FOUR YEAR UNDERGRADUATE PROGRAMME

(UNDER NEP 2020)

STRUCTURE AND SYLLABUS

(As recommended by the Board of Studies for Mathematics on 8th September, 2023)

(With effect from Academic Session 2023-2024)

DEPARTMENT OF MATHEMATICS

VISVA-BHARATI

SANTINIKETAN-731235

WEST BENGAL

COURSE STRUCTURE OF FOUR-YEAR UNDERGRADUATE PROGRAMME IN MATHEMATICS

Sem	Major Courses	Minor Courses	Multi Courses	AECC	SEC	CVAC	Research*	Internship	Total Credits	
I	2x4cr=8	MnA 1x4cr=4	1x3cr=3	ENG/MIL1 1x2cr=2	1x3cr=3	TS 1x3cr=3	---		23	B.Sc. Certificate
II	2x4cr=8	MnB 1x4cr=4	1x3cr=3	ENG/MIL2 1x2cr=2	1x3cr=3	ES 1x3cr=3	---		23	
YEAR 1	2x8=16cr	2x4=8cr	2x3=6cr	2x2=4cr	2x3=6cr	2x3=6cr	---	Sum 4cr	46+4	
After successful completion of ONE YEAR Course (Semesters - I & II) securing 46 credits + 4 credits vocational summer internship, students may exit with B.Sc. Certificate in MATHEMATICS or continue further.										
III	2x4cr=8	MnA 1x4cr=4	1x3cr=3	MIL/ENG1 1x2cr=2	1x3Cr=3	---	---		20	B.Sc. Diploma
IV	4x4cr=16	MnB 1x4cr=4	---	MIL/ENG2 1x2cr=2	---	---	---		22	
YEAR 2	10x4=40cr	4x4=16cr	3x3=9cr	4x2=8cr	3x3=9cr	2x3=6cr	---	Sum 4cr	88+4	
After successful completion of TWO YEAR Course (Semesters - I to IV) securing 88 credits + 4 credits vocational summer internship, students may exit with B.Sc. Diploma in MATHEMATICS or continue further.										
V	3x4cr=12	MnA 1x4cr=4	---	---	---	---	---		16	B.Sc. Degree
VI	3x4cr=12	MnB 1x4cr=4	---	---	---	---	---		16	
YEAR 3	16x4=64cr	6x4=24cr	3x3=9cr	4x2=8cr	3x3=9cr	2x3=6cr	---	Sum 4cr	120+4	
After successful completion of THREE YEAR COURSE (Semesters - I to VI) securing 120 credits + 4 credits vocational summer internship, students may exit with B.Sc. Degree in MATHEMATICS or continue further.										
VII	4x4cr=16	MnA 1x4cr=4	---	---	---	---	---		20	B.Sc. Honours Degree
VIII	4x4cr=16	MnB 1x4cr=4	---	---	---	---	---		20	
YEAR 4	24x4=96cr	8x4=32cr	3x3=9cr	4x2=8cr	3x3=9cr	2x3=6cr	---	Sum 4cr	160+4	
After successful completion of FOUR YEAR COURSE (Semesters - I to VIII) securing 160 credits + 4 credits vocational summer internship, students may obtain B.Sc. Honours in MATHEMATICS.										
OR										
VII	3x4cr=12	MnA 1x4cr=4	---	---	---	---	1x4cr=4*		20	B.Sc. Honours (with Research)
VIII	2x4cr=8	MnB 1x4cr=4	---	---	---	---	2x4cr=8*		20	
YEAR 4	21x4=84cr	8x4=32cr	3x3=9cr	4x2=8cr	3x3=9cr	2x3=6cr	3x4=12cr	Sum 4cr	160+4	
After successful completion of FOUR YEAR COURSE (Semesters - I to VIII) securing 160 credits + 4 credits vocational summer internship, students may obtain B.Sc. Honours (with Research) in MATHEMATICS.										

Dissertation can be opted by students who attain at least CGPA 75% in 3 years and desire the Research degree. The students pursuing B.Sc. Honours (with Research) have to secure 12 credits (4 credits in Semester VII and 8 credits in Semester VIII). However, students pursuing only B.Sc. Honours have to study additional three Major Courses in MATHEMATICS securing 12 credits (4 credits in Semester VII and 8 credits in Semester VIII) in lieu of the Dissertation.

Learning Outcome based Curriculum for
FOUR-YEAR UNDERGRADUATE PROGRAMME in MATHEMATICS following NEP 2020

MAJOR COURSES in MATHEMATICS [Discipline-Specific Core Courses]

Course Code	Course Type	Course Title	Credits	Marks	Hours
SEMESTER I					
MJMA01	Theory	Algebra I and Analysis I	(2+2)	(50+50)	
MJMA02	Theory	Analytical Geometry and Vector Calculus	(2+2)	(50+50)	
		Total	**8 credits**	**200**	
SEMESTER II					
MJMA03	Theory	Algebra III	4	100	
MJMA04	Theory	Differential Equations I	4	100	
		Total	**8 credits**	**200**	
ONE-YEAR CERTIFICATE PROGRAMME TOTAL 4 MAJOR COURSES			**16 credits**	**400**	
SEMESTER III					
MJMA05	Theory	Analysis III	4	100	
MJMA06	Theory	Differential equation II and Dynamics of a Particle	(2+2)	(50+50)	
		Total	**8 credits**	**200**	
SEMESTER IV					
MJMA07	Theory	Algebra IV	4	100	
MJMA08	Theory	Analysis IV	4	100	
MJMA09	Theory	Differential Equations III and Dynamics of a Rigid Body	(2+2)	(50+50)	
MJMA10	Theory	Mathematical Probability and Statistics	(2+2)	(50+50)	
		Total	**16 credits**	**400**	
TWO-YEAR DIPLOMA PROGRAMME TOTAL 10 MAJOR COURSES			**40 credits**	**1000**	
SEMESTER V					
MJMA11	Theory	Analysis V	4	100	
MJMA12	Theory	Linear Programming Problem and Mathematical Modelling	(2+2)	(50+50)	
MJMA13	Theory	Numerical Analysis	4	100	
		Total	**12 credits**	**300**	
SEMESTER VI					
MJMA14	Theory	Algebra V	4	100	
MJMA15	Theory	Analysis VI	4	100	
MJMA16 (Unit I)	Theory	C Programming	2	50	
MJMA16 (Unit II)	Practical	Laboratory for C Programming	2	50	
		Total	**12 credits**	**300**	
THREE-YEAR DEGREE PROGRAMME 16 MAJOR COURSES			**64 credits**	**1600**	

SEMESTER VII					
MJMA17	Theory	Real Analysis and Complex Analysis	(2+2)	(50+50)	
MJMA18	Theory	Algebra VI and Functional Analysis	(2+2)	(50+50)	
MJMA19	Theory	Ordinary Differential Equations and Partial Differential Equations	(2+2)	(50+50)	
		Total	**12 credits**	**300**	
Additional 4 Credit Course for Students having Marks < 75% to Obtain B.Sc (Hons) Degree Without Research					
MJAMA01	Theory	Statics and Hydrostatics	(2+2)	(50+50)	
			4 credits	**100**	
SEMESTER VIII					
MJMA20	Theory	Algebra VII and Classical Mechanics	(2+2)	(50+50)	
MJMA21	Theory	Methods of Applied Mathematics	4	100	
		Total	**8 credits**	**200**	
Additional 8 Credit Course for Students having Marks < 75% to Obtain B.Sc (Hons) Degree Without Research					
MJAMA02	Theory	Algebra VIII and Topology	(2+2)	(50+50)	45
MJAMA03	Theory	Dynamical Systems and Advanced Numerical Analysis	(2+2)	(50+50)	30
			8 credits	**200**	
FOUR-YEAR HONOURS PROGRAMME 21 COURSES + 3 COURSES* (Optional in lieu of Dissertation for Honours students) FOUR-YEAR HONOURS with RESEARCH PROGRAMME 21 COURSES + DISSERTATION			**84 credits + 12 credits**	**2100 + 300**	

MINOR COURSES in MATHEMATICS [Discipline-Specific Minor Courses]

Course Code	Course Type	Course Title	Credits	Marks	Hours
SEMESTER I					
MNMA01	Theory	Calculus (Differential Calculus, Integral Calculus and Vector Calculus)	2+1+1	50+25 +25	
SEMESTER II					
MNMA01	Theory	Calculus (Differential Calculus, Integral Calculus and Vector Calculus)	2+1+1	50+25 +25	
ONE-YEAR CERTIFICATE PROGRAMME		**TOTAL 1 MINOR COURSE**	**4**	**100**	
SEMESTER III					
MNMA02	Theory	Introduction to Algebra (Introduction to Abstract Algebra and Linear Algebra over $\mathbb{R}$)	2+2	50+50	
SEMESTER IV					
MNMA02	Theory	Introduction to Algebra (Introduction to Abstract Algebra and Linear Algebra over $\mathbb{R}$)	2+2	50+50	
TWO-YEAR DIPLOMA PROGRAMME		**TOTAL 2 MINOR COURSES**	**8**	**200**	

SEMESTER V					
MNMA03	Theory	Differential Applications and their Applications	4	100	
SEMESTER VI					
MNMA03	Theory	Differential Applications and their Applications	4	100	
THREE-YEAR DEGREE PROGRAMME TOTAL 3 MINOR COURSES			**12**	**300**	
SEMESTER VII					
MNMA04	Theory	Linear Programming Problems and Numerical Methods	2+2	50+50	
SEMESTER VIII					
MNMA04	Theory	Linear Programming Problems and Numerical Methods	2+2	50+50	
FOUR-YEAR HONOURS PROGRAMME TOTAL 4 MINOR COURSES			**16**	**400**	

SKILL ENHANCEMENT COURSES in MATHEMATICS

Course Code	Course Type	Course title	Credits	Marks	Hours
SEMESTER I					
SEC01	Theory	Algebra II and Analysis II	2+1	50+25	
SEMESTER II					
SEC02	Theory	Analysis II and Tensor Calculus	1+2	25+50	
ONE-YEAR CERTIFICATE PROGRAMME TOTAL 2 COURSES			**6 credits**	**150**	
SEMESTER III					
SEC03	Theory	Number Theory	3	75	
TWO-YEAR DIPLOMA PROGRAMME TOTAL 3 COURSES			**9 credits**	**225**	

MULTIDISCIPLINARY COURSE in MATHEMATICS

Course Code	Course Type	Course title	Credits	Marks	Hours
SEMESTER I /II/III					
MDMA 01	Theory	Fundamentals of Mathematical Logic	3	75	

DETAILS SYLLABUS OF THE MAJOR COURSES IN MATHEMATICS

SEMESTER I

MJMA 01 : Algebra I and Analysis I
Credit: 2+2 **Marks: 50+50**

UNIT I : ALGEBRA I

Inequalities, AM $\geqslant$ GM $\geqslant$ HM and the Cauchy-Schwarz inequality (statement only); their applications.

Introduction to groups, definition and examples of groups including symmetries of a square, dihedral groups, permutation groups, quaternion groups (illustration through matrices) and matrix groups. Elementary properties of groups, order of an element in a group. The groups Z_n of all integers modulo n and U_n of all units modulo n.

Definition and examples of subgroups, subgroup tests. Centralizer, normalizer, center of a group. Algebra of subgroups, product of two subgroups. Subgroups generated by a subset; generators and defining relations. Subgroups of the group of all integers.

Properties of cyclic groups, generators. Characterizations of the subgroups of a cyclic group.

Properties of permutations, cycle notation for permutations, even and odd permutations, alternating group. Order and conjugates of a permutation.

Books Recommended:

1. M. Artin, Abstract Algebra, 2nd Ed., Pearson, 2011.
2. David S. Dummit and Richard M. Foote, Abstract Algebra, 3rd Ed., Wiley, 2011.

3. John B. Fraleigh, A First Course in Abstract Algebra, 7th Ed., Pearson, 2013.
4. Joseph A. Gallian, Contemporary Abstract Algebra, 9th Ed., Cengage, 2020.
5. I. N. Herstein, Topics in Algebra, 2nd Ed., Wiley, 2006.
6. Charles C. Pinter, A Book of Abstract Algebra, 2nd Ed., Dover, 2010.
7. Vivek Sahai and Vikas Bist, Algebra, 4th Ed., Alpha Science, 2018.
8. M. K. Sen, Shamik Ghosh, Parthasarathi Mukhopadhyay and Sunil Kumar Maity, Topics in Abstract Algbera, 4th Ed., Universities Press, 2022.

UNIT II : ANALYSIS I

Real Numbers: Algebraic and order axioms of the real number system without construction. Existence of nth roots of positive real numbers. The completeness property of R. Archimedean Principle, Dense property, Existence of nth roots of positive real numbers. Countable and uncountable sets, Countability of rational numbers and uncountability of real numbers.

Real sequences: Bounded sequences, Monotone sequences, Convergent sequences, Limit of a sequence, Theorem on limits of sequences, Cauchy's limit theorems, Cauchy's sequence and Cauchy's general principle. Sub-Sequence, Existence of monotone subsequence, Cluster points, Nested interval theorem, Bolzano-Weierstrass theorem for sequences, Limit superior and limit inferior.

Limit points, Bolzano-Weierstrass theorem, Theorem of lub and glb.

Functions of single real variable: Basic properties of limits, Bounded functions, Continuous functions.

