Five Year Integrated M.Sc. Examination, 2022
Semester-III
Subject: Mathematics III
Course Code: MT-2-3-1

Time: 3 Hours **Full Marks: 60**

Questions are of value as indicated in the margin.
Attempt any four questions.

1. Prove or disprove of the following (any five)
(a) The functions $\sin(x), \cos(x), 2\sin(x) + 3\cos(x)$ are linearly independent.
(b) $e^{2y} - y\cos(xy) + (2xe^{2y} - x\cos(xy) + 2y)\dfrac{dy}{dx} = 0$ is not exact.
(c) The order and degree of $\left[1 + \left(\dfrac{dy}{dx}\right)^4\right]^{1/3} = \sin\left(\dfrac{d^2y}{dx^2}\right)$ are 1 and 2 respectively.
(d) The differential equation satisfied by the family of curves given by $c^2 + 2cy - x^2 + 1 = 0$ is $(1 - x^2)p^2 + 2xyp + x^2 = 0$ where $p \equiv \dfrac{dy}{dx}$.
(e) The value of $\frac{1}{aD+m}X$ is $e^{\frac{mx}{a}} \int e^{-\frac{mx}{a}} X$ where a, m are constants and $D \equiv \dfrac{d}{dx}$.
(f) $\phi = ar\cos(kr) + br\sin(kr)$ satisfies the equation $\phi'' - \left(1 + \dfrac{1}{r}\right)\phi' + \left(\dfrac{2}{r^2} + ak^2\right)\phi = 0$.
(5 × 3=15)

2. (a) Show that the Wronskian of the functions x^2 and $x^2 \log x$ is nonzero. Can these functions be independent solutions of an ordinary differential equation?
(b) Show that $\{x(x^2 - y^2)\}^{-1}$ is an integrating factor of the differential equation $(x^2 + y^2) - 2xy\dfrac{dy}{dx} = 0$, hence solve the equation.
(c) Obtain the differential equation of all circles each of which touches the axis of x at the origin. ((3+1)+5+(1+5)=15)

3. (a) Define wellposed and illposed problem. Define Lipschitz continuity of a function $f(x, y)$. State and prove sufficient condition of Lipschitz continuity of $f(x, y)$. Show that $f(x, y) = x + 3y; (x, y) \in \mathbb{R}^2$ satisfies Lipschitz continuity.
(b) Which of the following is not an integrating factor of $xdy - ydx = 0$?
(i) $1/x^2$ (ii) $1/(x^2 + y^2)$ (iii) $1/(xy)$ (iv) x/y
(c) If $y = x$ is a solution of $x^2y'' + xy' - y = 0$, then the second linearly independent solution of the above equation is
(i) $1/x$ (ii) x^2 (iii) x^{-2} (iv) x^n

(d) (i) The number of arbitrary constants in the general solution of a differential equation of order three is————.

(ii) The integrating factor of $\dfrac{dy}{dx}+y=\dfrac{1+y}{x}$ is ————-. ((2+2+4+3)+2+2=15)

4. (a) Solve by the method of undetermined coefficients $\dfrac{d^2y}{dx^2}+6\dfrac{dy}{dx}+9y=24e^{-3x}$.

(b)If u and v are two independent solutions of $\dfrac{d^2y}{dx^2}+P\dfrac{dy}{dx}+Qy=0$ then the Wronskian $W(u,v)$ is given by $W(u,v)=Ae^{-\int Pdx}$ where A is constant.

(c) Solve $x^2\dfrac{d^2y}{dx^2}-3x\dfrac{dy}{dx}+4y=2x^2$. (5+5+5=15)

5. (a) Solve $D^4+2D^3-3D^2)y=3e^{2x}$, $\left(D=\frac{d}{dx}\right)$.

(b) Solve the differential equation $\dfrac{d^2y}{dx^2}+a^2y=\sec ax$ by the method of variation of parameters.

(c) Prove that the equation $x^2\dfrac{d^2y}{dx^2}+3x\dfrac{dy}{dx}+y=\dfrac{1}{(x-1)^2}$ is exact and hence find its solution. (5+5+5=15)

6. (a) State Fubini's theorem on double integral. Evaluate the iterated integrals for $\int_1^2\int_0^3 x^2y\,dxdy$.

(b) Find the volume of the solid S that is bounded by the elliptic paraboloid $x^2+2y^2+z=16$, the planes $x=2$ and $y=2$, and the three coordinate planes.

(c) Evaluate $\int\int_D(x+2y)dA$, where D is the region bounded by the parabolas $y=2x^2$ and $y=1+x^2$. (5+5+5=15)