Five Year Integrated M.Sc. Examination, 2022
Subject: Mathematics-I
Course Code: MT-1-1-1
Full Marks.-60
Time: 3 Hrs.

Answer question No.1 and any three from the rest.

1. Prove or disprove. Justify your answer briefly.
(a) The relation "||" (parallel) on the set L of lines in a plane is an equivalence relation
(b) Every bijective function is an onto function but the converse is not true
(c) Every relation is a function
(d) Let $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = |x|, x \in \mathbb{R}$. The function is neither injective nor surjective
(e) The union of two equivalence relations is again an equivalence relation.
(f) Every continuous function is differentiable. 6 $\times$ 3=18

2. (i) Find the power set of the set $\{a, b, c\}$.
(ii) Find the value of $f_o g_o h_o$ where $f(x) = \dfrac{x}{x+1}$, $g(x) = x^{10}$, $h(x) = x + 3$.
(iii) Prove that the intersection of two equivalence relations is again an equivalence relation.
(iv) Check whether the relation {(1, 2), (2, 2), (3, 3), (3, 2) (2, 3), (1, 2) } on the set A={1,2,3} is reflexive, symmetric or transitive. 2+4+4+4=14

3. (i) Show that $f : \mathbb{Z} \to \{-1, 1\}$ given by $f(x) = (-1)^x, x \in \mathbb{Z}$ is surjective but not injective.
(ii) Does the limit of the sequence $\{(-1)^n\}$ exist at $n \to \infty$.
(iii) Using $\epsilon - \delta$ definition prove that $\lim_{x\to 1}$(2x+1)=3.
(iv) If f is differentiable at a, then f is continuous at a. 4+2+4+4=14

4. (i) Define Fibonacci sequence.
(ii) Using definition show that for the sequence $\left\{\dfrac{n-1}{n+1}\right\}$, $\lim_{n\to\infty} \left\{\dfrac{n-1}{n+1}\right\} = 1$.
(iii) Show that the sequence $a_n = \left\{\dfrac{n}{n^2+1}\right\}$ is decreasing.
(iv) Show by means of an example that $\lim_{x\to a}[f(x)+g(x)]$ and $\lim_{x\to a}[f(x)g(x)]$ may exist even though neither $\lim_{x\to a} f(x)$ nor $\lim_{x\to a} g(x)$ exists. 2+4+4+4=14

5. (i) State and prove Cauchy-Riemann equations. Does the limit $\lim\limits_{z\to 0} \dfrac{\overline{z}}{z}$ exist?
(ii) Is there a number a such that $\lim\limits_{x\to 2} \dfrac{3x^2 + ax + a + 3}{x^2 + x - 2}$ exist? If so, find the value of

a and the value of the limit.
(iii) State Squeeze theorem. Hence show that $\lim_{x\to 0} x^2 \sin\frac{1}{x} = 0$. (2+4)+4+(1+3)=14

6. (i) Find the domain and range of $g(x,y) = \sqrt{9 - x^2 - y^2}$.
(ii) Show that the repeated limits for $\dfrac{xy}{x^2+y^2}$ exist at (0,0) but the double limit does not exist.
(iii) Define graphs and level curve. Sketch the level curves of the function $g(x,y) = \sqrt{9 - x^2 - y^2}$. 4+6+4=14

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