Books Recommended:.

1. S. N. Mukhopadhyay and A. Layek, *Mathematical Analysis:Vol-I*, U. N. Dhar & Sons. Pvt. Ltd., 2nd Edition, 2009.
2. R. G. Bartle and D. R. Sherbert, *Introduction to Real Analysis*, Wiley India Pvt. Ltd, 4th Edition, 2021.
3. S. K. Mapa, *Introduction to Real Analysis*, Sarat Book Distributors, Revised 8th Edition, 2021
4. R. K. Ghosh and K. C. Maity, *An introduction to Analysis: Differential Calculus: Part-I*, New Central Book Agency, 13th revised Edition, 2011.

5. S. Ponnusamy, *Foundations of Mathematical Analysis*, Birkhauser, 2011.
6. Corey M. Dunn, *Introduction to Analysis*, CRC Press, 2017.

MJMA 02 : Analytic Geometry and Vector Calculus
Credits: 2+2 **Marks: 50+50**

UNIT I : ANALYTICAL GEOMETRY

Two-dimensions Geometry:
Transformation of Axes: Translation, Rotation, Invariants, Rigid motion, Oblique axes.

General Equation of Second Degree: Classification of quadratic equations representing lines, Parabola, Ellipse, Hyperbola. Tangent and Normal, Poles and Polars. Normal forms.

Polar equations: Polar co-ordinates, Polar equation of straight line, Conics, Tangent, Normal, Chord of contact.

Three-dimensional Geometry:
Brief review of direction cosines, direction ratios, plane and straight line.

Sphere: Equation in different forms, Section of a sphere with a given plane, Section of two spheres. Radical plane and line, Co-axial systems of spheres.

Cone: Cone with vertex at origin, Condition for general equation of second degree to represent a cone, Right circular cone, Intersection of cone by a plane, Condition for three mutually perpendicular generators.

Conicoids: Paraboloid, Ellipsoid, Hyperboloid and their tangent planes and normals.

Generating Lines: Ruled surfaces, Generating lines of hyperboloid of one sheet, Hyperbolic paraboloid, Properties of generators, Condition for a straight line to be a generator of a given conicoid.

Other Co-ordinate Systems: Spherical polar co-ordinates, cylindrical polar co-ordinates.

Books Recommended:

1. E. H. Askwith, The analytical geometry of the conic sections, Radha Publishing House, Kolkata, 1988.
2. M. C. Chaki, A text book of analytical geometry, Calcutta Publishers, Kolkata.
3. S. L. Loney, The elements of co-ordinate geometry, Macmillan and Company, London.
4. R. J. T. Bell, An elementary treatise on co-ordinate geometry of three dimensions, Macmillan India Ltd.
5. Shanti Narayan, Analytical solid geometry, S. Chand & Co. (Pvt.) Ltd, New Delhi.

UNIT II : VECTOR CALCULUS

Products: Brief review of scalar and vector products**.** Scalar triple products and vector triple products, Properties of scalar and vector triple products, Scalar and vector products of four vectors, reciprocal system of vectors. Applications.

Limit and Continuity of Vector Functions.

Vector Differentiation: Derivative of a vector function and its properties. Tangent and normal vectors. Serret-Frenet formula. Directional derivative, gradient, divergence, curl, Laplacian and their properties. Applications.

Vector Integration: Introduction to double and triple integrals**,** Line integrals, surface integrals, volume integrals, Green's theorem, Stokes' theorem, Gauss's divergence theorem. Applications.

Orthogonal Curvilinear Coordinates: Arc length, surface area and volume element in curvilinear coordinate system. Gradient, divergence, curl and Laplacian in curvilinear coordinate system**.** Cylindrical Co-ordinate system, spherical polar coordinate system. Applications.

Books Recommended:

1. Vector Calculus, S. J. Colley. (Pearson)
2. Vector Calculus, J. E. Marsden and A. J. Tromba (Freeman, W. H. & Company)
3. Vector Analysis, M. R. Spiegel (Schaum's Outlines)
4. Vector and Tensor Analysis, U. Chatterjee and N. Chatterjee (Academic Publisher)
5. Vector Analysis, J.G. Chakravorty and P.R. Ghosh (U. N. Dhur & Sons)
6. Vector Analysis with Applications, A. A. Shaikh and S. K. Jana (Narosa Publishing House Pvt. Ltd., New Delhi)

7. A Text Book of Vector Analysis, Shanti Narayan (S. Chand Publishing)

SEMESTER II

MJMA 03 : Algebra III
Credits: 4 **Marks: 100**

Properties of cosets, Lagrange's theorem and its consequences. Index of a subgroup, multiplicative property of index.

External direct product of groups; examples and basic properties. Product of cyclic groups.

Normal subgroups and factor groups.

Group homomorphisms, properties of homomorphisms. Isomorphisms, definition and examples. Properties of isomorphisms. Cayley's theorem. Isomorphism classes of cyclic groups and groups up to order 6. Isomorphism theorems and the correspondence theorem.

Automorphism, inner automorphism, automorphism groups, automorphism groups of finite and infinite cyclic groups.

Definition and examples of rings, properties of rings, subrings. Integral domains and fields, characteristic of a ring.

Ideal, operations on ideals and ideal generated by a subset of a ring. Factor rings; the chinese remainder theorem. Prime and maximal ideals.

Ring homomorphisms, properties of ring homomorphisms. Isomorphism theorems and the correspondence theorem. The field of quotients.

Books Recommended:

1. M. Artin, Abstract Algebra, 2nd Ed., Pearson, 2011.
2. David S. Dummit and Richard M. Foote, Abstract Algebra, 3rd Ed., Wiley, 2011.

3. John B. Fraleigh, A First Course in Abstract Algebra, 7th Ed., Pearson, 2013.
4. Joseph A. Gallian, Contemporary Abstract Algebra, 9th Ed., Cengage, 2020.
5. I. N. Herstein, Topics in Algebra, 2nd Ed., Wiley, 2006.
6. Charles C. Pinter, A Book of Abstract Algebra, 2nd Ed., Dover, 2010.
7. Vivek Sahai and Vikas Bist, Algebra, 4th Ed., Alpha Science, 2018.
8. M. K. Sen, Shamik Ghosh, Parthasarathi Mukhopadhyay and Sunil Kumar Maity, Topics in Abstract Algbera, 4th Ed., Universities Press, 2022.

MJMA 04 : Differential Equations I
Credits: 4 **Marks: 100**

Ordinary Differential Equations:
Basic Concepts:
Definitions of ordinary differential equation (ODE). Formation of ODEs by elimination of arbitrary constants. Formation of ODEs from real-world problems. Meaning of the solution of ODEs (Geometrical and Physical). Concept of linear and nonlinear ODEs. Concept of initial value problems and boundary value problems in ODEs. Concept of direction field and isocline.

First Order ODEs of first Degree:
Existence and uniqueness theorem for 1st order and higher order differential equations (Statement only). General solution and particular solution of 1st order IVP. Separable equations and equations reducible to this form. Homogeneous equations. Non-homogeneous equations. Exact equations and condition for exactness. Integrating factor and methods of finding integrating factors. Linear equations. Bernoulli equations. Equations reducible to 1st order linear equations. Solution of 1st order ODE by variation of parameter method. Orthogonal and oblique trajectories. Riccati's equation, Method of solving Riccati's equation. Applications of 1st order differential equations.

First Order ODEs of Higher Degree:
Equations solvable for p. Equations solvable for x. Equations solvable for y. Clairaut’s equation. Equations reducible to Clairaut’s form. Lagrange's Equation. p-discriminant and c-discriminant relations. Nodal locus. Cuspidial locus. Tac Locus. Singular solution. Geometrical significance of singular solutions.

Higher Order Linear ODEs with Constant Coefficients:
Linearly dependent and independent solutions. Wronskian and its properties. General solution of homogeneous equation of second order. Principle of superposition for homogeneous equation. Finding out Particular Integrals (P.I.) by symbolic operator D. Solution of ordinary differential equations by variation of parameters method. Applications of higher order differential equations.

Higher Order Linear ODEs with Variable Coefficients:
Cauchy-Euler homogeneous equations. Method of variation of parameters. Criterion of an exact differential equation, linear and nonlinear exact equations. Change of dependent variable. Reduction to normal form. Change of independent variable. Reduction of order. Factorization of the operators.

Books Recommended:

1. Elementary Differential Equations, E. D. Rainville , P. E. Bedient, R. E. Bedient. (Pearson)
2. A First Course in Differential Equations with Modeling Applications, D.G. Zill (Cengage)
3. Differential Equations, S. L. Ross (Wiley)
4. Differential Equations, P. R. Ghosh and J. G. Chakraborty (U. N. Dhar & Sons)
5. An introduction to Differential Equations, R. K. Ghosh, K. C. Maity, (New Central)
6. Elementary Differential Equations and Boundary Value Problems, W. E. Boyce, and R. C. DiPrima (Wiley)
7. Theory of Ordinary Differential Equations, E.A. Coddington, N. Levinson (Tata McGraw-Hill)
8. Differential Equations, H. T. H. Piaggio (G. Bell and Sons)

SEMESTER III

MJMA 05 : Analysis III
Credits: 4 **Marks: 100**

Continuous functions on an interval, intermediate value theorem, location of roots theorem, preservation of intervals theorem. Boundedness theorem,

Discontinuous functions, Classification of discontinuities.Monotone functions and its points of discontinuity. Uniform continuity, Sequential criteria for non-uniform continuity, uniform continuity theorem.

Series: Convergence and Divergence, Series of non-negative terms, p-series.
Test for convergence: Comparison test, D'Alembert ratio test, Cauchy's root test, Kummer's test, Raabe's test, Logarithmic test, Gauss test.
Alternating series: Leibnitz's test, Absolute convergence, Conditional convergence.

Riemann integration: Upper and lower sums, Riemann conditions of integrability, Riemann sum and definition of Riemann integral through Riemann sums, equivalence of two definitions, Riemann integrability of monotone and continuous functions and discontinuous functions, Properties of the Riemann integral, Mean value theorems for Riemann integral, Fundamental theorem of Calculus.

Functions of bounded variations, Simple properties, Jordan decomposition theorem.

Riemann-Stieltjes Integral: Definition and Properties, Existence of the integral.

Improper integrals; Convergence and Absolute Convergence, Cauchy's Criterion (Statement only), Comparison tests, Abel's and Dirichlet's tests, Convergence of Beta and Gamma functions.

Books Recommended:

1. W. Rudin, *Principles of Mathematical Analysis*, TMH, Third Edition, Indian Edition, 2013.
2. T. M. Apostal, *Mathematical Analysis*, Narosa Book Disributors Pvt. Ltd., 2nd Edition, 2000.
3. S. C. Malik & S. Arora, *Mathematical Analysis*, New Age International Publishers, 4th Edition, 2010.
4. R. G. Bartle and D. R. Sherbert, *Introduction to Real Analysis*, Wiley India Pvt. Ltd, 4th Edition, 2021.
5. R.K. Ghosh and K.C. Maity, *An introduction to Analysis: Differential Calculus: Part-I*, New Central Book Agency, 13th revised Edition, 2011.
6. S. N. Mukhopadhyay and S. Mitra – *Mathematical Analysis* – Vol-II (U. N. Dhar & Sons. Pvt. Ltd.), 2014.
7. Konrad Knopp, *Infinite sequences and series*, Dover Publication, 1990.

8. Shanti Narayan, *A course of Mathematical Analysis*, S. Chand and Co. Ltd., 1st Edition, 2005.
9. S. K. Mapa, *Introduction to Real Analysis*, Sarat Book Distributors, Revised 8th Edition, 2021.
10. S. Ponnusamy, *Foundations of Mathematical Analysis*, Birkhauser, 2011

MJMA 06 : Differential Equations II and Dynamics of a Particle

Credits: 2+2 **Marks: 50+50**

Unit I : Differential Equations II

Simultaneous Linear Differential Equations:

Methods of solving simultaneous equations of Type I. Methods of solving simultaneous equations of Type II. Geometrical Interpretation.

Total Differential Equations:

Conditions for integrability. Methods of solving total differential equations. Geometrical Interpretation.

Series Solution of ODEs and Special Functions:

Ordinary point, Singular point, Series solutions (Power series, Frobenius method), Series solution of Legendre's equation. Bessel's equation. Hypergeometric equation. Laguerre's equation and Hermite's equation.

Legendre functions. Legendre polynomials and their zeros. Rodrigues' formula. Generating function. Legendre coefficients. Recurrence relations. Orthogonality.

Bessel functions. Bessel function of first kind and of second kind of integer order. Generating function. Identities. Recurrence relations. Orthogonality. Bessel integral formula.

Laguerre polynomials. Generating function. Recurrence relations. Orthogonality.

Hermite polynomials. Generating function. Recurrence relations. Orthogonality.

Systems of Linear Differential Equations:

Types of linear systems, differential operators, an operator method for linear systems with constant coefficients, Basic theory of linear systems in normal form, homogeneous linear systems with constant coefficients: Two equations in two unknown functions. Theory of n-th order homogeneous linear differential equations. Theory of n-th order non-homogeneous linear differential equations. Wronskian, Abel-Liouville formula.

