligned}$$

   Find other basic feasible solutions, if available. [4]

3. (a) Prove that a basic feasible solution to an LPP must correspond to an extreme point of the convex set of all feasible solutions. [4]

   (b) Obtain the optimal solution to the LPP

   Minimize $Z = x_1 + x_2$

   subject to

$$\begin{aligned} 2x_1 + x_2 &\geq 4, \\ x_1 + 7x_2 &\geq 7, \\ x_1, x_2 &\geq 0. \end{aligned}$$

   if it exists, by Big-M or the two-phase method. Justify the answer by solving the same by graphical method. [6]

4. (a) Use duality to solve the LPP
Maximize $Z = 8x_1 + 6x_2$
subject to

$$\begin{aligned} x_1 - x_2 &\le \frac{3}{5}, \\ x_1 - x_2 &\ge 2, \\ x_1, x_2 &\ge 0. \end{aligned}$$

if the optimal solution exists. [5]

(b) In a factory, there are four workers A, B, C, D and five machines M_1, M_2, M_3, M_4, M_5. The operating cost for each worker to handle each machine is given in the table below. Find the optimal assignment and the assignment cost. Also, find which machine will remain idle.

	M_1	M_2	M_3	M_4	M_5
A	12	15	14	11	16
B	15	12	14	13	15
C	16	18	12	15	15
D	12	14	13	13	14

[5]

5. (a) For $C = [c_{ij}]_{n\times n}$, a cost matrix for an assignment problem, if another matrix is constructed in the form $C^* = [c^*_{ij}]_{n\times n}$ where $c^*_{ij} = c_{ij} + \alpha_i + \beta_j$, α_i, β_j being arbitrary constants, prove that the optimal solution of C^* will be identical with that of C. [3]

(b) Can pure strategies be applied to obtain the solution of a game with the following payoff matrix? Justify the answer. Then use dominance to reduce the payoff matrix and find the value of the game.

	B			
A	−1	−1	2	1
	2	2	0	1
	3	−2	1	−2
	2	1	−3	2

[2+5]

Unit-II (Full Marks: 30)

(Mathematical Statistics)

Answer *any three* questions.

1. (a) Define a *random sample* of a population. Show that if the size of a random sample is large, the distribution of the sample approximates to that of the population. [1+2]

(b) Define an *estimate* of a population characteristic. When is an estimate said to be consistent and biased? [1+(1+1)]

(c) If Γ is an unbiased estimate of a population parameter γ, show that Γ^2 is a biased estimate of γ^2; but if Γ is a consistent estimate of γ then show that Γ^2 is also a consistent estimate of Γ^2. [2+2]

2. (a) For a continuous population explain what are meant by *class intervals*, *class limits*, and *grouping of data*. Describe how to obtain a graphical representation of *grouped data*. [3+2]

(b) Define a *statistic* and sampling distribution of a *statistic*. Find the sampling distribution of the mean for the binomial (N, p) population. [(1+1)+3]

3. (a) A random variable X can take all non-negative integral values and $P(X = i) = p(1-p)^i$, $i = 0, 1, 2, ...$, where p $(0 < p < 1)$ is a parameter. Find the maximum likelihood estimate of p on the basis of a sample of size n from the population of X. Examine whether the estimate is consistent. [3+2]

(b) Show that approximate confidence limits for large samples for the parameter μ of a Poisson population having confidence coefficient $1-\epsilon$ are given by the roots of the quadratic equation in μ: $n(\bar{x}-\mu)^2 = u_\epsilon^2 \mu$, which, to the order of $1/\sqrt{n}$, are approximately $\bar{x} \pm u_\epsilon \sqrt{\bar{x}/n}$, where $\bar{x}$ is the mean of a sample of size n and u_ϵ is given by

$$\int_{-u_\epsilon}^{u_\epsilon} \phi(x)dx = 1 - \epsilon$$

with $\phi(x)$ denoting the standard normal density function. [3+2]

4. (a) Define (i) Simple hypothesis, (ii) Composite hypothesis, and (iii) Null hypothesis and give examples of each of them. [2+2+2]

