Five Year Integrated M.Sc. 2022
Subject: Mathematics III
Course Code: MT-2-3-1
Full Marks.-60
Time: 3 Hrs.

Attempt **Question No. 1** and any **three** from the rest.

1. (a) Define Wronskian. If $y_1(x) = \sin(3x)$ and $y_2(x) = \cos(3x)$ are two solutions of differential equation $y'' + 9y = 0$, show that y_1 and y_2 are linearly independent solutions.
(b) Reduce the equation $\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2 - 1)y = -3e^{x^2}\sin(2x)$ into the normal form and hence solve it.
(c) Obtain the differential equation of all circles each of which touches the axis of x at the origin. (5++5=15)

2. (a) Define wellposed and illposed problem. Define Lipschitz continuity of a function $f(x, y)$. State the sufficient condition to gurantee Lipschitz continuity of $f(x, y)$. Show that $\frac{dy}{dx} = 2\sqrt{y}, y(0) = 0$ has non-unique solutions.
(b) Which of the following is not an integrating factor of $xdy - ydx = 0$?
(i) $1/x^2$ (ii) $1/(x^2 + y^2)$ (iii) $1/(xy)$ (iv) x/y
(c) If $y = x$ is a solution of $x^2y'' + xy' - y = 0$, then the second linearly independent solution of the above equation is
(i) $1/x$ (ii) x^2 (iii) x^{-2} (iv) x^n ((2+2+2+5)+2+2=15)

3. (a) Evaluate $\int_C y^2dx + xdy$, where (i) $C = C_1$ is the line segment from (-5, -3) to (0, 2) and (ii) $C = C_2$ is the arc of the parabola $x = 4 - y^2$ from (-5, -3) to (0, 2). Remark on the different results.
(b) Find the tangent plane to the surface with parametric equations $x = u^2, y = v^2, z = u + 2v$ at the point (1, 1, 3).
(c) Find a unit normal vector $\widehat{n}$ of the cone of revolution $z^2 = 4(x^2 + y^2)$ at the point P(1,0,2).
(d) Discuss the singularities of the equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$ at $x = 0$ and $x = \infty$. (5+3+3+4=15)

4. (a) Reduce the equation $xy(y - px) = x + py$ to Clairaut's form using $u = x^2$ and $v = y^2$ and obtain the complete primitive.
(b) Define Ordinary and singular point. Find the series solution of $(x^2+1)y_2+xy_1-xy = 0$ about $x = 0$. (5+10=15)

5. (a) If the given equation $Mdx + Ndy = 0$ is homogeneous and $(Mx + Ny) \neq 0$, then $\dfrac{1}{Mx + Ny}$ is an integrating factor.

(b) Show that $\dfrac{1}{3x^3y^3}$ is an integrating factor of $y(xy+2x^2y^2)dx+x(xy-x^2y^2)dy = 0$

(c) Solve $y = p^2x + p$. (5+5+5=15)

6. (a) Solve $D^4 + 2D^3 - 3D^2)y = 3e^{2x}$, $\left(D = \frac{d}{dx}\right)$.

(b) Solve the differential equation $\dfrac{d^2y}{dx^2} + y = \cos x$ by the method of variation of parameters.

(c) Prove that the equation $(2x^2 + 3x)\dfrac{d^2y}{dx^2} + (6x + 3)\dfrac{dy}{dx} + 2y = (x + 1)e^x$. is exact and hence find its solution. (5+5+5=15)