Books Recommended:

1. Differential Equations, S. L. Ross (Wiley)
2. Special Functions for Scientists and Engineers, W.W. Bell (Dover).
3. Special Functions & Their Applications, N. N. Lebedev, R. A. Silverman (Dover)
4. Introduction to Bessel Functions, F Bowman (Dover)
5. An Introduction to Special Functions, C. Viola (Springer)
6. Special Functions, Z. X. Wang, D. R. Guo (World Scientific)
7. Elementary Differential Equations and Boundary Value Problems, W. E. Boyce and R. C. DiPrima (Wiley)
8. Differential Equations and Boundary Value Problems: Computing and Modeling, C. H. Edwards and D. E. Penny (Pearson Education India)

UNIT II : Dynamics of a Particle

Fundamental definitions and principles.

Motion in a straight line, Harmonic oscillator, Damped forced oscillation.

Motion in a plane: Tangential and Normal Accelerations, Radial and Cross-radial accelerations, Path for a given law of force.

Motion of a particle under central forces, Planetary motion.

Motion in a resisting medium.

Books Recommended:

1. S. L. Loney, An Elementary Treatise on the Dynamics of a Particle and of Rigid Bodies, McGraw-Hill

2. F. Chorlton, A Text Book of Dynamics, (John Wiley & Sons, New York) 2nd Edition

3. A. S. Ramsey, Dynamics (Vols. I & II), Cambridge University Press

4. M. Spiegel, Theoretical Mechanics, Schaum's Outline Series

SEMESTER IV

MJMA 07 : Algebra IV
Credits: 4 **Marks: 100**

Vector spaces, subspaces, algebra of subspaces, direct sum of subspaces, quotient spaces. Linear combination of vectors, linear span. Linear independence. Basis and dimension, dimension of subspaces. Coordinates; change of coordinate matrices.

Row rank and column rank of a matrix; their equivalence. Rank of a matrix; properties of rank. Full rank factorization and it's consequences. Rank inequalities.

Linear transformations. Algebra of linear transformations. Isomorphisms. Projections, their sum, difference and product.

Matrix representations of a linear transformation; similarity of matrix representations. Rank of a linear transformation and its matrix representation. Determinant and trace of a linear transformation.

Linear functionals. Dual spaces, dual basis. Double dual, reflexivity of finite dimensional vector spaces. Transpose of a linear transformation and its matrix in the dual basis. Annihilator of a subspace.

Determinant functions; their existence and uniqueness. Properties of determinant. Determinant and rank of a matrix.

Books Recommended:

1. Sheldon Axler, Linear Algebra Done Right, 3rd Ed., Springer, 2014.
2. R. B. Bapat, Linear Algebra and Linear Models, 3rd Ed., Springer, 2012.
3. Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence, Linear Algebra, 5th Ed., Pearson, 2022.
4. Kenneth Hoffman, Ray Alden Kunze, Linear Algebra, 2nd Ed., Pearson, 2015.
5. Jin Ho Kwak and Sungpyo Hong, Linear Algebra, 2nd Ed., Birkhauser, 2004.
6. Vivek Sahai and Vikas Bist, Linear Algebra, 2nd Ed., Narosa, 2013.
7. Helene Shapiro, Linear Algebra and Matrices, American Mathematical Society, 2015.

8. Gilbert Strang, Introduction to Linear Algebra, 6th Ed., Wellesley-Cambridge Press, 2023.
9. Fuzhen Zhang, Matrix Theory : Basic Results and Techniques, 2nd Ed., Springer, 2011.

MJMA 08 : Analysis IV

Credits: 4 **Marks: 100**

Sequence and Series of functions: Pointwise and uniform convergence, Theorems on continuity, derivability and integrability of the limit function and sum function, Cauchy criterion for uniform convergence and Weierstrass M-Test.

Power series: Radius of convergence, Cauchy Hadamard Theorem, Differentiation and integration of power series, Abel's Theorem, Real analytic function, Weierstrass Approximation Theorem.

Functions of Several Variables: Limit and Continuity, Partial derivatives, Homogeneous Functions and Euler's Theorem, Differentiability, Chain rules, Total differentials, Schwarz's and Young's theorems, Theorem of existence and uniqueness of implicit functions, Jacobians and their simple properties, Statement of inversion theorem, Mean value theorem, Taylor's theorem, Maxima, Minima and saddle points, Lagrange's multiplier method, Differentiation under integral sign, Leibnitz rule.

Double integration over rectangular region, double integration over non-rectangular region, Double integrals in polar co-ordinates, Triple integrals, Triple integral over a parallelepiped and solid regions. Change of variables in double integrals and triple integrals, Volume by triple integrals, cylindrical and spherical co-ordinates**.**

Books Recommended:

1. Konrad Knopp, *Theory and application of Infinite series*, Dover Publication, 1990.
2. S. K. Mapa, *Introduction to Real Analysis*, Sarat Book Distributors, Revised 8th Edition, 2021.
3. R. R. Goldberg, *Methods of Real Analysis*, Oxford and IBH Publishing Co. Pvt. Ltd, 2021.
4. P. C. Bhakta, *A Course of Real analysis*, Sarat Book Distributors, 2014.
5. David. V. Widder, *Advanced Calculus*, Prentice Hall of India Private Limited.

6. S. C. Malik and Savita Arora, *Mathematical Analysis*, New Age International (P) Ltd. Publishers, 2005.
7. K. C. Maity and R. K. Ghosh, *An introduction to Analysis, Integral Calculus*, Books and Allied (P) Ltd, 2013.
8. S. N. Mukhopadhyay and S. Mitra – *Mathematical Analysis* – Vol-II (U. N. Dhar & Sons. Pvt. Ltd.), 2014.
9. S. Ponnusamy, *Foundations of Mathematical Analysis*, Birkhauser, 2011.

MJMA 09 : Differential Equations III and Dynamics of a Rigid Body

Credits : 2+2 **Marks : 50+50**

Unit I : Differential Equations III

Partial Differential Equations (PDEs):

First Order PDEs:
Introduction. Formation of PDE and geometrical interpretation. Genesis of first order PDEs. Classification of integrals. Canonical forms of first order linear equations. Lagrange's method to solve quasi-linear PDEs. Method of separation of variables for solving first order PDEs. Cauchy's problem for first order PDEs.

Compatible systems of first order PDEs. Charpit's method to solve first order nonlinear PDEs. Integral surface through a given curve. Monge cone, Characteristic strip. Cauchy's method of characteristic.

Second Order PDEs:
Second order PDEs with constant coefficients. Simple techniques to solve second order PDEs with variable coefficients.

Applications leading to PDEs.Well-posed PDEs. First-order PDEs. Integral surface through a given curve. Monge cone. Characteristic strip. Cauchy's method of characteristic.

Second-order PDEs with constant and variable coefficients: Monge's method of solution.

Books Recommended:
1. Partial Differential Equations: An Introduction, W.A. Strauss, (John Wiley & Sons Ltd.)

2. An Elementary Course in Partial Differential Equations, T. Amarnath, (Narosa)
3. Partial Differential Equations, F. Prasad and R. Ravindran (New Age International Publishers)
4. Elements of Partial Differential Equations, I. N. Sneddon, (Tata McGraw-Hill)
5. Linear Partial Differential Equations for Scientists and Engineers, Tyn Myint-U, L. Debnath (Birkhäuser)
6. Nonlinear Partial Differential Equations for Scientists and Engineers, L. Debnath (Birkhäuser)

Unit II : Dynamics of a Rigid Body

Kinetics of a rigid body: Moments and products of inertia of a rigid body; Simple cases of
moments of inertia; Momental ellipsoid; Principal axes, Examples.

Motion of a rigid body: D'Alembert's principles; General equations of motion of a rigid body
in space.

Motion about a fixed axis, Examples; Compound pendulum; Motion about a fixed axis under
repulsive forces; Centre of percussion.

Motion in two dimensions: Motion in a plane under finite forces; Kinetic energy, moment of a
force and angular momentum.

Books Recommended:

1. S. L. Loney, An Elementary Treatise on the Dynamics of a Particle and of Rigid Bodies (McGraw-Hill)
2. F. Chorlton, A Text Book of Dynamics, (John Wiley & Sons, New York) 2nd Edition, 1983.
3. A. S. Ramsey, Dynamics (Vols. I & II) , (Cambridge, 1973).
4. M. Spiegel, Theoretical Mechanics (Schaum's Outline Series)

MJMA 10 : Mathematical Probability and Statistics

Credits : 2+2 **Marks : 50+50**

Unit I : Mathematical Probability

Definition and basic concepts: Random experiments, events: simple and compound, event space, examples of the same. Probability of an event: classical definition of probability, limitations; statistical regularity, frequency definition of probability. Fundamental axioms: axiomatic definition of probability and properties; general addition rule: the match problem; conditional probability: general multiplication rule, Polya's urn problem; Bayes' theorem: Laplace's urn problem; Stochastic independence. Compound experiments, Repeated independent trials: Bernoulli trials, Binomial law. Poisson approximation to the binomial law, Poisson trials. The multinomial law. Infinite sequence of Bernoulli trials. Markov chains.

Probability distributions: Random variables: continuous and discrete; Distribution functions and their properties: continuous and discrete distributions in one and two dimensions; Bivariate continuous distribution; conditional distributions; transformation of random variables in two dimensions. Mathematical expectations: Mean, variance and moments of different distributions; Moment generating function of a random variable; Characteristic functions; Median, mode. Expectation for bivariate distributions, covariance, correlation coefficient; Joint characteristic function, Multiplication rule for expectations; reproductive properties of some distribution functions; Conditional expectation. Regression lines and parabolic curve fittings.

Convergence in probability: Tchebycheff's inequality and convergence in probability, Tchebycheff's theorem; Bernoulli's theorem; Law of large numbers.

Some ideas on limit theorem: De Moivre-Laplace limit theorem, Asymptotically normal distribution, Central limit theorem and limit theorem for characteristic functions.

Stochastic processes: Markov process. Poisson process. Pure-birth process. Birth and Death process. Formulation of simple models and their analyses.

Books Recommended:

1. A. Gupta, Groundwork of Mathematical Probability and Statistics, Academic Press
2. W. Feller, An Introduction to Probability Theory and its Applications, Vols. I and II, Wiley Eastern
3. J. V. Uspensky, Introduction to Probability, Tata McGraw-Hill
4. B.V. Gnedenko, The Theory of Probability, Tata McGraw-Hill

Unit II : Statistics

Random samples: Basic concept of populations and random samples, Distribution of the sample, table and graphical representations, Sample characteristics.

Sampling distribution: Sampling distribution of a statistic (mean and variance), Estimates- Consistent and Unbiased, Exact sampling distribution of the Normal population.

Some ideas on Special distribution: χ^2, t and F distributions.

Statistical Inference I: Estimation of parameters: Methods of finding point estimators of parameters: moment method, maximum likelihood method, least square method. Criterion for evaluating goodness of estimators: unbiased estimator, relatively efficient estimator, minimum variance unbiased estimator, sufficient estimator, consistent estimator. Interval estimation. Confidence intervals for Binomial and Normal population parameters. Approximate confidence Interval for parameter with MLE. Criteria for evaluating confidence intervals.

Statistical Inference II: Testing of hypothesis. Statistical hypothesis: Simple and Composite; Two types of errors. Methods of finding tests: Likelihood ratio tests, Invariant tests, Bayesian tests and Union-Intersection and Intersection-Union Tests. Best critical region for simple hypothesis, Neyman-Pearson theorem. Application to Normal population. Methods for evaluating the goodness of a test procedure: Powerfulness, Unbiasedness and Invariancy and Local Powerfulness. Goodness of fits tests: Pearson Chi-squared test.

Books Recommended:

1. Aitken, A. C. (1944). Statistical Mathematics. 3rd edn. Edinburgh and London: Oliver and Boyd
2. Gupta, A. (2015). Groundwork of Mathematical Probability and Statistics. Academic Press
3. Fisher, R. A. (1922), On the mathematical foundations of theoretical statistics. Reprinted in Contributions to Mathematical Statistics (by R. A. Fisher) (1950), J. Wiley & Sons, New York
4. Hogg, R. V. and Craig, A. T. (1978). Introduction to Mathematical Statistics. New York: Macmillan
5. Taylor, L. D. (1974). Probability and Mathematical Statistics. New York: Harper & Row
6. Sahoo, P. (2013). Probability and mathematical Statistics. Louisville, USA

7. Gupta, S. C. and Kapoor, V. K. (2000) Fundamentals of Mathematical Statistics, Sultan Chand and Sons, Delhi
8. Das, N. G. (2008) Statistical Methods (Part-II), McGraw Hill Education

SEMESTER V

MJMA 11 : Analysis V
Credits : 4 **Marks : 100**

Construction of real number system: Assuming N and induction construction of Z, Q and the cauchy construction of R, Cantor-Dedekind theorem.

Open sets, closed sets, Neighbourhood of a point, Limit points, Closure and interior of a set, Structure of open sets, Open covering, Lindelöf covering theorem, Heine-Borel theorem and its converse.

Fourier Series: Determination of Fourier coefficients, Convergence of Fourier Series, Dirichlet's Kernel, Parseval's theorem, Weierstrass Approximation Theorem for trigonometric polynomial. Sine and cosine series.

Axiom of choice, Cardinal number, Cardinal arithmetic, Schröder–Bernstein theorem, Cardinality of real numbers and natural numbers.

