Use separate answer script for each unit

B. Sc. (Honours) Examination-2022

Semester-VI (CBCS)
Mathematics
Course: CCMA-13
(Analysis-VI)

Time: 3 Hours **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I (Marks: 30)
(Complex Analysis)
Answer *any three* questions.

1. (a) Show that a complex sequence $\{z_n\}$ converges to z_0 if and only if $\{Re(z_n)\}$ converges to $Re(z_0)$ and $\{Im(z_n)\}$ converges to $Im(z_0)$.
   (b) When is a complex series $\sum_{n=1}^{\infty} z_n$ called conditionally convergent? Discuss the convergence of the series $\sum_{n=1}^{\infty} n^2 2^n (1+i)^{-n}$. [6+4]

2. (a)Find the points in $\mathbb{C}$ at which the function f defined by $f(z) = \begin{cases} \frac{\bar{z}^2}{z} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$ is differentiable.
   (b)Compute an analytic function $f(z) = u + iv$ such that $u - v = e^x(\cos y - \sin y)$. [5+5]

3. (a) Write down Cauchy-Riemann equations in polar form and derive them.
   (b) Obtain a condition on k such that the Möbius map $f(z) = \frac{z-3}{1-2z}$ maps the circle $|z-1| = k$ onto a straight line. Also find the equation of the straight line. [(1+5)+4]

4. (a) Find all Möbius transformations of the half-plane $Re(z) \leq 0$ onto the unit disc $|w| \leq 1$.
   (b) Evaluate $\oint_C \frac{e^{iz}}{z - \pi i} dz$, where C is the (positively oriented) curve $|z-2| + |z+2| = 6$. [6+4]

5. (a) Let f and g be entire functions such that $|f(z)| \leq |g(z)|$ and $g(z) \neq 0 \; \forall \; z \in \mathbb{C}$. Show that there is a constant c such that $f(z) = cg(z)$.
   (b) Let $P(z)$ be a polynomial of degree n and $a \in \mathbb{C}$. Then show that there are exactly n points in $\mathbb{C}$ which are mapped by $P(z)$ to a. [5+5]

Unit-II (Marks: 30)
(Metric Spaces)
Answer *any three* questions.

6. (a) Define a metric space.Prove that for any three points α, β, γ in a metric space (X, ρ), $|\rho(\alpha,\beta) - \rho(\beta,\gamma)| \leq \rho(\gamma, \alpha)$. [2+2]
   (b) Let $\rho : R^2 \times R^2 \to R$ be defined by $\rho(x,y) = |x_1 - y_1|$, where $x = (x_1, x_2), y = (y_1, y_2) \in R^2$. Examine if ρ is a metric on R^2. [2]
   (c) Prove that l_3 is a metric space. [4]

7. (a) Define an open sphere in a metric space (X, ρ) and prove that it is an open set. [1+2]
   (b) Show that finite intersection of open sets in a metric space (X, ρ) is open. [3]
   (c) Prove that a sequence $\{x_n\}$ converges to x in the metric space $c[a,b]$ if and only if the corresponding sequence of real valued functions $\{x_n(t)\}$ converges uniformly to $x(t)$ on $[a,b]$. [4]

8. (a) Define the closure $\bar{A}$ of a set $A \subset X$ in a metric space (X, ρ). The closure of a set A, consisting of a single point is the set A itself - justify it. [1+2]
   (b) Define a Cauchy sequence in a metric space (X, ρ). Prove that if a sequence $\{x_n\}$ converges, then it is a Cauchy sequence. Show by an example the converse is not always true. [1+2+1]
   (c) Define a closed set in a metric space. Examine if arbitrary union of closed sets in a metric space is closed. [1+2]

9. (a) Define a complete metric space. Prove that the space m, of all bounded sequences of real numbers with a suitable metric defined by you is complete. [1+4]

(b) Let X be the set of all polynomials $P(t), 0 \leq t \leq 1$ with real coefficients and the distance between $P(t), Q(t)$ in X be defined by $\rho(P,Q) = Sup\{|P(t) - Q(t)|; 0 \leq t \leq 1\}$. Show that (X, ρ) is not a complete metric space. [3]

(c) Let X be a complete metric space and let Y be a subset of X. Then prove that Y, considered as a metric subspace is complete if it is closed. [2]

