Five Year Integrated M.Sc. Examination, 2024
Semester-III
Subject: Mathematics III
Course Code: MT-2-3-1

Time: 3 Hours **Full Marks: 60**

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Attempt **Question Number 1** and any three from the rest.

1. Find out the correct answer for the following questions (any six).
(a) The integrating factor of $x \ln x \dfrac{dy}{dx} + y = 2x \ln x$ is
(i) $\ln x$ (ii) $\dfrac{1}{x}$ (iii) $\ln(\ln x)$ (iv) none
(b) Solution of $(D^2 - 2D + 1)y = 0$ is
(i) $c_1 e^x + c_2 e^{-x}$ (ii) $(c_1 + c_2)e^{-x}$ (iii) $(c_1 + c_2 x)e^{-x}$ (iv) $(c_1 + c_2 x)e^x$
(c) The order of the differential equation whose general solution is $y = A \sin x + B \cos x$ (A and B are constants) is
(i) 2 (ii) 3 (iii) 1 (iv) 0
(d) The differential equation which has $y = e^{a \sin x}$ as general solution (a is constant) is
(i) $\ln y = \tan x \dfrac{dy}{dx}$ (ii) $y \ln y = \tan x \dfrac{dy}{dx}$ (iii) $y \ln y = \cos x \dfrac{dy}{dx}$ (iv) $\ln y = \cos x \dfrac{dy}{dx}$
(e) The differential equation $2y dx - (3y - 2x)dy = 0$ is
(i) homogeneous and linear but not exact (ii) exact and linear but not homogeneous (iii) homogeneous, linear and exact (iv) homogeneous and exact but not linear
(f) An auxiliary equation can not have
(i) real and equal (ii) real and equal (iii) complex conjugate (iv) complex and equal roots
(g) The differential equation of the family of lines passing through origin is
(i) $x \dfrac{dy}{dx} + y = 0$ (ii) $\dfrac{dy}{dx} = y$ (iii) $\dfrac{dy}{dx} = x$ (i) $x \dfrac{dy}{dx} - y = 0$ (2 × 6=12)

2. Prove or disprove of the following
(a) The Wronskian of the functions x^2 and $x^2 \log x$ is nonzero.
(b) $e^{2y} - y\cos(xy) + (2xe^{2y} - x\cos(xy) + 2y)\dfrac{dy}{dx} = 0$ is not exact.
(c) The order and degree of $\left[1 + \left(\dfrac{dy}{dx}\right)^{1/2}\right]^2 = e^{\left(\frac{d^2y}{dx^2}\right)}$ are 1 and 2 respectively.
(d) The differential equation satisfied by the family of curves given by $c^2 + 2cy - x^2 +$

$1 = 0$ is $(1-x^2)p^2 + 2xyp + x^2 = 0$ where $p \equiv \dfrac{dy}{dx}$.
(e) The value of $\frac{1}{aD-m}X$ is $e^{-\frac{mx}{a}} \int e^{\frac{mx}{a}} X$ where a, m are constants and $D \equiv \dfrac{d}{dx}$.
(f) The length of the arc of the cycloid $x = r(\theta - \sin\theta), \quad y = r(1-\cos\theta) \quad 0 \leq \theta \leq 2\pi$ is 8r.
OR
The substitution $z = \log(1+2x)$ transforms the equation

$$(1+2x)^2 - 6(1+2x)\frac{dy}{dx} + 16y = 8(1+2x)^2$$

into

$$\frac{d^2y}{dz^2} - 4\frac{dy}{dz} + 4y = 2e^{2z}.$$

(2+3+2+3+3+3=16)

3. (a) What is the condition for being $Mdx + Ndy = 0$ as exact. Hence, show that the equation $(x^3 - 3x^2y + 2xy^2)dx - (x^3 - 2x^2y + y^3)dy = 0$ is exact and find the solution if $y = 1$ when $x = 1$.
(b) Solve $x\dfrac{dy}{dx} + y = x^2y^2$.
(c) Reduce the equation $xy(y - px) = x + py$ to Clairaut's form by the substitution $x^2 = u$ and $y^2 = v$ and hence obtain the complete primitive. (1+2+3)+4+6=16

4. (a) Show that the functions $y_1, y_2, ...y_n$ will be linearly independent if and only if Wronskian of $(y_1, y_2, ...y_n)$ is nonzero.
Hence, show that the functions x, x^2 and x^3 are linearly independent. What is the general solution? Determine the differential equation with these independent solutions.
(b) Prove that the system of confocal conics given by $\dfrac{x^2}{a^2+\lambda} + \dfrac{y^2}{b^2+\lambda} = 1$ is self orthogonal. (5+3+1+2+5=16)

5. (a) If $x^a y^b$ be an integrating factor of the equation $(-3x^{-1} - 2y^4)dx + (-3y^{-1} + xy^3)dy = 0$, then find a and b.
(b) What is singular solution? Solve the equation $y = px + \sqrt{a^2p^2 + b^2}$ and obtain the singular solution.
(c) Solve $(D^3 + 2D^2 - 3D)y = 4\sin x$. (5+(1+5)+5=16)

6. (a) Solve $x^2\dfrac{d^2y}{dx^2} - 3x\dfrac{dy}{dx} + 4y = 2x^2$.
(b) Prove that the equation $(1 + x + x^2)\dfrac{d^3y}{dx^3} + (3+6x)\dfrac{d^2y}{dx^2} + 6\dfrac{dy}{dx} = \dfrac{1}{(x-1)^2}$ is exact and hence find its solution.

(c) Write down the condition to remove the first derivative term from $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = X$ using the substitution $y = uv$ where u and v are functions of x.

OR

Evaluate $\int_C 2x ds$ along the arc C, where C consists of the arc C_1 of the parabola $y = x^2$ from (0, 0) to (1, 1) followed by the vertical line segment C_2 from (1, 1) to (1, 2). (5+(2+4)+5=16)