Five Year Integrated M.Sc. Examination, 2022-2023

Semester - III

Code: PH-2-3-1

Time: Three Hours — Subject: Electricity and Magnetism — Full Marks: 60

Questions are of values as indicated in the margin

Answer question number **one** and any **three** from the rest

1. Answer any **eight** questions

$8 \times 3 = 24$

(a) Using integration by parts show that

$$x\frac{d}{dx}(\delta(x)) = -\delta(x)\,.$$

(b) Prove that $\vec{\nabla}.\left(\frac{\hat{r}}{r^2}\right) = 4\pi\delta^3(\vec{r})$.

(c) State and explain Biot-Savart law. Is it valid for time varying current?

(d) Suppose a charged particle is moving under the influence of both electric and magnetic fields. How can the effect of the two fields on the motion of the particle be distinguished?

(e) Draw the field lines and equipotential surfaces for an electric dipole i.e. a system of $+q$ and $-q$ charges separated at a distance d.

(f) Two concentric spherical surfaces enclose a point charge q. The radius of the inner and outer sphere are respectively R and $3R$. Compare the electric fields and electric fluxes crossing the two surfaces.

(g) Show that magnetic forces do no work.

(h) Two particles have the same linear momentum, but particle A has four times the charge of particle B. If both particles move in a plane perpendicular to a uniform magnetic field, what is the ratio R_A/R_B of the radii of their circular orbits?

(i) Define Poynting vector. What is the physical significance of Poynting vector?

(j) If a current is passed through a spring, does the spring stretch or compress? Explain.

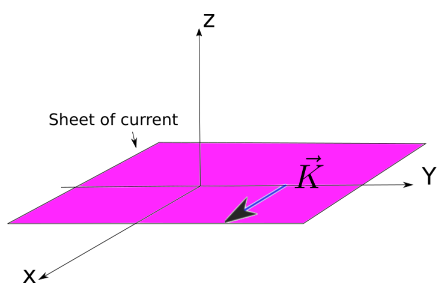
2. (a) Find the electric field a distance z above the midpoint of a straight line segment of length $2L$, which carries a uniform line charge λ. Taking appropriate limit calculate the electric field at the same point due to a infinite wire.

   (b) Suppose the electric field in some region is found to be

$$\vec{E} = k(x^2 + y^2 + z^2)(x\hat{i} + y\hat{j} + z\hat{k}) \, ,$$

   in Cartesian coordinates (k is some constant). Find the charge density at some arbitrary point $P(x, y, z)$. By using Gauss' law, calculate the total charge contained in a sphere of radius R, centred at origin $O(0, 0, 0)$.

   (c) Find the magnetic field of an infinite uniform surface current $\vec{K} = K\hat{i}$, flowing over the xy plane.



   Using symmetry argument explain why some of the components of the magnetic fields are zero.

4+4+4=12

3. (a) Calculate the force per unit length between two infinite parallel wires separated by distance d and carrying I_1 and I_2 currents respectively.

   (b) Define volume current density ($\vec{J}$). Is it a vector or scalar? Show that $\vec{J} = \rho\vec{v}$, where $\vec{v}$ is the velocity of the charges and ρ is the volume charge density.

   (c) An electron moving with a velocity $\vec{v} = (4.0\hat{i} + 3.0\hat{j} + 2.0\hat{k}) \times 10 \, m/s$ enters a region where there is a uniform electric field and a uniform magnetic field. The magnetic field is given by $\vec{B} = (1.0\hat{i} - 2.0\hat{j} + 4.0\hat{k}) \times 10^{-2} \, T$. If the electron travels through a region without being deflected, what is the electric field?

4+(1+1+2)+4=12

4. (a) Derive equation of continuity for electrical charges.
   (b) Explain the limitations of Ampere's law with the help of capacitor charging experiment. Derive the expression for displacement current.
   (c) Starting from the Maxwell's equations prove that the electromagnetic fields satisfy wave equations in vacuum.

4+4+4=12

5. (a) Starting from appropriate Maxwell's equations define vector potential and scalar potential for electromagnetic fields.
   (b) What do you mean by Gauge transformation? Show that the magnetic field and electric field remain invariant under Gauge transformation.
   (c) Suppose the electric field of a plane electromagnetic wave is given by

$$\vec{E}(z,t) = E_0 \sin(kz + \omega t)\hat{j}.$$

      i. Find the corresponding Magnetic field $\vec{B}(z,t)$.
      ii. Find the direction of wave propagation.

4+4+(3+1)=12