10. (a) Define a contraction mapping over a metric space.
Let $X = (0, \frac{1}{4})$ be a metric space with the usual metric of R. Let $T : X \to X$ be given by $Tx = x^2$. Show that T is a contraction maping having no fixed point in X. [1+3]

(b) Let (X, ρ) and (Y, ρ_1) be two metric spaces and $f : X \to Y$ be a function. Then prove that $f(x)$ is continuous at $x \in X$ if and only if for every sequence $\{x_n\}, x_n \in X$ converging to $x \in X$ implies $f(x_n) \to f(x)$ as $n \to \infty$. [6]

B. A./B. Sc. (Hons) Examination-2022

Semester-VI

Mathematics

CCMA 14 (Algebra IV)

Time: Three Hours

Full Marks: 60

Questions are of values as indicated in the margin.
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Group - A (Group Theory)

Answer *any four* questions.

1. (a) Suppose that $f : \mathbb{Z}_{50} \to \mathbb{Z}_{50}$ is an automorphism with $f(11) = 13$. Determine a formula for $f(x)$.

   (b) Let f be a homomorphism of a group G into a group G^1. Show that $ker\ f$ is a normal subgroup of G.

2. (a) Show that any infinite cyclic group is isomorphic to $\mathbb{Z}$.

   (b) Show that $(\mathbb{Q}, +)$ is not isomorphic to $(\mathbb{Q}^+, \cdot)$.

3. (a) Show that there is only one noncommutative group of order 6 up to isomorphism.

   (b) How many subgroups are there in the group $\mathbb{Z}/21\mathbb{Z}$?

4. (a) Suppose that $\mathbb{Z}_{10}$ and $\mathbb{Z}_{15}$ are homomorphic images of a finite group G. What can be said about $|G|$?

   (b) Let H and K be two subgroups of a group G, with H normal in G. Then show that $K/(H \cap K)$ is isomorphic to HK/H.

5. (a) Let ϕ be an automorphism on a group G. Show that $H = \{a \in G : \phi(a) = a\}$ is a subgroup of G.

   (b) Show that $|Inn(S_3)| = 6$.

6. (a) Let G be a group such that $\mathbb{Z}(G) = \{e\}$. Show that $\mathbb{Z}(Aut(G)) = \{e\}$.

   (b) State Cauchy's theorem on finite groups.

Group - B (Ring Theory)

Answer *any four* questions.

1. (a) State and prove the First Isomorphism Theorem for rings.

   (b) Let R be a commutative ring of characteristic 2. Then show that the mapping $a \to a^2$ is a ring homomorphism from R to R.

2. (a) Show that the ring $2\mathbb{Z}$ is not isomorphic to the ring $3\mathbb{Z}$.

   (b) Give an example of a simple ring which is not a field. Justify your answer.

3. (a) Show that every ideal of $\mathbb{Z}$ are of the form $m\mathbb{Z}$ for $m \in \mathbb{Z}$.

   (b) Show that there does not exist any proper ideal I in a a ring R that contains a unit.

4. Find the number of elements in the quotient ring $\mathbb{Z}[i]/ < 3 + i >$.

5. (a) Let R be a commutative ring with unity 1. If R has a prime ideal P such that P contains no zero divisors, then show that R is an integral domain.

   (b) In the ring $\mathbb{Z} \times \mathbb{Z}$, give an example of a prime ideal which is not maximal. Justify your answer.

6. Let R be a commutative ring with unity 1. Then show that every proper ideal of R is contained in a maximal ideal of R.

Group - C (Linear Algebra)

Answer *any four* questions.

1. Let A be a $m \times n$ matrix over $\mathbb{R}$, Show that row rank of A is equal to the column rank of A.

2. (a) Let $A \in M_{m\times n}(\mathbb{R})$ and $B \in M_{n\times p}(\mathbb{R})$. If rank of AB= rank of B, then show that there exists $D \in M_{n\times m}(\mathbb{R})$ such that $DAB = B$.

   (b) Let $A \in M_{5\times 3}(\mathbb{R})$ and $B \in M_{3\times 5}(\mathbb{R})$. If rank of A=rank of $B = 3$, find rank of AB.