Lebesgue measure on real line: Outer measure of a set. Properties of outer measure. Measurable sets and Lebesgue measure. Properties of measurable sets. Borel sets, Existence of non-measurable sets, Cantor sets.

Books Recommended:

1. S. C. Malik and Savita Arora, *Mathematical Analysis*, New Age International (P) Ltd. Publishers, 2005.
2. S. K. Mapa, *Introduction to Real Analysis*, Sarat Book Distributors, Revised 8th Edition, 2021.
3. R. R. Goldberg, *Methods of Real Analysis*, Oxford and IBH Publishing Co. Pvt. Ltd, 2021.
4. D. Soma Sundaram and B. Choudhury, *A first Course in Mathematical Analysis*, Norosa Publishing House, 1996.
5. R. G. Bertle and D. R. Sherbert, *Introduction to Real Analysis*, John Wiley & Sons, Inc.
6. H. H. Sohrab, *Basic Real Analysis*, Birkhauser, 2003.

7. P. K. Jain, V. P. Gupta, P. Jain, *Lebesgue measure and integration*, 3rd Edition,New Age International Publishers 2019.

MJMA 12 : Linear Programming Problems and Mathematical Modeling
Credits : 2+2 **Marks: 50+50**

Unit I : Linear Programming Problems

Introduction to Linear Programming Problem (LPP), Mathematical formulation of LPP, Graphical method of solution, Nature of solutions.

Basic Concepts: Basis, replacing a vector in a basis. Solution of a system of linear equations- Basic solution, Basic feasible solution. Convex set- Extreme points. Matrix formulation of L.P.P. Standard form of LPP. Relation among the optimal solution of a LPP, Basic feasible solution and extreme point of the convex set of all feasible solutions.

Fundamental theorem of LPP, Theory of Simplex method, Reduction of a feasible solution to a B.F.S.; improving a basic feasible solution; Optimality checking, Unboundedness; Existence of alternative optimal solution. Simplex algorithm and the simplex tableau, Slack, Surplus and Artificial variables-introduction and its subsequent removal, Inconsistency and redundancy.

Two-phase Simplex method. Concept of degeneracy- cycling, Methods of solving L.P.P. involving Degeneracy and Cycling. Charnes' perturbation method. Problems having degeneracy at (i) the initial stage, (ii) a subsequent stage.

Concept of Duality: Its economic interpretation. Construction of dual. The relation between feasible solution of dual and primal problems. Fundamental properties of Dual Problems. Weak duality theorem, Strong duality theorem, Fundamental duality theorem. Computational aspects of Simplex method and duality. Dual simplex method-The Algorithm and Difference between Regular Simplex method and Dual Simplex method.

Transportation Problems: Mathematical Formulation; finding an initial basic feasible solution; The Transportation Algorithm-checking for optimality, improving a B.F.S. Degeneracy in Transportation Problem. Resolution of degeneracy in the initial stage. Resolution of degeneracy during solution stage. Unbalanced Transportation Problems. Maximization in Transportation Problems.

Assignment Problems: Mathematical Formulation; Hungarian Assignment Method, Unbalanced Assignment Problem. Maximization in Assignment Problem. Travelling Salesman Problem.

Books Recommended:

1. Linear Programming and Game theory, Chakraborty & Ghosh; Moulik Library, Kolkata
2. Operations Research: An introduction, Hamdy A. Taha, 10th Edition, Pearson, 2011
3. Operations Research: Principles and Practice-Ravindran, Phillips, Solberg; Wiley India
4. Operations Research-Richard Bronson, Govindasami Naadimuthu, Tata McGraw-Hill
5. Principles of Operations Research, Harvey M. Wagner, PHI
6. Operations Research: Theory and Applications, J.K.Sharma, Macmillian India Ltd
7. An introduction to Linear Programmig, Mukhopadhyay, Maity & Mazumdar, Kalimata
Pustakalaya, Kolkata
8. Game Theory: A nontechnical introduction, M. D. Davis, Basic Books, N.Y
9. Introduction to the theory of games, J.C.C. Mckinsey, Tata McGraw-Hill B.C., N.Y
10. Linear Programming, S.I.Gass, Tata McGraw-Hill B.C., N.Y

Unit II : Mathematical Modeling

PART-A: Physical Systems

Introduction, Emergence of Mathematical Modelling on simple situations; Basic steps of Mathematical Modelling - its needs; Process / technique of Mathematical Modelling; Some characteristics of Mathematical Models; Importance of the usage of mathematical models over physical models; Classification of mathematical models; Deterministic and Stochastic models and their distinctive features with illustrations; Limitations of Mathematical Modelling.

Formulation of some mathematical models and their analyses for (i) harmonic oscillator, (ii) damped and forced oscillator. Simple pendulum; Compound pendulum; Electric circuits (L-R, R-C, L-R-C). Derivation of Heat equation, Wave equation (linear wave and non-linear wave), Laplace equation. Boundary value problems for infinite and semi-infinite string. Nonlinear problems and perturbation technique.

PART-B: Biological Systems

Autonomous dynamical system and its classification, Jacobian matrix, System reducible to autonomous system, Time-dependent system, Fixed points and their characterization - node, saddle point, focus, centre and concept of limit cycle with simple illustrations, Stability of fixed points.

Population Models: (i) Single-species models – Malthus, Logistic, Smith and Qiwu's population growth models; Stochastic birth and death processes; Discrete-time models; Trajectories (ii) Interacting populations – A classical predator-prey model; Stability of equilibrium points; Derivation of Lotka-Volterra model; Two competing species model and its stability analysis; Mutualism model and its stability; Biological applications of bifurcation theory in various ecological models.

Idea of diffusion in biological system and reaction-diffusion PDE equation models: Fick's laws of diffusion; Diffusion equation – one and two dimensional forms; Predator-prey model with diffusion; Competition model with diffusion; Influence of diffusion on stability of both predator-prey and competition models.

Books Recommended:

1. Mathematical Models, Mechanical Vibrations, Population Dynamics, and Traffic Flow: An Introduction to Applied Mathematics, Richard Haberman, Prentice-Hall, Inc., Englewood Cliffs, New Jersey (1977)
2. Introduction to Differential Equations with Dynamical System, Stephen L Campbell and Richard Haberman, Princeton University Press (2008)
3. Concept of Mathematical Modeling, W. Meyer, McGraw-Hill, New York (1994)
4. Mathematics for Dynamic Modeling, E. Beltrami, Academic Press, Orlando, Florida (1987)
5. Mathematical Modeling with case Studies, A Differential Equation Approach using Maple and MATLAB, Belinda Barnes and Glenn R. Fulford, Taylor and Francis Group (2009)
6. Differential Equations and Boundary Value Problems: Computing and Modeling, C. H. Edwards and D. E. Penny, Pearson Education India (2005)
7. Mathematical Biology, J. D. Murray, Springer-Verlag, Berlin (1989)
8. Elements of Mathematical Ecology, M. Kot, Cambridge University Press (2003)
9. Dynamical Systems in Population Biology, X.-Q. Zhao, Canadian Mathematical Society (2017)

10. Mathematical Models in Biology & Medicine, J. N. Kapur, East West Press Pvt. Ltd. (1985)
11. Differential Equations and Dynamical Systems, L. Perko, Springer (2001)

MJMA 13 : Numerical Analysis
Credits: 4 **Marks : 100**

Accuracy of Approximate Calculations:
Approximate numbers, Floating point form of numbers, Normalized scientific form of numbers, Overflow and underflow. Binary machine numbers, Significant figures, Rounding off numbers, Absolute, Relative and Percentage errors, Loss of significance. General formula for errors and its applications, Propagation of round off errors in arithmetic operations. Inherent errors in numerical computations.

Interpolation:
Weierstrass' approximation theorem (statement only), Polynomial interpolation, Existence and uniqueness of interpolating polynomial, Derivation of the error in interpolation, Finite differences: Forward and Backward, Difference operators (Forward and Backward), Shifting operator, Properties and Relations between these operators; Difference table, Error in the entry values noise level, Differences of a polynomial, Newton's forward and backward interpolation formulae, Lagrange's interpolation formula, Inverse interpolation, Divided difference and their properties. Generalized Newton's divided difference formula. Confluent divided difference.

Numerical Differentiation:
Differentiation formulae based on Newton's forward and backward interpolation formulae. Error in differentiation.

Numerical Integration:
Newton-Cotes quadrature formula (without error), Degree of precision, Trapezoidal rule, Simpson's one-third rule, Weddle's rule, Composite rules, Derivation of the error for Trapezoidal and Simpson's 1/3 rd rules.

Numerical Solution of Nonlinear Equations:
Tabulation method, Bisection method, Regula-Falsi method, Secant method, Fixed point iteration method, Newton-Raphson method, Geometrical significance and convergence of these methods.

Numerical Solution of a System of Linear Equations:

Directs methods: Gaussian elimination, LU factorization, Gauss-Jordan elimination; Operation counts, Matrix factorization, Matrix inversion.
Iterative methods: Norms of vector and matrices, Jacobi, Gauss-Seidel, SOR and their convergence.

Initial Value Problems:
Solution of first order ordinary differential equations: Picard's method, Taylor's method, Euler's method and its modified form (concept of Predictor-Corrector method),
Error estimate and its convergence, Runge-Kutta method of second and fourth orders and their significance.

Books Recommended:

1. M. K. Jain, S.R. K. Iyengar and R.K. Jain, Numerical Methods for Scientific and Engineering Computation, Wiley Eastern Limited, (1991)
2. D. Kincaid, W. Cheney, Numerical Analysis: Mathematics of Scientific Computing, American Mathematical Society, Rhode Island (2002)
3. R. L. Burden, J. D. Faires, Numerical Analysis: Theory and Applications, Cengage Learning, New Delhi (2005)
4. P. Niyogi, Numerical Analysis and Algorithms, Tata McGraw-Hill Publishing Company
   Limited, (2003)
5. A. Gupta and S. C. Bose, Introduction to Numerical Analysis, Academic Publisher, (1989)
6. N. Dutta and R. N. Jana, Introductory Numerical Analysis, Shreedhar Prakashani, (2001)

SEMESTER VI

MJMA 14 : Algebra V
Credits : 4 **Marks : 100**

Eigen values and eigen vectors of a matrix, characteristic polynomial. Eigen spaces and diagonalizability of a matrix. Eigen values, eigen vectors and the characteristic polynomial of a linear operator. Invariant subspaces and the Cayley-Hamilton theorem.

Diagonal forms, minimal polynomials. Invariant direct sum and the primary decomposition theorem. Equivalent characterizations of diagonalizability. The spectral theorem for the diagonalizable linear operators.

Jordan canonical forms. Generalized eigen spaces and the fundamental theorem of linear operators. Nilpotent operators, Weyr and segre characteristics of a nilpotent complex matrix. Invariants of a nilpotent operator, uniqueness of the invariants. Jordan canonical form of a general matrix. Similarity of matrices and Jordan canonical forms. Reduction of real and complex matrices to Jordan canonical form.

Polynomial rings over commutative rings. The division algorithm for polynomials and consequences. Units in a polynomial ring.

Divisibility in integral domain, prime and irreducible elements. Euclidean domain, principal ideal domain and unique factorization domain; greatest common divisor in principal ideal domains and unique factorization domains. The ring of Gaussian integers.

Factorization of polynomials, Gauss's theorem. Unique factorization in Z[x]. Irreducibility tests for polynomials, Eisenstein's criterion. The chinese remainder theorem in the polynomial rings over fields.

Books Recommended:

1. Michael Artin, Algebra, 2nd Ed., Pearson, 2011.
2. Sheldon Axler, Linear Algebra Done Right, 3rd Ed., Springer, 2014.
3. R. B. Bapat, Linear Algebra and Linear Models, 3rd Ed., Springer, 2012.
4. David S. Dummit and Richard M. Foote, Abstract Algebra, 3rd Ed., Wiley, 2011.
5. John B. Fraleigh, A First Course in Abstract Algebra, 7th Ed., Pearson, 2013.
6. Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence, Linear Algebra, 5th Ed., Pearson, 2022.
7. Joseph A. Gallian, Contemporary Abstract Algebra, 9th Ed., Cengage, 2020.
8. I. N. Herstein, Topics in Algebra, 2nd Ed., Wiley, 2006.
9. Kenneth Hoffman, Ray Alden Kunze, Linear Algebra, 2nd Ed., Pearson, 2015.
10. Jin Ho Kwak and Sungpyo Hong, Linear Algebra, 2nd Ed., Birkhauser, 2004.
11. Charles C. Pinter, A Book of Abstract Algebra, 2nd Ed., Dover, 2010.
12. Vivek Sahai and Vikas Bist, Algebra, 4th Ed., Alpha Science, 2018.
13. Vivek Sahai and Vikas Bist, Linear Algebra, 2nd Ed., Narosa, 2013.
14. M. K. Sen, Shamik Ghosh, Parthasarathi Mukhopadhyay and Sunil Kumar Maity, Topics in Abstract Algbera, 4th Ed., Universities Press, 2022.

15. Helene Shapiro, Linear Algebra and Matrices, American Mathematical Society, 2015.
16. Gilbert Strang, Introduction to Linear Algebra, 6th Ed., Wellesley-Cambridge Press, 2023.
17. Fuzhen Zhang, Matrix Theory : Basic Results and Techniques, 2nd Ed., Springer, 2011.

