

Five Year Integrated M.Sc. Examination 2021

Semester-I

Subject: Mathematics-I (MT-1-1-1)

Time: Three Hours

Full Marks: 60

Answer **Question number 1** and any **three** from the rest.

1. (a) Use a graph to find a number δ such that if $|x - 1| < \delta$ then $|x^2 - 1| < 0.5$.
(b) Prove that $\lim_{x \rightarrow 3} (4x - 5) = 7$ using $\epsilon - \delta$ definition.
(c) If $4x - 9 \leq f(x) \leq x^2 - 4x + 7$ for $x \geq 0$, find $\lim_{x \rightarrow 4} f(x)$.
(d) Is there a number a such that $\lim_{x \rightarrow 2} \frac{3x^2 + ax + a + 3}{x^2 + x - 2}$ exists? If so, find the value of ' a ' and find the value of the limit.
(e) Show by means of an example that $\lim_{x \rightarrow a} [f(x) + g(x)]$ may exist even though neither $\lim_{x \rightarrow a} [f(x)]$ nor $\lim_{x \rightarrow a} [g(x)]$ exists. 5+5+3+5+3=21
2. (a) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = |x|, x \in \mathbb{R}$. Show that f is neither injective nor surjective
(b) Let $X = a, b, c$. Write down the power set of X . Does every set admit a subset? Give an example of a set that has only one proper subset?
(c) Let A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then prove that $B = C$. 3+7+3=13
3. (a) Find the value of $f \circ g \circ h$ where $f(x) = \frac{x}{x+1}, g(x) = x^{10}, h(x) = x + 3$.
(b) Given $F(x) = \cos^2(x + 9)$, find functions f, g , and h such that $F = f \circ g \circ h$.
(c) What is a one-to-one function? How can you tell from the graph of a function whether it is one-to-one? If $g(x) = 3 + x + e^x$, find $g^{-1}(4)$. 3+3+7=13
4. (a) Let $\Omega = (a, b) \in \mathbb{Z} \times \mathbb{Z} | 3a + 4b = 7n$ for some $n \in \mathbb{Z}^+$. Show that it is an equivalence relation.
(b) Show that the relation " \perp " on the set L of lines in a plane is symmetric.
(c) If $f(x) = x^2 - 2x + 3$, evaluate the difference quotient $\frac{f(a+h) - f(a)}{h}$, where ' a ' is a constant. 5+5+3=13
5. (i) Define graph of a function of two variables.
(ii) Sketch the level curves of the function $g(x, y) = \sqrt{9 - x^2 - y^2}$.
(iii) Find the limit, if it exists, or show that the limit does not exist:
(a) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 y^2}{x^5 + y^7}$ (b) $\lim_{(x,y) \rightarrow (0,0)} \frac{3x^2 y}{x^2 + y^2}$ 2+3+8=13
6. (a) Show that the function

$$f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2 + y^2}}, & (x, y) \neq 0, \\ 0, & (x, y) = 0. \end{cases}$$

is continuous at $(0, 0)$ and $f_x(0, 0)$ and $f_y(0, 0)$ both exist.

(b) Show that any function of the form $z = f(x + at) + g(x - at)$ is a solution of the wave equation

$$\frac{\partial^2 z}{\partial t^2} = a^2 \frac{\partial^2 z}{\partial x^2}.$$

(c) Define tail of a sequence. For what value of n , the sequence $\left\{\frac{1}{n}\right\}$ can be made smaller than 0.001.

$$7+2+4=13$$

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