3. (a) Let $B \in M_n(R)$. Define $T : M_n(R) \rightarrow M_n(R)$ by $T(A) = AB - BA$. Show that T is a linear transformation and find its kernal. [2+

(b) Give an example of a linear transformation which is not one-one but onto. Justify your answer.

4. (a) Let V be a vector space over the field F and β an γ two finite bases of V. For a linear transformation $T : V \longrightarrow V$ show that $[T]_\beta$ and $[T]_\gamma$ are similar.

(b) Suppose U, V and W be three finite dimensional vector spaces over the field F. Let $T : U \longrightarrow V$ and $S : V \longrightarrow W$ be two linear transformations. Then show that $rank(ST) = rank(T) - \dim(Im\ T \cap \ker S)$.

5. (a) Let V be a vector space over the field F and $T : V \rightarrow V$ be a linear operator such that $T^2 = cT$ for some $c \in F$. Prove that $V = \text{Im}T \oplus \ker T$.

(b) Find a basis for $L(P_2(\mathbb{R}), \mathbb{R}^2)$.

6. (a) Let $T : P_2(\mathbb{R}) \rightarrow M_2(\mathbb{R})$ be the linear transformation defined by,

$$T(p(t)) = \begin{pmatrix} f''(0) & 3f(2) \\ 0 & f'(1) \end{pmatrix}.$$

Find the matrix representation of T relative to the standard bases of the given vector spaces.

(b) Prove that the linear operator $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $T(x, y) = (2x, 0)$ is not a projection oparetor.

S.B

B.Sc. (Honours) Examination, 2022

Semester-VI (CBCS)
Mathematics
Course: DSEMA-3A
(Computer Fundamentals)

Time: Two Hours **Full Marks: 20**

Questions are of values as indicated in the margin.
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Answer ***any four*** questions.

1. (a) Given a set of 100 positive integers, design a flowchart (using standard symbols) to evaluate [4]
   (i) the total number of odd integers.
   (ii) the sum of all odd integers.

   (b) Write the full form of EBCDIC. [1]

2. (a) Name different types of variables with appropriate examples which are recognised by FORTRAN 77 . [2]

   (b) Given the following declaration statement [3]
   CHARACTER*10 P, Q*5, R*5, C*1, S1*3, S2*5
   where the variables S1 and S2 hold the values 'FOR' and 'TRAIL' respectively. Examine if the following statements are valid. Find the values assigned to the variables P, Q and R in the following assignment statements, if valid.
   (i) P=S2 // 'ING' (ii) Q=S1 // S2(: 1) (iii) R='ONE' //22

3. (a) Write down the FORTRAN expression of the following mathematical expression: [1.5]
   $$\cos\left(\frac{(a^2+5d-\log d)}{\tan(b^2-c^2)\operatorname{cosec}(x+c)}\right)+\frac{1}{\sqrt{2\pi k}}\,e^{-\frac{(x-c)^2}{2k^2}}$$

   (b) State two main differences between STOP statement and END statement. [2]

   (c) Write a set of logical IF statements which are equivalent to: [1.5]
   GO TO(47, 33, 55, 77), K

4. (a) Write a FORTRAN 77 program which reads the elements of a square matrix M of order $n(\leq 5)$ and check whether it is symmetric or not. [3]

   (b) Find the label of the statement to which control is transferred for each of the following FORTRAN 77 segments. [1+1]

   (i)
   ```
   J=2
   J=J+2
   GO TO(23, 47, 16, 94), J
   ```

   (ii)
   ```
   INTEGER ROLL
   K=2
   ROLL=2*K
   GO TO(51, 61, 71, 81), ROLL
   ```

5. (a) Find the output (mentioning all the necessary steps involved) of the following program: [3]
   ```
   INTEGER A(10)
   DO 100 I=1, 10, 2
   A(I) = 2*I-3
   A(I+1) =I**2-5
   100 CONTINUE
   DO 200 I= 2, 10, 3
   A(I) =A(I+2)-A(I)
   200 CONTINUE
   WRITE (6, 10) A
   10 FORMAT(1X, 5I8)
   STOP
   END
   ```

(b) If LG=.FALSE., I=2, J=3, K=4 and L=5, find the value of the following expression by indicating the order of the evaluation: [2]

(LG .OR. K .NE. J) .AND. .NOT. (I * J .LT. L ** 2)