MJMA 15 : Analysis VI

Credits : 2+2 **Marks : 100**

Unit I : Complex Analysis II

Differentiability of Functions: Definition and Examples, The Cauchy-Riemann Equations, Sufficient conditions for differentiability, Cauchy-Riemann equations in polar form.
Analytic Functions: Definition and examples, Harmonic functions, Conjugate harmonic functions, Construction of an analytic function whose real part is given (Related theorem is to be assumed).

Infinite Series, Absolutely Convergent Series, Power Series, Radius of convergence, Cauchy-Hadamard theorem (Statement only).

Bilinear Transformations: Definition, Cross ratio, Fixed points of a bilinear transformation, Normal form of a bilinear transformation, Inverse points, Circles, Orientation principle.

Definite integrals of functions. Contours, Contour integrals and its examples, Upper bounds for moduli of contour integrals. Cauchy-Goursat theorem (Statement only), Cauchy integral formula. Derivatives of Cauchy integral formula, Liouville's theorem and the fundamental theorem of algebra.

Unit II : Metric Spaces

Metric Spaces: Definition, Example of Metric Spaces. Convergence of a sequence, Spherical Neighbourhood, Closure of a Set, Closed Sets, Open Sets, Interior and Boundary Points.

Complete Metric Spaces: Cauchy sequences, Example of complete metric spaces, Example of incomplete metric Spaces.

Diameter of a set, Bounded Set, Cantor's Intersection Theorem, Baire's category theorem, Isometry, Completion of metric spaces.

Continuous mappings and related results.

Fixed Point Theorems: Contraction mapping, Idea of fixed point, Banach's contraction principle Theorem and its simple applications.

Compact metric spaces, Uniform continuity, Total boundedness, sup and inf on compact sets. Sequential compactness.

Books Recommended:

1. B. K. Lahiri, *Complex Analysis*, The World Press Private Limited.
2. R. V. Churchil and J. W. Brown, *Complex Variables and Applications*, McGraw-Hill, New York, 2013.
3. S. Ponnusamy, *Foundations of Complex Analysis*, Narosa Publishing House, 2005
4. Sobhakar Ganguly, *Elements of Complex Analysis*, Academic Publisher, 2020.
5. E. T. Copson, *An introduction to the theory of functions of a Complex Variable*, Oxford University Press, 1970.
6. L. V. Ahlfors, *Complex Analysis*, McGraw-Hill, New York, 2017.
7. B. K. Lahiri, *Metric Spaces*, The World Press Private Limited.
8. G. F. Simmons, *Introduction to Topology and Modern Analysis*, McGraw-Hill, 2017.
9. P. K. Jain and Khalil Ahmad, *Metric Spaces*, Narosa Publishing House, 2004.
10. M. N. Mukherjee, Elements of *Metric Spaces*, Academic Publishers, 2006.

MJMA 16 : C Programming and Laboratory for C Programming

Credits : 2+2 **Marks : 50+50**

Unit I : C Programming

Computer Language: Algorithm, Flowchart, Concept of programming languages, Machine language, Assembly language, Low and high-level

languages, Interpreter, Compiler, Assembler, Source and Object programs. Importance of C programming.

Constants, Variables and Data type of C-Program: Character set. Constants and variables data types, expression, assignment statements, declaration.

Operation and Expressions: Arithmetic operators, relational operators, logical operators.

Decision Making and Branching: decision making with if statement, if-else statement, Nesting if statement, switch statement, break and continue statement.

Control Statements: While statement, do-while statement, for statement.

Arrays: One-dimension, two-dimension and multidimensional arrays, declaration of arrays, initialization of one and multi-dimensional arrays.

User-defined Functions: Definition of functions, Scope of variables, return values and their types, function declaration, function call by value, Nesting of functions, passing of arrays to functions, Recurrence of function.

Pointers: Pointer declarations, Passing pointers to a function, Pointers and one dimensional and multidimensional array, Arrays of pointers.

File handling: Why files? Opening and closing of data file, Reading and writing a data file, Processing a data file.

Introduction to Library functions: stdio.h, math.h, string.h stdlib.h, time.h etc.

Unit II : Laboratory for C Programming

PART-A:
Basic C program: Writing basic C programs, executing and debugging a C Program, managing input and output operations.

Operators and Expressions: Writing C programs with arithmetic operators, relational operators, logical operators, and assignment operators.

Decision Making and Branching: Writing decision making C programs using control statements like if, if..else, if..else ladder, nested if, and switch cases.

Looping: Writing C programs using do, while, do-while, and for loops.

Arrays: Writing C programs using one- and two-dimensional arrays with special emphasis on matrices.

Functions: Writing C programs using user-defined functions- function calls, function declaration, no arguments and no return values; arguments but no return values, arguments with return values, no arguments but returns a value, passing arrays to functions.

Pointers and Structures: Writing C programs with pointers and structures.

File Management: Writing C programs using files for reading input and storing output.

Dynamic Memory Allocation: Writing C programs using malloc and calloc functions.

PART-B: Solutions of different mathematical problems by integrating the above concepts.

Books Recommended:

1. B. Gottfried and J. Chhabra, Programming With C, Tata Mcgraw Hill, 2005.
2. B. W. Kernighan and D. M. Ritchi , The C-Programming Language, 2nd Edi.(ANSI Refresher), Prentice Hall, 1977.
3. E. Balagurnsamy, Programming in ANSI C, Tata McGraw Hill, 2004.
4. Y. Kanetkar , Let Us C ; BPB Publication, 1999.
5. C. Xavier , C-Language and Numerical Methods, New Age International.
6. V. Rajaraman, Computer Oriented Numerical Methods, Prentice Hall of India, 1980

SEMESTER VII

MJMA 17 : Lebesgue Integration and Complex Analysis III

Credits : 2+2 **Marks : 50+50**

Unit I : Lebesgue Integration

Measurable functions and their properties, Equivalent functions, Simple functions, Measurable functions as limit of simple function, Sequence of measurable functions. Almost everywhere convergence, convergence in measure and almost uniform convergence, Riesz theorem and Egoroff's theorem, Lusin's theorem and Frechet's theorem.

Lebesgue's criterion for Riemann integrability, Lebesgue integral of bounded function and its properties. Comparison of Riemann integral and Lebesgue integral. Passage to the limit under the sign of integration. Bounded convergence theorem.

Summable functions. Integrals of summable functions. Properties of the integrals of summable functions. Fatou's Lemma, Monotone convergence theorem, Dominated convergence theorem.

Unit II : Complex Analysis III

Simply connected region and primitives of analytic functions, Morera's theorem. Maximum and Minimum modulus theorem, Schwarz's lemma, Taylor's theorem.

Zeros of an analytic functions, Uniqueness of analytic functions, Singularities and classification of singularities, Laurent's theorem, Limit points of zeros and poles, Riemann's theorem on removable singularity, Casorati-Weierstrass theorem.

Cauchy's residue theorem, Evaluation of integration by residues theorem, Argument principle, Rouche's theorem, Open mapping theorem.

Behavior of a function at the point at infinity, Conformal mapping, Riemann mapping theorem (Statement only), Introduction to analytic continuation, Riemann Surface.

Books Recommended:

1. H. L. Royden and P. Fitzpatrick, *Real Analysis*, Prentice Hall, 2010.
2. P. R. Halmos, *Measure theory*, Prentice Hall, 1978.
3. P. K. Jain & V. P. Gupta, *Lebesgue Measure and Integration*, Wiley Eastern Ltd., 1986.
4. B. Z. Vulikh, *A Brief course in the Theory of Functions of a Real Variable*, Mir Publishers, 1976.

5. I. P. Natanson, *Theory of Functions of a Real variable*, Vol. 1, Frederick Ungar Pub. Co., 2016.
6. I. K. Rana, *An introduction to measure and integration*, Narosa Publishing House, 2007.
7. W. Rudin, *Real and Complex Analysis*, McGraw Hill, 1987.
8. R. V. Churchill and J. W. Brown, *Complex variables and Applications*, McGraw Hill, 2013.
9. E. T. Copson, *Theory of functions of a complex variable, Oxford University Press, 1970.*
10. E. C. Titchmarsh, *The theory of functions*, Oxford University Press, 1976.
11. Shanti Narayan, *Theory of functions of a Complex variable*, S. Chand & Co., New Delhi, 2005.
12. John B. Conway, *Functions of one complex variable*, Springer Interanational, 1995.
13. J. K. Lu, S. G. Zhong & S. Q. Liu, *Introduction to the theory of Complex Functions*, World Scientific Publishing Company, 2002.

MJMA 18 : Algebra VI and Functional Analysis

Credits: 2+2 **Marks : 50+50**

Unit I : Algebra VI

Inner product spaces and norms. Gram-Schmidt orthogonalisation process, Orthogonal and orthonormal basis. Orthogonal complements and projection onto a subspace. Linear operators on inner product spaces, adjoint of a linear operator; matrix representation of the adjoint. Normal, self adjoint operators, unitary and orthogonal operators; and their matrices. Orthogonal projections, the spectral theorems and consequences.

Hermitian matrices, properties and characterizations of Hermitian matrices. Congruence and diagonalization. Positive definite and semi-definite matrices, properties and characterizations, Sylvester's criterion. The polar decomposition of positive semi-definite matrices.

Bilinear and quadratic forms, characterization and classification theorems. Definite and semidefinite real quadratic forms. Congruence and Sylvester's Law of Inertia. Classification of conic sections and surfaces.

Books recommended:

1. Sheldon Axler, Linear Algebra Done Right, 3rd Ed., Springer, 2014.
2. R. B. Bapat, Linear Algebra and Linear Models, 3rd Ed., Springer, 2012.

3. Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence, Linear Algebra, 5th Ed., Pearson, 2022.
4. Kenneth Hoffman, Ray Alden Kunze, Linear Algebra, 2nd Ed., Pearson, 2015.
5. Jin Ho Kwak and Sungpyo Hong, Linear Algebra, 2nd Ed., Birkhauser, 2004.
6. Vivek Sahai and Vikas Bist, Linear Algebra, 2nd Ed., Narosa, 2013.
7. Helene Shapiro, Linear Algebra and Matrices, American Mathematical Society, 2015.
8. Gilbert Strang, Introduction to Linear Algebra, 6th Ed., Wellesley-Cambridge Press, 2023.
9. Fuzhen Zhang, Matrix Theory : Basic Results and Techniques, 2nd Ed., Springer, 2011.

Unit II : Functional Analysis

Normed Linear Space: Definition, Convergent sequence, Cauchy sequence, Continuity of norm function, Definition of Banach space, Examples of Banach spaces, Properties of finite dimensional Banach spaces, Equivalent norms, Riesz lemma.

Linear operators on normed linear spaces: Boundedness and Continuity of linear operators, Norm of a bounded linear operator and its formulae, Bounded linear functional, Dual space.

Four fundamental theorems (Statement only): Hahn-Banach theorem, Open mapping theorem, Closed graph theorem, Uniform boundedness principle theorem.

Inner product space: Definition, Cauchy-Schwartz inequality, Continuity of inner product function, Parallelogram law, Relation between inner product space and normed linear space. Definition of Hilbert space and some examples, Orthogonal and orthonormal set and their simple properties, Bessel's inequality, Riesz representation theorem.

Books Recommended:

1. E. Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley & Sons, 1989.
2. G. Bachman & L. Narici, *Functional Analysis*, Academic Press, 2003.
3. C. Gottman & G. Pedrick, *First Course in Functional Analysis*, Prentice Hall of India, 2002.

4. A. H. Siddiqui, *Functional Analysis with Applications*, Tata-McGraw Hill, 1986.
5. G. F. Simmons, *Introduction to Topology and Modern Analysis*, McGraw Hill, 1963.
6. B. K. Lahiri, *Elements of Functional Analysis*, The World Press, 1982.
7. V. K. Krishnan, *Text Book of Functional Analysis*, Prentice Hall, 2004.
8. B. V. Limaye, *Functional Analysis*, Wiley Eastern, 2004.

MJMA 19 : Differential Equations IV

Credits : 2+2 **Marks : 50+50**

Unit I : Ordinary Differential Equations

Existence and uniqueness**:** Continuity theorem, strong continuity theorem, existence and uniqueness theorem for 1st order and higher order differential equations, existence and uniqueness theorems for systems of differential equations, Dependence of solutions on initial conditions and on the function.

Adjoint and self-adjoint linear differential equations: Abel's identity, oscillatory solutions. Nonoscillating solutions, estimation of distance between two successive zeros of solutions. Separation and comparison theorems.

Sturm-Liouville problems: Orthogonality of characteristic functions, expansion of a function in a series of orthogonal functions.

Green's function: Solutions of second order linear differential equations by Green's function approach for initial and boundary value problems.

Fundamental matrix. Exponential matrix function and their properties. Method of solving systems of linear ordinary differential equations by fundamental matrix and exponential matrix function.