(b) The probability density function of a population with parameter α is given by

$$f(x, \alpha) = \alpha e^{-\alpha x}, \ 0 \leq x < \infty, \ \alpha > 0.$$

The null hypothesis $H_0 : \alpha = 2$ will be tested against the alternative $H_1 : \alpha > 2$ on the assumption that H_0 should be rejected for a sample $x \geq 6$. Find the probability of Type-I error and the power of the test. [2+2]

5. State the Neyman-Pearson theorem of statistical testing. Using it construct a test of the null hypothesis $H_0 : m = m_0$ against an alternative $H_1 : m = m_1$ for a normal (m, σ) population when m_0, m_1, and σ are known and $m_0 \neq m_1$. [2+8]

B.Sc. (Honours) Examination, 2022

Semester-V (CBCS)

Mathematics

Course: CCMA-12

(Numerical Analysis)

Time: Three Hours | Full Marks: 60

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer *any six* questions.

1. (a) What do you mean by significant figure of an approximate number? State the rules of significant figures with suitable examples. [1+4]

   (b) Find the decimal equivalent to the following floating point machine number:
   1 10000100 10100001111000000000000 [3]

   (c) Calculate $\sqrt{25.11} - \sqrt{25.1001}$ correct to three significant figures, providing necessary steps. [2]

2. (a) If the 3rd order differences of $f(x)$ be constant and $f(-1) = -1,\ f(0) = 0,\ f(1) = 1,\ f(2) = 8$ and $f(3) = 27$, find $f(4)$ using difference table. [3]

   (b) Prove that $\delta(E^{\frac{1}{2}} + E^{-\frac{1}{2}}) \equiv \Delta E^{-1} + \Delta$. [3]

   (c) What do you mean by 'round-off errors' in numerical data? Show how these errors propagated in a difference table. List your observations from the difference table. [4]

3. (a) Establish Lagrange's interpolation formula when the functional values $y = f(x)$ are known at $(n+1)$ points. Also derive the error involved in this formula [6]

   (b) The population of a city are given below:

Year	1971	1981	1991	2001	2011	2021
Population (in lakh)	46.52	66.23	81.01	93.70	101.58	120.92

   Using suitable interpolation formula estimate the population of the city for the year 2015. [4]

4. (a) Why we need to study numerical differentiation? Deduce numerical differentiation formula from Newton's forward interpolation formula at an interpolating point. Also derive the error involved in this formula. [1+3+2]

   (b) Show that $\nabla y_{n+1} = h\left[1 + \frac{1}{2}\nabla + \frac{5}{12}\nabla^2 + \cdots\right] Dy_n$, where D is the differential operator. [4]

5. (a) Establish Trapezoidal rule from Newton-Cotes quadrature formula (closed type). Estimate the error involved in this rule. [6]

   (b) The following table gives the values of acceleration (f) of a particle in cm/sec^2 at equal interval of time (t) in sec. Find the velocity of the particle at 2 secs.

t	0.0	0.5	1.0	1.5	2.0
f	0.3989	0.3521	0.2420	0.1295	0.0540

   [4]

6. (a) Explain the Regula-Falsi method to determine approximately one real root of an equation $f(x) = 0$. Give its geometric interpretation and discuss its convergence. [2+2+4]

(b) The equation $f(x) = 3x^3 + 4x^2 + 4x + 1 = 0$ has a root in $(-1, 0)$. Determine an iteration function $\phi(x)$ such that the sequence $x_{k+1} = \phi(x_k)$, $x_0 = -\frac{1}{2}$, $k = 0, 1, 2, \cdots$, converges to the root. [2]

7. (a) Can bisection method be used to find the roots of the equation $f(x) = \sin x + 1 = 0$? Why or why not? [2]

(b) What do you mean by degree of precision of a quadrature formula? Find the degree of precision of the following quadrature formula

$$\int_{-1}^{1} f(x)dx = f\Big(-\frac{\sqrt{3}}{3}\Big) + f\Big(\frac{\sqrt{3}}{3}\Big).$$

[1+3]

(c) Construct the quadratic polynomial that interpolates the function $f(x) = \sin\frac{\pi x}{2}$ at $x = -1, 0, 1$. Also show that $|f(x) - P(x)| \leq \frac{\pi^3}{72\sqrt{3}}$ for $|x| \leq 1$. [1+3]