6. (a) Find errors, if any, in each of the following subprogram defining statements: [1+1]

(i) SUBROUTINE NEW(X, Y(3), Z)

(ii) FUNCTION ADD(X, Y, Z)

```
Z=X+Y
RETURN
END
```

(b) Define a statement function to calculate $R = \sqrt{u^2 + v^2 + w^2}$ and write assignment statements to calculate $A = \frac{x}{\sqrt{x^2 + y^2 + z^2}}$, $B = \sqrt{x^4 + y^4 + z^4}$, $C = \sqrt{4x^2 + 9y^2 + 4z^2}$. [2]

(c) Why is a subprogram called a complete program? [1]

B. Sc. (Honours) Examination-2022
Semester-VI
Mathematics
Course: DSE-4
(Mathematical Modelling)

Time: Three Hours Full Marks: 60

Questions are of values as indicated in the margin.
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Answer question no. 1 and any five fofm the rest (taking at least two from each group)

1. Choose the correct alternative.
   (a) Mathematical models provide (i) estimated results (ii) accurate results (iii) wrong results (iv) approximate results. [1]
   (b) Scientists used mathematical models to predict the growth of world population, and computers converted that data into a (i) sample (ii) model (iii) design (iv) structure. [1]
   (c) To solve engineering problem we need to formulate the pattern as math expression in terms of variables, functions and equations. Such expression is called (i) function model (ii) math model (iii) variable model (iv) math equation. [1]
   (d) In the standard logistic growth recursion equation, the change in N_t is proportional to (i) $(b - N_t)\, N_t$ (ii) $a\,(b - N_t)\, N_t$ (iii) $a\,(1 - N_t)\, N_t$ (iv) $(1 - bN_t)\, N_t$. [2]
   (e) If in a population of bacteria the birth and death rates are proportional to the number of individuals present, then at $t = 0$ the number of bacteria (i) remains stable (ii) grows exponentially (iii) decays exponentially (iv) changes randomly. [2]
   (f) Kirchoff's voltage law is a consequence of the conservation of (i) both charge and energy (ii) energy only (iii) charge only (iv) both charge and current. [1]
   (g) If the energy of an oscillating particle is continuously dissipated, then oscillations are experiencing (i) resonance (ii) damping (iii) harmonic (iv) beats. [1]
   (h) One of the set of variables that are used to describe the mathematical 'state' of a dynamical system is a (i) static variable (ii) dynamic variable (iii) statistical variable (iv) state variable. [1]

Group A (Physical System)

2. Derive the equation of motion of a simple pendulum in presence of a frictional force. Hence discuss the characteristics of underdamped and overdamped motions of the pendulum. [4+6]

3. State the Fourier's law of heat transfer. Use it to construct a mathematical model for one-dimensional heat flow in a uniform rod. [2+8]

4. (a) Build a mathematical model to describe the linear dynamics of a particle of mass m attached to an elastic string with its one end fixed. Hence discuss the nature of the solution of the evolution equation. [3+2]
   (b) For the propagation of electromagnetic waves in vacuum derive the wave equation. What is the phase velocity of the wave? [4+1]

5. (a) Define a continuous dynamical system. When is it said to be autonomous and non-autonomous? Give an example for each of them. [1+2+2]
   (b) For an one-dimensional autonomous system explain the notions of phase line, phase point and phase portrait. [1.5+1.5+2]

6. (a) Define fixed point of a dynamical system. When is it said to be stable and asymptotically stable? [2+2]

(b) Find the fixed point(s) and analyze the local stability of the systems: (i) $\dot{x} = x + x^3$ (ii) $\dot{x} = -x - x^3$. [3+3]

Group-B (Biological System)

7. Describe the Gompertz model. Why is it used? What is the 'carrying capacity' and how does it differ from that of the logistic model? [4+2+4]

8. Construct a stochastic model for the linear birth process of a single species population, and hence find the expected value and the coefficient of variation of the number of individuals at time t. [6+(2+2)]

9. Why harvesting models are used? Discuss the dynamics and optimization approaches to model harvesting. [2+(4+4)]

10. Use Fick's law to construct a single species reaction-diffusion (RD) model in one space dimension. Give examples of linear and nonlinear RD models. [7+3]