Books Recommended:

1. Differential Equations, S. L. Ross (John Wiley and Sons)
2. Elementary Differential Equations and Boundary value problems, Boyce and DiPrima (Wiley)
3. A First Course in Ordinary Differential Equations, R.E. Langer (Wiley)
4. Differential Equations with Boundary-Value Problems, Zill and Wright (CENGAZE)
5. Ordinary Differential Equations, Ince (Dover)

6. Differential Equations, H. T. H. Piaggio (G. Bell and Sons)
7. Nonlinear Ordinary Differential equations, Jordon and Smith (Cambridge)
8. Differential Inequalities, J Szarski, PWN (1965).
9. Special Functions for Scientists and Engineers, W.W. Bell (Dover).
10. Special Functions & Their Applications, N. N. Lebedev, R. A. Silverman (Dover)
11. Introduction to Bessel Functions, F Bowman (Dover)
12. An Introduction to Special Functions, C. Viola (Springer)
13. Special Functions, Z. X. Wang, D. R. Guo (World Scientific)

Unit II : Partial Differential Equations

Second order PDEs, The Cauchy-Kowalewskaya theorem. Idea of initial and boundary value problems. Non-homogeneous PDEs.

Classification of second order PDE. Elliptic, Hyperbolic and Parabolic PDEs. The Canonical forms for Hyperbolic, Parabolic and Elliptic equations.

Laplace equation: Harmonic function. Basic properties of harmonic functions, Laplace PDE. Boundary value problems, Uniqueness of solutions of Dirichlet and Neumann problems. Mean value theorem for harmonic functions. Maximum and minimum principles for harmonic functions. Uniqueness and stability of various BVPs.

Methods of solving PDEs: Representation of solutions. Separation of variables. Fourier and Laplace Transforms. Green's function of solving PDEs.

Boundary value problems of Dirichlet and Neumann: Dirichlet's principle, Dirichlet problem for a rectangle, The Neumann problem for a rectangle, Interior Dirichlet problem for a circle, Exterior Dirichlet problem for a circle, Interior Neumann problem for a circle, Green's function solution of Dirichlet's and Neumann's problem for sphere, formulation of Dirichlet's problems as a problem of integral equation, Poisson's integral solution.

Laplace's equation in three dimensions: Solution of Laplace equation in Cartesian co-ordinates, cylindrical co-ordinates and spherical polar coordinates, Surface and solid harmonics.

Books Recommended:

1. I. N. Sneddon , Elements of Partial Differential Equations (McGraw Hill)
2. T. Amarnath, An Elementary Course in Partial Differential Equations (Narosa)

3. Lawrence C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, Vol. 19 (American Mathematical Society)
4. Kenneth S. Miller, Partial Differential Equations (Krieger Pub. Co.)
5. F. John, Partial Differential Equations (Springer)
6. P. Prasad, R. Ravindran, Partial Differential Equations (John Wiley and Sons Ltd.)
7. J. David Logan, Applied Partial Differential Equations (Springer)
8. E. DiBenedetto, Partial Differential Equations (Birkhäuser)
9. A. D. Polianin, V. F. Zaitsev, Alan Moussiaux, Handbook of first Order Partial Differential
Equations (CRC Press, Taylor & Francis Group)
10. Tyn Myint-U, L. Debnath, Linear Partial Differential Equations for Scientists and Engineers (Birkhäuser)

MJAMA 01 : Statics and Hydrostatics
Credits : 2+2 **Marks : 50+50**

Unit I : Statics

Coplanar forces: Conditions of equilibrium; Astatic equilibrium and Astatic centre.

Principle of virtual work for a single particle and rigid body, Examples.

Forces in three dimensions: Moment of a force about a line; Axis of a couple; Resultant of any couples acting on a body; Conditions of equilibrium of a system of forces acting on a body; Poinsot's central axis; Wrench, Pitch, Intensity and Screw.

Books Recommended:

1. S. L.Loney-An Elementary Treatise on Statics, Radha Publishing House, Kolkata
2. M.C.Ghosh-Analytical Statics, Sreedhar Prakashani, Kolkata
3. A.S.Ramsay-Statics
4. H.Lamb-Statics and Hydrostatics
5. Murray Spiegel-Theoretical Mechanics, McGraw Hill-Schaum

Unit II : Hydrostatics

Definition of fluid, Classification of fluids, Notion of stress.
Pressure at a point in a fluid. Pressure of heavy fluids in equilibrium under gravity.
Equilibrium of fluids in a given field of force: Pressure gradient; Surface of equipressure; Condition of equilibrium for a system of given forces.
Thrust on plane surfaces.
Centre of pressure; Depth of centre of pressure of a plane area under gravity; Effect of additional depth without rotation.

Books Recommended:

1. J. M. Kar, Hydrostatics, K. P. Basu Publishing Co., 2016
2. Md. M. Rahaman, Hydrostatics, New Central Book Agency Ltd., Kolkata, 2012
3. B. Bhattacharyya, Hydrostatics, New Central Book Agency Ltd., Kolkata, 2013
4. A. Mukherjee and N. K. Bej, Advanced Hydrostatics, Shreetara Prakashani, 2012

SEMESTER VIII

MJMA 20 : Algebra VII and Classical Mechanics
Credits : 2+2 **Marks : 50+50**

Unit I : Algebra VII

Internal direct product of normal subgroups, necessary and sufficient conditions. Equivalence of internal and external direct products.

Structure of finite abelian groups. The primary decomposition theorem and the fundamental theorem of finite abelian groups. Invariant factors and elementary divisors.

Conjugation and the conjugacy class equation. Cauchy's Theorem for finite groups. The converse of the Lagrange's theorem for finite abelian groups. Center of a p-group is nontrivial.

Group action, orbits and isotropy group. The class equation of a finite group. Burnside counting principle.

Sylow's Theorems on finite groups. Applications of Sylow's theorems to test the simplicity of finite groups. If $1 \leqslant n < 60$ and n is not prime then no groups of order n is simple. Any simple group of order 60 is isomorphic to A_5.

Solvable groups. Subgroups and homomorphic images of solvable groups. Commutator subgroups. Solvability of the symmetric groups S_n.

Books recommended:

1. Michael Artin, Algebra, 2nd Ed., Pearson, 2011.
2. David S. Dummit and Richard M. Foote, Abstract Algebra, 3rd Ed., Wiley, 2011.
3. John B. Fraleigh, A First Course in Abstract Algebra, 7th Ed., Pearson, 2013.
4. Joseph A. Gallian, Contemporary Abstract Algebra, 9th Ed., Cengage, 2020.
5. I. N. Herstein, Topics in Algebra, 2nd Ed., Wiley, 2006.
6. Charles C. Pinter, A Book of Abstract Algebra, 2nd Ed., Dover, 2010.
7. Vivek Sahai and Vikas Bist, Algebra, 4th Ed., Alpha Science, 2018.
8. M. K. Sen, Shamik Ghosh, Parthasarathi Mukhopadhyay and Sunil Kumar Maity, Topics in Abstract Algbera, 4th Ed., Universities Press, 2022.

Unit II : Classical Mechanics

Generalized coordinates, Lagrange's equations, Hamilton's canonical equations, Hamilton's principle and principle of least action, Two-dimensional motion of rigid bodies, Euler's dynamical equations for the motion of a rigid body about an axis, theory of small oscillations.

Canonical Transformations. Generating Functions. Poisson's Bracket. Jacobi's Identity. Poisson's Theorem. Jacobi-Poisson Theorem.

Books Recommended:

1. H. Goldstein, Classical Mechanics. Narosa Publishing House, New Delhi, (1980)
2. F. Gantmacher, Lectures in Analytical Mechanics, MIR Publishers, Moscow (1975)
3. J.L. Synge and B.A. Griffith, Principles of Mechanics, McGraw-Hill, N.Y. (1970)
4. N.C. Rana and P.S. Joag, Classical Mechanics, Tata McGraw Hill Pub. Company Ltd., New Delhi (1998)
5. N.H. Louis and Janet D. Finch, Analytical Mechanics, C.U.P. (1998)

6. E.T. Whittaker, A Treatise of Analytical Dynamics of Particle and Rigid Bodies, C.U.P. (1977)
7. S.W. McCusky, An Introduction to Advanced Dynamics, Addison-Wesley Publ. Co. Inc.
Massachusetts (1953)
8. A.S. Ramsey, Dynamics Part-II, C.U.P. (1972)
9. L. Elsgolts, Differential Equations and the Calculus of Variations, MIR Publishers. Moscow
(1973)
10. R.H. Dicke and J.P. Wittke, Introduction to Quantum Mechanics, Addison Wesley, (1960)
11. Rydnik, ABC of Quantum Mechanics, Peace Publisher, Moscow
12. F. Chorlton, Textbook of Fluid Dynamics, CBS Publications, Delhi, 1985
13. A.S. Ramsey, Newtonian Attractions. (Cambridge)
14. O.D. Kellog, Foundations of Potential Theory, Dover (1963)

MJMA 21 : Methods of Applied Mathematics
Credits : 4 **Marks: 100**

Integral Transforms:
Laplace transform. Convergence. Continuity requirements. Exponential order. Existence and Uniqueness. Properties of Laplace transform. Heaviside's unit step function. Convolution theorem. Complex inversion formula. Inverse Laplace transform. Properties of inverse Laplace transform.

Fourier transform. Derivation of Fourier transform from Fourier series. Properties of Fourier transform. Convolution theorem. Inverse Fourier transform. Properties of inverse Fourier transform. Parseval's Identity.
Applications of Laplace and Fourier transforms.

Integral Equations:
Basic Introduction. Conversion of ordinary differential equations into integral equations. Conversion of integral equations into ordinary differential equations.

Linear integral equations of Volterra type. Solution of Volterra integral equations by successive substitutions, successive approximations, series solution, Adomian decomposition, modified Adomian decomposition, and Laplace transform methods. Resolvent kernel.

Linear integral equations of Fredholm type. Solution of Fredholm integral equations by successive substitutions, successive approximations, series solution, Adomian decomposition and modified Adomian decomposition methods. Resolvent kernel. Neumann series. Solution of integral equations with separable kernels. Fredholm Alternative theorem. Hilbert-Schmidt theory of integral equations for symmetric kernels.

Singular integral equations. Solution of singular integral equations by Laplace transform method. Existence and uniqueness of integral equations.
Applications.

Calculus of Variations:
Variational problems with fixed boundaries: The concept of variation and its properties. Euler's equation. Functionals dependent on higher order derivatives. Functionals dependent on functions of several independent variables. Applications.
Variational problems with moving boundaries: Transversality conditions. Orthogonality conditions. Variational problem with a moving boundary for a functional dependent on two functions. One sided variations. Applications.
Sufficient conditions for extremum: Field of extremals. Weak and strong extremum. Jacobi condition. Weirstrass function. Legendre condition. Applications.
Variational problems with subsidiary conditions: Isoperimetric problems. Applications.

Books Recommended:
1. F.G. Tricomi, Integral Equations (Inter - Science)
2. A. M. Wazwaz, Linear and Nonlinear Integral Equations. (Springer)
3. R. P. Kanwal, Linear Integral Equations (Academic Press)
4. P.P.G. Dyke, An introduction to Laplace Transforms and Fourier Series. (Springer)
5. M.G. Spiegel, Laplace Transforms (Schaum's OuTlines series)
6. J. L. Schiff, The Laplace Transform (Springer)
7. S. L. Ross, Ordinary Differential Equations. (Wiley)
8. R.L. Bracewell, The Fourier Transforms and Its Applications (McGraw-Hill)
9. I. M. Gelfand and S. V. Fomin, Calculus of Variations (Prentice Hall)
10. A. S. Gupta, Calculus of Variations with Applications (Prentice Hall)
11. R. Weinstock, Calculus of Variations (Dover Publications)

MJAMA 02 : Algebra VIII and Topology
Credits : 2+2 **Marks : 50+50**

Unit I : Algebra VIII

Inequalities in R. AM $\geqslant$ GM $\geqslant$ HM and the Cauchy-Schwarz inequality.

Real polynomials; nature of the zeros and their positions; Descarte's rule of sign. Relation between roots and coefficients, symmetric functions of roots of a polynomial equation. Cardan's and Ferrari's method to solve cubic and biquadratic equations.

Ordered sets, partially ordered, fully ordered and well ordered sets; examples and basic properties. Monotone maps, duality principle.

Lattices as posets and as algebraic structures. Sublattices, products and homomorphisms. Ideals and filters. Complements of an element in a lattice.

Modular and distributive lattices; examples and basic properties. Boolean algebras as complemented distributive lattices; examples and basic properties. Boolean
polynomials; their normal forms and applications to switching circuits.

Books recommended:

1. Barnard and Child, Higher Algebra, Arihant, 2016.
2. G. Birkhoff, Lattice Theory, AMS Colloquium Publications, 1940.
3. B. A. Davey and H. A. Priestley, Introduction to Lattices and Order, 2nd Ed., Cambridge University Press, 2009.
4. R. L. Goodstein, Boolean Algebra, Dover, 2007.
5. Steven Givant and Paul Halmos, Introduction to Boolean Algebras, Springer, 2010.
6. G. Gr¨atzer, General Lattice Theory, 2nd Ed., Birkh¨auser, 2002.
7. S. K. Mapa, Higher Algebra : Classical, 9th Ed., Levant, 2019.
8. S. Roman, Lattices and Ordered Sets, Springer, 2010.
9. J. Eldon Whitesitt, Boolean Algebra and Its Applications, Dover, 2010.

Unit II : Topology

Topological spaces; examples, union and intersection of Topologies, Discrete and Indiscrete Topologies, Topologies, Base for a Topology, Necessary and sufficient condition for a base of a Topology, Sub-base, Neighbourhood system at a point, limit point of a set, closed sets and their algebra; Derived set, closure of a set, Interior, boundary of a set - their relations, Kuratowski closure operator and resulting topology, subspace and relative topology. First and

second axiom spaces, Lindeloff Theorem.