8. (a) Show that the estimate of error of the linear iteration method in the approximate value x_n of one real root of the equation $f(x) = 0$ is given by

$$|\xi - x_n| \leq \frac{m}{1-m}|x_n - x_{n-1}|,$$

where ξ is a root of the equation and m is positive proper fraction. [4]

(b) Solve the differential equation $\frac{dy}{dx} = \frac{x^2}{1+y^2}$ with the condition $y(0) = 0$ by Picard's method to find $y(0.25)$ correct to 3 decimal places. [3]

(c) Find an expression for the iteration matrix Q_{GS} by Gauss-Seidel iteration method $\vec{x}^{(k)} = Q_{GS}\vec{x}^{(k-1)} + \vec{c}, k \geq 1$ for solving the linear algebraic system of equations given by the single matrix equation $A\vec{x} = \vec{b}$. You must mention all the matrices involved in the process. [3]

9. (a) What do you mean by partial pivoting? Carry out the total operational count required for solving a system of n linear equations in n unknowns by Gaussian elimination method. [5]

(b) Solve the following system of equations

$$5.09x_1 + 3.46x_2 + 1.09x_3 = 1.28$$
$$2.82x_1 + 6.46x_2 - 4.27x_3 = 4.65$$
$$1.27x_1 - 3.09x_2 + 7.54x_3 = 2.19$$

using Gauss-Jordon elimination method. [5]

10. (a) Derive the second order Runge-Kutta method for solving the initial value problem $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$. [5]

(b) Compute $y(1.2)$, from the differential equation $\frac{dy}{dx} = 0.05x^{-2} - y^2, y(1) = 1$, taking $h = 0.05$, by Modified Euler's method, correct to four decimal places. [5]

Use separate answer script for each unit

B. Sc. (Honours) Examination-2022

Semester-V (CBCS)
Mathematics
Paper: CCMA 11
(Analysis-V and Differential Equations-III)

Time: 3 Hours **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I (Analysis-V)
(Full Marks: 30)
Answer ***any three*** questions.

1. (a) Prove that the improper integral $\int_a^b \frac{dx}{(x-a)^n}$ converges if and only if $n < 1$.

   (b)When is an improper integral $\int_a^b f dx$ called absolutely convergent and conditionally convergent? Show that the improper integral $\int_0^\infty \frac{\sin x}{x} dx$ is conditionally convergent. [4+(2+4)]

2. (a) Show that the improper integral $\Gamma(n) = \int_0^\infty x^{n-1}e^{-x}dx$ is convergent if and only if $n > 0$. Also derive that $\Gamma(n+1) = n\Gamma(n)$.

   (b) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be uniformly continuous on $\mathbb{R}$. For each n, define $f_n : \mathbb{R} \longrightarrow \mathbb{R}$ by $f_n(x) = f(x + \frac{1}{n})$. Prove that $\{f_n\}$ converges uniformly to f on $\mathbb{R}$. [(4+2)+4]

3. (a) Prove or disprove: The limit function of a uniformly convergent sequence of continuous functions on $\mathbb{R}$ is continuous on $\mathbb{R}$.

   (b) Let $f_n(x) = n^2x(1-x^2)^n$, $x \in [0,1]$. Show that $\{f_n\}$ converges pointwise to a function f (say) on $[0,1]$. Compute, if possible, $\int_0^1 f_n dx$ and $\int_0^1 f dx$, and check if $\{f_n\}$ converges uniformly to f on $[0,1]$. [4+6]

4. (a) Let f be the limit function of a pointwise convergent series $\sum f_n$ of continuously differentiable functions on a closed and bounded interval $[a,b]$ and let $\sum f_n'$ converge uniformly to a function g on $[a,b]$. Prove that $f'(x) = g(x)\ \forall x \in [a,b]$.

   (b) Prove that the series $\sum(-1)^n \frac{x^2+n}{n}$ is uniformly convergent on any closed and bounded interval $[a,b]$ but is conditionally convergent on $\mathbb{R}$. [5+5]

5. (a) Determine the radius of convergence and interval of convergence of the power series $\sum_{n=2}^\infty \frac{(n!)^2}{(2n)!}(x-4)^n$.

