Five Year Integrated M.Sc. Examination, 2023
Semester-III
Subject: Mathematics III
Course Code: MT-2-3-1

Time: 3 Hours **Full Marks: 60**

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Attempt **question number 1** and any three from the rest.

1. (i) The number of arbitrary constants in the general solution of an ordinary differential equation of order four is————.
(ii) General solution of the differential equation of the type $\frac{dx}{dy} + Px = Q$ is given by————.
(iii) The integrating factor of $\frac{dy}{dx} + y = \frac{1+y}{x}$ is ————-.
(iv) The solution of the differential equation $2x\frac{dy}{dx} - y = 3$ represents a family of —————.
(v) The singular solution of $\frac{dy}{dx} = y^2 - 4$ is———.
(vi) Complementary function of $(D^3 - 8)y = e^x$ is———- 6 × 2=12

2. Prove or disprove of the following (any four)
(i) The functions e^{2x}, e^{3x} are linearly independent solution of $y'' - 5y' + 6y = 0$.
(ii) $\frac{1}{3x^3y^3}$ is an integrating factor of $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$
(iii) The order and degree of $\left[1 + \left(\frac{dy}{dx}\right)^3\right]^{2/3} = \sin\left(\frac{d^2y}{dx^2}\right)^2$ are 1 and 2 respectively.
(iv) $y = a\log x + b$ is a solution of the differential equation $x\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$.
(v) The solution of $x^3\frac{dy}{dx} + 4x^2y = 6$ is $yx^4 = 3x^2 + c$, where c is a constant.
(vi) $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$ is exact. 4 × 4=16

3. (i) Find the domain in which the solution is defined for $\frac{dy}{dx} = y^2, y(0) = b > 0$.
(ii) Show that $f(x, y) = x + 3y; (x, y) \in \mathbb{R}^2$ satisfies Lipschitz continuity.
(iii) Find the general solution and singular solution of $\frac{dy}{dx} = (y - 3)^2$.
(iv) Show that $\frac{dy}{dx} = 2\sqrt{y}, y(0) = 0$ has nonunique solutions. 3+4+5+4=16

4. (i) Find a general Maclaurin series solution of the ODE: $y_1 - 2xy = 0$.

   (ii) Define Ordinary, regular and irregular singular points with examples. Find the series solution of $(x^2+1)y_2+xy_1-xy = 0$ about $x = 0$. 4+(6+6)=16

5. (i) Show that the Wronskian of the functions x^2 and $x^2 \log x$ is nonzero. Can these functions be independent solutions of an ordinary differential equation?
   (ii) Show that $\{x(x^2 - y^2)\}^{-1}$ is an integrating factor of the differential equation $(x^2 + y^2) - 2xy\dfrac{dy}{dx} = 0$, hence solve the equation.
   (iii) Obtain the differential equation of all circles each of which touches the axis of x at the origin. (3+1)+(3+3)+(1+5)=16

6. (i) Form the differential equation of the family of curves $\dfrac{x^2}{a^2+\lambda} + \dfrac{y^2}{b^2+\lambda} = 1$ for different values of λ. Determine whether the differential equation is linear or nonlinear.
   (ii) Solve the differential equation $(x^3 - 3x^2y + 2xy^2)dx - (x^3 - 2x^2y + y^3)dy = 0$ if $y = 1$ when $x = 1$.
   (iii) Solve $(D^3+3D^2+3D+1)y = e^{-x}$. (5+1)+5+5=16