Continuous functions, open functions, closed functions, homeomorphism, product of finite number of spaces, sum and quotient spaces, separation axioms T_0, T_1, T_2, regular spaces, completely regular spaces, normal spaces, completely normal spaces.

Open cover, compact spaces, compact sets, F.I.P. (finite intersection property), characterization of compact sets of reals with usual topology, continuous image of compact spaces.

Connected spaces, connected sets of reals with usual topology, union of connected sets, continuous image of connected spaces, Components.

Books Recommended:

1. J. R. Munkres, *Topology*, Prentice Hall Inc., 1999.
2. S. T. Hu, *Introduction to General Topology*, Holden-Day Inc., 1966.
3. K. D. Joshi, *Introduction to General Topology*, Wiley Eastern Ltd., 2004.
4. J. Dugungi, *Topology*, Prentice Hall, New Delhi, 1966.
5. G. Simmons, I*ntroduction to Topology and Modern Analysis*, McGraw Hill Education, 2017.
6. W. Thorn, *Topological Structure*, Holt, Rinehart & Winston of Canada Ltd., 1966.
7. W. J. Pervin,*Foundations of General Topology*, Academic Press, 2014.

MJAMA 03 : Dynamical Systems and Advanced Numerical Analysis

Credits : 2+2 **Marks : 50+50**

Unit I : Dynamical Systems

Continuous Dynamical Systems:

Nonlinear Systems: Linearization of Non-linear Systems, Limitations, Hartman–Grobman Theorem, Local Stability, Global Stability, Lyapunov Function, Lyapunov Theorem on Stability, LaSalle Invariance Principle.

Oscillations: Limit Set, Attractors, Periodic Orbits, Limit Cycle, Poincare-Bendixson Theorem, Bendixson-Dulac Criterion.

Discrete Dynamical Systems:

Maps and Flows, Composition of Maps, Orbits, Phase Portrait, Fixed Points, Stable and Unstable Fixed Points, Basin of Attraction and Basin Boundary, Linear Stability Analysis, Cobweb Diagram, Periodic Points, Periodic Cycles,

Stability of Periodic Point and Periodic Cycle, Hyperbolic Points, Non-Hyperbolic Points, Schwarzian Derivative.

Some Maps: Tent Map, Logistic Map, Dynamics of Quadratic Maps, Shift Map, Baker's Map, Circle Map, Henon Map, Smale Horseshoe Map.

Conjugacy of Maps: Conjugacy, Topological semi-conjugacy, Homeomorphism, Topological conjugacy, Conjugacy between different maps.

Chaos: Sensitive Dependence on Initial Conditions (SDIC), Sarkovskii's Theorem, Period-Three Implies Chaos for 1-D Maps. Some Chaotic Maps, Universal sequence, Feigenbaum Number, Poincaré Section, Lyapunov Exponents, Routes of Chaos, Some Examples of Chaos.

Books Recommended:

1. S Strogatz, Nonlinear Dynamics And Chaos: With Applications To Physics, Biology, Chemistry, And Engineering, Hachette (2014)
2. G C Layek, An Introduction to Dynamical Systems, Springer (2015)
3. P Glendinning, Stability, Instability and Chaos, Cambridge (1994)
4. S Wiggins, Introduction to Applied Nonlinear Dynamical Systems and Chaos, Springer (1996)
5. M W Hirsch, S Smale, R L Devaney, Differential Equations, Dynamical Systems, and an
Introduction to Chaos, Academic Press (2012)
6. R A Holmgren, A First Course in Discrete Dynamical Systems, Springer (1996)
7. J Hale and H Koack, Dynamics and Bifurcations, Springer (2012)

Unit II : Advanced Numerical Analysis

Interpolation: Piecewise polynomial interpolation, Hermite interpolation formula, Error in Hermite interpolation. Cubic spline interpolation, B-splines: Basic theory and applications.

Approximation of Functions: Norms, Least-squares polynomial approximation, Orthogonal polynomials, Gram-Schmidt orthogonalization, Chebyshev polynomials, Uniform approximation in mini-max sense of error, Lanczos economization.

Numerical Integration: Errors in Newton-Cotes quadrature formula, Gauss-Legendre and

Gauss-Chebyshev quadrature formulae, Romberg integration.

Iterative Techniques for solving linear systems: Norms of vectors & matrices, matrix eigenvalues & eigenvectors, The spectral radius, Matrix splitting and classical stationary iterative methods, Krylov subspace methods; Conjugate-gradient, biconjugate-gradient (BiCG), BiCG Stab methods, GMRES, preconditioning techniques.

Numerical Solution of Initial Value problems (IVP) for ODE: First Order Equations: Multi-step Predictor-Corrector Methods: Adams-Bashforth method, Adams-Moulton method, Milne's method --- Convergence and stability.

Numerical Solution of Partial Differential Equations (PDE) by Finite Difference Methods: Classification of second order PDE, Representation of derivatives by forward, backward and central differences, Thomas Algorithm, Parabolic Equation (Heat-Conduction Equation) (1D) by explicit finite difference and implicit Crank-Nicolson methods, consistency, stability and convergence, Lax equivalence theorem, Poisson Equation on a rectangular region, Hyperbolic Equation (Wave Equation) in one space dimension.

Books Recommended:

1. M. K. Jain, S. R. K. Iyengar, R. K. Jain, Numerical Methods for Scientific and Engineering Computation, New Age International Publisher, New Delhi (2007)
2. K. E. Atkinson, An Introduction to Numerical Analysis, John Wiley and Sons (1989)
3. A. Gupta and S. C. Bose, Introduction to Numerical Analysis, Academic Press (1989)
4. P. Niyogi, Numerical Analysis and Algorithms, Tata McGraw-Hill, New Delhi (2004)
5. E. Isaacson, H. B. Keller, Analysis of Numerical Methods, Dover Publications, Inc., New York (1966)
6. D. Kincaid, W. Cheney, Numerical Analysis: Mathematics of Scientific Computing, American Mathematical Society, Rhode Island (2002)
7. R. L. Burden, J. D. Faires, Numerical Analysis: Theory and Applications, Cengage Learning, New Delhi (2005)
8. S. D. Conte and Carl de Boor, Elementary Numerical Analysis - An Algorithmic Approach, 3rd Edition, McGraw Hill (1980)
9. P. Niyogi, S. K. Chakrabartty, M. K. Laha, Introduction to Computational Fluid Dynamics, Pearson, 2005
10. G. D. Smith, Numerical Solutions to Partial Differential Equations, Oxford University Press, 3rd Edition, 1986

11. Joe D. Hoffman, Numerical methods for Engineers and Scientist, McGraw-Hill, 1993
12. C. T. Kelley, Iterative methods for linear and nonlinear equations, SIAM, 1995

DETAILS SYLLABUS OF THE MINOR COURSES IN MATHEMATICS

MNMA 01 : Calculus
Credits : 2+1+1 **Marks : 50+25+25**

Unit I : Differential Calculus

Rolle's theorem (Statement only). Lagrange's and Cauchy's mean value theorems. Taylor's and Maclaurin's theorems (Statement only). Necessary conditions for existence of Power series. Expansion of functions --e^x, sin x, cos x, log (1+x), $(1+x)^n$. Indeterminate forms , L'Hospital's rule.

Curve tracing in Cartesian and polar coordinates of standard curves , Maxima and Minima.

Envelope. Asymptotes. Curvature. Concavity and convexity. Points of inflexion.
Functions of Several Variables: Limit and continuity. Partial derivatives. Differentiability.

Statement of Schwarz's and Young's theorems. Homogeneous Functions. Euler's theorem (For functions of two variables).

Unit II ; Integral Calculus

Reduction formulae, derivations and illustrations of reduction formulae of the type $\int \sin nx\, dx$, $\int \cos nx\, dx$, $\int \tan nx\, dx$, $\int \sec nx\, dx$, $\int (\log x)^n\, dx$, $\int \sin^n x \cos^n x\, dx$.
Parameterizing a curve, Quadrature, Rectification, Volumes and Surfaces of Revolution.

Improper integrals, Beta function and Gamma function.

Unit III : Vector Calculus

Triple product, introduction to vector functions, operations with vector-valued functions, limits and continuity of vector functions, differentiation and integration of vector functions.

Concepts of scalar and vector fields, Directional derivative. Gradient of a scalar field, Divergence and Curl of a vector field.

Line Integrals as integrals of vectors, circulation, irrotational vector, work done, conservative force. Surface integrals-parametric representations of a surface, area of a parametric surface.

Volume integrals. Statements and verifications of Green's theorem, Stokes' theorem and Gauss' Divergence theorem. Applications.

Books Recommended:

1. G. B. Thomas, M. D. Weir, J. R. Hass, Thomas Calculus , Pearson, 12th Edition, 2010.
2. M.J. Strauss, G.L. Bradley and K. J. Smith, Calculus, 3rd Ed., Dorling Kindersley (India) P. Ltd. (Pearson Education), Delhi, 2007.
3. H. Anton, I. Bivens and S. Davis, Calculus, 7th Ed., John Wiley and Sons (Asia) Pvt. Ltd.,Singapore, 2002.
4. R. Courant and F. John, Introduction to Calculus and Analysis (Volumes I & II), Springer-Verlag, New York, Inc., 1989.
5. L. J Goldstein, David Lay, N.I.Asmar, David I. Schneider, Calculus and Its Applications, Pearson, New International Edition, 2014
6. Shantinarayan, P.K. Mittal, Integral Calculus, S. Chand Publishing, 10th Edition, 2012
7. Shantinarayan, Mathematical Analysis, S. Chand and Company Ltd, 1st Edition, 2005.
8. J. Edwards, Differential Calculus for Beginners, MacMilan, 1996.
9. B. Williamson, An Elementary Treatise on the Integral Calculus, D. Appleton and Co., 1977
10. A. A. Shaikh & S. K. Jana, Vector Analysis with Applications, Narosa Publishing House Pvt. Ltd., New Delhi, 2009.
11. L. Brand, Vector Analysis, Dover Publications Inc., 2006.
12. Shanti Narayan**,** A Text Book of Vector Analysis, S.Chand publishing, 19th Edition,2013.

13. M. Spiegel, S. Lipschutz, D. Spellman, Vector Analysis, Tata McGraw-Hill, 2nd Edition, 2009.

MNMA 02 : Introduction to Algebra
Credits : 2+2 **Marks : 50+50**

Unit-I : Introduction to Abstract Algebra

Real polynomials; nature of the zeros and their positions; Descarte's rule of sign. Relation between roots and coefficients, symmetric functions of roots of a polynomial equation. Cardan's and Ferrari's method to solve cubic and biquadratic equations.

Introduction to groups, definition and examples of groups including symmetries of a square, dihedral groups, permutation groups and matrix groups. Elementary properties of groups, order of an element in a group. Cyclic groups, definition and examples. Elementary properties. Definition and examples of subgroups, subgroup tests. Centralizer, normalizer, center of a group. Algebra of subgroups.

Definition and examples of rings, properties of rings, subrings. Integral domains and fields, characteristic of a ring.

Unit-II : Linear Algebra on $\mathbb{R}$

Systems of linear equations, homogeneous and non-homogeneous systems. The matrix equation AX=b, row reduction and echelon forms, uniqueness of reduced echelon form. Pivot positions, basic and free variables, parametric description of the solution set. Rank of a matrix (in terms of non-zero rows in the unique reduced row echelon form); existence and uniqueness theorem for solutions of a system of linear equations.

Vector spaces, subspaces, algebra of subspaces, direct sum of subspaces. Linear combination of vectors, linear span. Linear independence. Basis and dimension, dimension of subspaces. Geometric classification of the subspaces of R^2 and R^3 .

Eigenvalues and eigenvectors of a matrix, Characteristic polynomial. Eigen spaces and diagonalizability of a matrix. The Cayley-Hamilton theorem for matrices(statement only) and it's applications.

Books recommended:

1. Barnard and Child, Higher Algebra, Arihant, 2016.
2. Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence, Linear Algebra, 5th Ed., Pearson, 2022.
3. Joseph A. Gallian, Contemporary Abstract Algebra, 9th Ed., Cengage, 2020.
4. Kenneth Hoffman, Ray Alden Kunze, Linear Algebra, 2nd Ed., Pearson, 2015.
5. David C. Lay, Steven R. Lay and Judi J. McDonald Linear Algebra and its Applications, 5th Ed., Pearson, 2023.
6. S. K. Mapa, Higher Algebra : Classical, 9th Ed., Levant, 2019.
7. S. K. Mapa, Higher Algebra : Abstract and Linear, 15th Ed., Levant, 2020.
8. Charles C. Pinter, A Book of Abstract Algebra, 2nd Ed., Dover, 2010.
9. M. K. Sen, Shamik Ghosh, Parthasarathi Mukhopadhyay and Sunil Kumar Maity, Topics in Abstract Algbera, 4th Ed., Universities Press, 2022.
10. Gilbert Strang, Linear Algebra and its Applications, 4th Ed., Cengage, 2005.

MNMA 03 : Differential Equations and their Applications
Credits: 4 **Marks : 100**

Basic Concepts:
Definitions of ordinary differential equation (ODE). Formation of ODEs by elimination of arbitrary constants. Formation of ODEs from real-world problems. Meaning of the solution of ODEs (Geometrical and physical).
Concept of linear and nonlinear ODEs. Concept of initial value problems and boundary value problems in ODEs.