   (b) Find the Fourier series of the function $f : (-\pi, \pi) \longrightarrow \mathbb{R}$ defined by

$$f(x) = \begin{cases} 0 & \text{if} \quad -\pi < x < 0 \\ 1 & \text{if} \quad 0 < x < \pi \end{cases}$$

   on $(-\pi,\pi)$ and deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$. [4+(4+2)]

Unit-II (Differential Equations-III)
(Full Marks: 30)
Answer question number 1 and *any two* from the rest.

1. Answer *any five* questions.
(a) Define with example of each of the following types of first order partial differential equation (PDE) in two variables:
(i) Quasilinear (ii) Nonlinear.
(b) Form the partial differential equation by the elimination of arbitrary function ψ from $\frac{x^2+y^2+z^2}{z} = \psi(\frac{y}{z})$; $z = z(x,y)$.
(c) Find the value of $z(1,3)$ corresponding to the Cauchy value problem $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$; $z(0,y) = -e^{y^2}$.
(d) Find the family of surfaces orthogonal to the family of surfaces whose PDE is $(y+z)\frac{\partial z}{\partial x} + (z+x)\frac{\partial z}{\partial y} = x+y$.
(e) Show that $z = c_1x + (2022 - \sqrt{c_1})^2 y + c_2$ is the complete integral of the PDE $\sqrt{\frac{\partial z}{\partial x}} + \sqrt{\frac{\partial z}{\partial y}} = 2022$.
(f) Show that the PDE $z = px + qy$ is compatible with any PDE of the form $f(x,y,z,p,q) = 0$ which is homogeneous in x, y, z.
(g) What are the limitations of Charpit's method for solving a first-order nonlinear partial differential equation?
(h) Find the particular integral (PI) of the PDE

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^3 z}{\partial y^3} = \sin(2x+y).$$

[5 × 2=10]

2. (a) For the PDE

$$z^2 - z\frac{\partial z}{\partial x} + z\frac{\partial z}{\partial y} + (x+y)^2 = 0,$$

find the general integral and the integral surface through the curve $z(x,0) = \sqrt{2x+1}$.
(b) Using Charpit's method, find the complete integral of the following PDE:

$$2\left(x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right) = y\left(\frac{\partial z}{\partial x}\right)^2 - 2z.$$

[(3+2)+5]

3. (a) Show that the PDEs $[(\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2]y = z\frac{\partial z}{\partial y}$ and $(\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2 = 4$ are compatible on a specified domain and hence find the one-parameter family of common solutions.
(b) Find the singular integral, if any of the following PDE:

$$z = x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} - \frac{1}{2022}\sqrt{\frac{\partial z}{\partial x}\frac{\partial z}{\partial y}}.$$

[(2+3)+5]

4. (a) Find the initial strip of the following PDE passing through the straight line $C: x_0 = 2s, y_0 = 2s, z_0 = 5s$ in the context of Cauchy's method of characteristic:

$$\frac{\partial z}{\partial x}\frac{\partial z}{\partial y} = 1.$$

(b) Let $z(x,y)$ be the solution of the following Cauchy problem:

$$\frac{\partial z}{\partial y} - x\frac{\partial z}{\partial x} + z - 1 = 0 \text{ where } -\infty < x < \infty,\ y \geq 0 \text{ and } z(x,0) = \sin(x).$$

Find the value of $z(0,1)$ and hence comment on the nature of the solution.
(c) Solve:

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial y} - z = \cos(x+2y).$$

[2+4+4]

Use separate answer script for each unit

B.Sc. (Honours) Examination-2022

Semester-V (CBCS)
Mathematics
Course: DSEMA-1
(Mechanics-II)

Time: Three Hours **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I (Full Marks: 40)

(Dynamics of a rigid body)

Answer ***any four*** questions.

1. (a) What is meant by equimomental bodies? Show that a uniform plane triangular lamina is equimomental with a system of three particles placed at the middle points of the sides, each equal to one-third the mass of the lamina. [1+4]

   (b) Show that the equation of the momental ellipsoid at the centre of a solid ellipsoid is

$$\left(b^2+c^2\right)x^2+\left(c^2+a^2\right)y^2+\left(a^2+b^2\right)z^2=constant.$$

[5]

2. (a) If α, β, γ be the distances of the vertices of a triangle from a straight line in its plane, then show that the moment of inertia of the triangle about this line is

$$\frac{M}{6}\left(\alpha^2+\beta^2+\gamma^2+\alpha\beta+\beta\gamma+\gamma\alpha\right),$$

   where M denotes its mass. [5]

   (b) Show that at the centre of the quadrant of an ellipse, the principal axes in its plane are inclined at an angle $\frac{1}{2}\tan^{-1}\left(\frac{4}{\pi}\frac{ab}{a^2-b^2}\right)$ to the axes. [5]

3. (a) State D'Alembert's principle. Obtain the general equations of motion of a rigid body from D'Alembert's principle. [1+4]

   (b) A plank of mass M is initially at rest along a straight line of greatest slope of a smooth plane inclined at an angle α to the horizon and a man of mass M' starting from the upper end walks down the plank so that it does not move. Show that he gets to the other end in time

$$\sqrt{\frac{2M'a}{(M+M')g\sin\alpha}}$$

   where a is the length of the plank. [5]

4. (a) Obtain the equation of motion of a rigid body about the axis of rotation. [5]

   (b) A uniform rod of mass m and length $2a$ can turn freely about one fixed end. It is started with an angular velocity ω from the position in which it hangs vertically. Find its angular velocity at any instant. [5]

5. (a) Find the moment of momentum of a body moving in two dimensions about the origin. [5]

   (b) Show that the length of the simple equivalent pendulum of an elliptic lamina of semi-major axis a and eccentricity e when the axis of oscillation is a latus rectum is $\frac{a}{2e}\left(1+2e^2\right)$. [5]

6. (a) An imperfectly rough sphere moves from rest down a plane inclined at an angle α to the horizon. Discuss the motion. [5]

   (b) A uniform solid cylinder is placed with its axis horizontal on a plane, whose inclination to the horizon is α. Show that the least coefficient of friction between it and the plane is $\frac{1}{3}\tan\alpha$, when it may roll but not slide. [5]

Unit-II (Full Marks: 20)
(Hydrostatics)
Answer ***any two*** questions.

1. (a) Explain the terms "perfect fluid" and pressure at a point in the fluid. [1+1]
   (b) Prove that if the fluid is in equilibrium, the pressure at a point is the same in every direction. [4]
   (c) A fine circular tube in a vertical plane contains a column of liquid of density σ_1, which subtends a right angle at the centre and a column of density σ_2, subtending an angle ϕ. Prove that the radius through the common surface makes with the vertical an angle $\tan^{-1}\dfrac{\sigma_1 - \sigma_2 + \sigma_2\cos\phi}{\sigma_1 + \sigma_2\sin\phi}$. [4]
2. (a) Define the surface of equi-pressure. Show that for a conservative field of forces, the surfaces of equi-pressure, equi-density and equi-potential energy coincide. [1+4]
   (b) The side AB of a triangle ABC is in the surface of a heavy homogeneous liquid and the points D, E are taken on AC such that the thrusts on the triangles BAD, BDE and BEC are equal. Show that $AD : DE : EC = 1 : (\sqrt{2} - 1) : (\sqrt{3} - \sqrt{2})$. [5]
3. (a) A uniform solid sphere, of radius r_1 and density σ is surrounded by a concentric shell of liquid of outer radius r_2 and density ρ. The liquid is subject to an attractive force $\dfrac{g}{r_2\rho}\left[\dfrac{(\sigma-\rho)r_1^3}{r^2} + r\rho\right]$ per unit mass towards the centre, where r is the distance from the centre. Assuming the outer boundary of the liquid to be free from pressure, calculate the pressure at any point of the surface of the sphere. [5]
   (b) Define centre of pressure of a given area in a fluid in equilibrium. An ellipse of eccentricity $\dfrac{1}{4}$ is just immersed in water with its major axis vertical. Show that the centre of pressure coincides with a focus. [1+4]