First Order ODEs of first Degree:
Existence and uniqueness theorem for 1st order and higher order differential equations (Statement only). General solution and particular solution of 1st order IVP. Separable equations and equations reducible to this form.

Homogeneous equations. Exact equations and condition for exactness. Integrating factor and methods of finding integrating factors. Linear equations. Equations reducible to 1st order linear equations. Orthogonal and oblique trajectories. Applications of 1st order differential equations.

First Order ODE of Higher Degree:
Equations solvable for p, x, and y. Clairaut's equation. Singular solution.

Higher Order Linear ODEs with Constant Coefficients:
Linearly dependent and independent solution. Wronskian and its properties. General solution of homogeneous equation of second order. Principle of super position for homogeneous equation. Finding out Particular Integrals (P.I.) by symbolic operator D. Solution of 2nd order ODEs by variation of parameters. Linear simultaneous differential equations. Applications of higher order differential equations.

Homogeneous Linear ODEs with Variable Coefficients:
Euler's Equation, Exact higher order ODEs and some special forms.

Partial Differential Equations (PDEs):
Definitions. Order and degree of partial differential equations. Concept of linear and non-linear partial differential equations. Formation of first order partial differential equations.
Linear partial differential equation of first order. Lagrange's method. Charpit's method.
Applications of partial differential equations.

Books Recommended:

1. Differential Equations, P. R. Ghosh and J. G. Chakraborty (U. N. Dhar & Sons)
2. Elementary Differential Equations, E. D. Rainville , P. E. Bedient, R. E. Bedient. (Pearson)
3. A First Course in Differential Equations with Modeling Applications, D.G. Zill (Cengage)
4. Introduction to Ordinary Differential Equations, S. L. Ross (Wiley)
5. Elementary Differential Equations and Boundary Value Problems, W. E. Boyce, and R. C. DiPrima (Wiley)
6. Theory of Ordinary Differential Equations, E.A. Coddington, N. Levinson (Tata McGraw-Hill)
7. Elements of Partial Differential Equations, I. N. Sneddon, (Tata McGraw-Hill)

8. Linear Partial Differential Equations for Scientists and Engineers, Tyn Myint-U, L. Debnath (Birkhauser)
9. Nonlinear Partial Differential Equations for Scientists and Engineers, L. Debnath (Birkhauser)

MNMA 04 : Linear Programming Problems and Numerical Methods
Credits : 2+2 **Marks : 50+50**

Unit I : Linear Programming Problems

Introduction: Linear Programming Problem (LPP), Mathematical formulation of L.P.P., Graphical method of solution, Nature of solutions.

Basic Concepts: Basis, replacing a vector in a basis. Solution of a system of linear equations-Basic solution, Basic feasible solution. Convex set- Extreme points. Matrix formulation of L.P.P. Standard form of LPP. Relation among the optimal solution of a LPP, Basic feasible solution and Extreme point of the convex set of all feasible solutions.

Fundamental theorem of LPP-Simplex method, Reduction of a feasible solution to a B.F.S.; improving a basic feasible solution; Optimality checking; Unboundedness; Existence of alternative optimal solution. Simplex algorithm and the simplex tableau; Slack, Surplus and Artificial variables-introduction and its subsequent removal; Inconsistency and redundancy. Two-phase Simplex method.

Concept of Duality- Its economic interpretation. Construction of dual. The relation between feasible solution of dual and primal problems. Fundamental properties of Dual Problems. Weak duality theorem, Strong duality theorem, Fundamental duality theorem. Computational aspects of Simplex method and duality. Dual simplex method-The Algorithm and Difference between Regular Simplex method and Dual Simplex method.

Transportation Problems-Mathematical Formulation; finding an initial basic feasible solution; The Transportation Algorithm-checking for optimality, improving a B.F.S. Degeneracy in Transportation Problem. Resolution of degeneracy in the initial stage. Resolution of degeneracy during solution stage. Unbalanced Transportation Problems. Maximization in Transportation Problems.

Assignment Problems-Mathematical Formulation; Hungarian Assignment Method; Unbalanced Assignment Problem. Maximization in Assignment Problem. Travelling Salesman Problem.

Books Recommended:

1. Linear Programming and Game theory, Chakraborty & Ghosh, Moulik Library, Kolkata.
2. Operative Research: An introduction, Hamdy A. Taha, Pearson.
3. Operations Research: Principles and Practice, Ravindran, Phillips, Solberg, Wiley India.
4. Operations Research, Richard Bronson, Govindasami Naadimuthu, Tata Mc Graw-Hill.
5. Principles of Operations Research, Harvey M. Wagner, PHI.
6. Operations Research: Theory and Applications, J.K.Sharma, Macmillian India Ltd.

Unit II : Numerical Methods

Approximate numbers. Significant figures, Rounding off numbers. Errors – Absolute, Relative and percentage errors. The general formula for errors and its applications
Finite differences, Difference table, propagation of error in a difference table. Differences of a polynomial.

Polynomial interpolation. Error in polynomial interpolation. Newton's forward and backward interpolation formulae. Lagrange's interpolation formula.

Numerical integration : Trapezoidal rule, Simpson's one-third rule. Composite rules. Geometrical significance. (No derivation of error term)

Numerical solution of algebraic and transcendental equations. Bisection method, Method of fixed point iteration, Newton Raphson's method. Geometrical significance and convergence.

Numerical solution of a system of linear equations : Iterative methods : Gauss Jacobi and Gauss Seidel.

Books Recommended:

1. P. Niyogi, Numerical Analysis and Algorithms, Tata McGraw-Hill Publishing Company Limited, 2003.
2. A. Gupta and S.C. Bose, Introduction to Numerical Analysis, Academic Publisher, 1989.
3. M.K. Jain, S.R. K. Iyengar and R.K. Jain, Numerical Methods for Scientific and Engineering Computation, Wiley Eastern Limited, 1991.
4. N. Dutta and R.N. Jana, Introductory Numerical Analysis, Shreedhar Prakashani, 2001.
5. J. B. Scarborough, Numerical Mathematical Analysis, Oxford and IBM Pub. Co.,1966.

DETAILS SYLLABUS OF THE SKILL ENHANCEMENT COURSES IN MATHEMATICS

SEC 01 : Algebra II and Analysis II
Credits : 2+1 **Marks : 50+25**

Unit I : Algebra II

Transformations of polynomial equations. Cardan's and Ferrari's method to solve cubic and biquadratic equations.(without proof)

Systems of linear equations, homogeneous and non-homogeneous systems. The matrix equation AX=b, row reduction and echelon forms, uniqueness of reduced echelon form. Pivot positions, basic and free variables, rank of a matrix (in terms of unique reduced echelon form) and parametric description of the solution set. Existence and uniqueness theorem for solutions of a system of linear equations.

Vectors in R^n. Vector form of a linear system and the column picture. Question of existence of solutions and linear combination of vectors. Question of uniqueness of solution and linear independence of vectors. independent subsets. Geometry of linear span and linear independence.

Subspaces of R^n. Existence of basis and dimension of a subspace (statements only). Othogonality in R^n . Orthogonality of row space and null space of a

matrix. Orthogonal complement of a subspace. The dimension of the solution space of AX = 0 and the rank of A.

Inverse of a matrix, equivalent characterizations of invertible matrices. Eigen values, eigen vectors and the characteristic equation of a real matrix. Diagonalizability of real matrices.

Books Recommended:

1. Howard Anton and Chris Rorres, Elementary Linear Algebra, 11th Ed., Wiley, 2016.
2. Bernard Kolman and David Hill, Elementary Linear Algebra with Applications, 9th Ed., Pearson, 2019.
3. David C. Lay, Steven R. Lay and Judi J. McDonald Linear Algebra and its Applications, 5th Ed., Pearson, 2023.
4. Gilbert Strang, Linear Algebra and its Applications, 4th Ed., Cengage, 2005.

Unit II : Analysis II

Review of derivative, Intermediate value property for derivatives.
Mean Value Theorems: Rolle's theorem, Lagrange's and Cauchy's mean value theorems and applications. Indeterminate forms: L'Hospital's rule and its applications.
Higher order derivatives, Leibniz rule and its applications.
Taylor's and Maclaurin's theorems with different forms of remainder, Maclaurin's series, Maxima and Minima, Expansion of functions e^x, sin x, cos x, log(1+ x), $(1+x)^n$.
Envelope, Asymptotes, Curvature, Quadrature, Rectification, Volumes and Surfaces of Revolution.

Books Recommended:

1. R. G. Bartle and D. R. Sherbert, *Introduction to Real Analysis*, Wiley India Pvt. Ltd, 4th Edition, 2021.
2. S. K. Mapa, *Introduction to Real Analysis*, Sarat Book Distributors, Revised 8th Edition 2021.
3. R. K. Ghosh and K. C. Maity, *An introduction to Analysis: Differential Calculus: Part-I*, New Central Book Agency, 13th revised Edition, 2011.
4. S. N. Mukhopadhyay and A. Layek, *Mathematical Analysis:Vol-I*, U. N. Dhar & Sons. Pvt. Ltd., 2nd Edition, 2009.

SEC 02 : Complex Analysis I and Tensor Calculus

Credits : 1+2 **Marks : 25+50**

Unit I : Complex Analysis I

Functions of a Complex Variable: Limits of Functions, Theorems on Limits, Continuous Functions, Continuity in terms of Real and Imaginary parts.

Sequences: Sequences of complex numbers, Limit and limit points of a sequence, Cauchy's General Principle of convergence.

Stereographic projection and extended complex plane.

Books Recommended:

1. B. K. Lahiri, *Complex Analysis*, The World Press Private Limited.
2. Sobhakar Ganguly, *Elements of Complex Analysis*, Academic Publisher, 2020.
3. A. I. Markushevich, *Theory of functions of a Complex Variable*, Prentice Hall, Inc., 2001.

Unit II : Tensor Calculus

Contravariant and Covariant vectors and tensors. Mixed Tensors. Algebra of tensors. Quotient laws. Fundamental and Associate tensors. Christoffel's symbols. Covariant differentiation of vectors and tensors. Riemann-Christoffel tensors. Permutation tensors.

Books Recommended:

1. I. S. Sokolnikoff, Tensor Analysis, Theory and applications to Geometry and Mechanics of Continua, John Wiley & Sons Inc.
2. B. Spain, Tensor Calculus, Oliver & Boyd.
3. T. J. Willmore, An introduction to Differential and Riemann Geometry, Oxford University Press.
4. J. A. Thorpe, Introduction to Differential Geometry, Springer-Verlag.
5. B. O. Neil, Elementary Differential Geometry, Academic Press.
6. S. Sternberg, Lectures on Differential Geometry. Prentice-Hall.
7. M. C. Chaki, Tensor Calculus, Calcutta Publishers.

SEC 03 : Number Theory
Credits : 3 **Marks : 75**

Review of elementary number theory, divisibility of integers, fundamental theorem of arithmetic, infinitude of prime numbers.

Congruence, definition of congruence as an equivalence relation on integers, Illustration of finding remainder of an integer upon division by another integer, linear congruence, Chinese remainder theorem for Z and its applications.

Arithmetic functions, definition of arithmetic functions, multiplicative and completely multiplicative functions, Möbius function, divisor function, Euler's ϕ function, prime counting function, divergence of the sum of reciprocals of primes.

The group structure of units of Z/nZ, n-th power residues, quadratic residues mod p, Legendre symbols, quadratic reciprocity law (proof included). Applications of quadratic reciprocity in Diophantine equations.

Books Recommended:

1. D. M. Burton, Elementary Number Theory, 7th Ed., Mc Graw Hill, 2017.
2. K. Ireland and M. Rosen, A Classical Introduction to Modern Number Theory, 2nd Ed., Springer, 1998.

DETAILS SYLLABUS OF THE MULTIDISCIPLINARY COURSES IN MATHEMATICS

MDMA 01 : Fundamentals of Mathematical Logic
Credits : 3 **Marks : 75**

Understanding Logic, proposition, argument and reasoning; sentences and statements.

Deductive and inductive arguments. Truth and validity of proposition and argument.

Symbolic logic --- simple and compound statements. Truth functionality of compound statement. Using symbols to abbreviate simple statements --- symbolize compound statements.

Conditional statements. Validity of arguments (using truth tables). Concept of tautology, contradictory and contingent statement forms.

Formal proof of validity of a given argument --- rule of inference, logically equivalent forms and rule of replacement. Difference between substitution and replacement.

Invalidity of an argument. Proof of invalidity of argument.

Method of natural deduction. Formal proof procedures --- indirect method, conditional and strengthened rule of conditional proof procedures, *reductio ad absurdum*. Examples on constructing proof of validity of arguments.

Books Recommended:

1. I. M. Copi; Symbolic logic; Coillier Macmillan Publishers; London (1979).
2. Elliott Mendelson; Introduction to mathematical logic; Chapman & Hall; London (1997).
3. S. C. Kleene; Introduction to Metamathematics; Amsterdam; Elsevier (1952).
4. J. H. Gallier; Logic for Computer Science; John.Wiley & Sons (1987).
5. H. B. Enderton; A mathematical introduction to logic; Academic Press; New York (1972).
6. A. G. Hamilton; Logic for Mathematicians; Cambridge University Press; England (1988).