

TWO-YEAR POSTGRADUATE PROGRAMME IN MATHEMATICS
Under National Education Policy (NEP-2020) STRUCTURE AND SYLLABUS

(As recommended by the Board of Studies for Mathematics on 21st April 2026)
(With effect from the Academic Session 2026-2027)



DEPARTMENT OF MATHEMATICS
VISVA-BHARATI
SANTINIKETAN-731235
WEST BENGAL, INDIA

**SYLLABUS STRUCTURE FOR TWO-YEAR PG PROGRAMME* IN
MATHEMATICS
UNDER NEP 2020 w.e.f. from the academic session 2026-27**

NcrF Credit Level	Semester	Common Courses (CCs)	Optional Courses (OCs)	Project Work	Total Credits	Total Marks	Degree
400 (Sem I)	I	MA401, MA402, MA403, MA404 (4x4 credits)	NA	MA405 (Dissertation) (1x4 credits)	20	500	
400 (Sem II)	II	MA406, MA407, MA408 (3x4 credits)	NA	MA409 & MA410 (Dissertation) (2x4 credits)	20	500	PG Diploma
500 (Sem II)	II (To continue to Sems. III & IV)	MA501, MA502, MA408 (Level 400)(3x4 credits)	NA	MA503 & MA504 (Dissertation) (2x4 credits)	20	500	
500 (Sem III & IV)	III	MA505, MA506, MA507, and MA508A & MA508B (4x4 credits)	MA509 (1x4 credits)	NA	20	500	
	IV	NA	NA	MARES510	20	500	PG

**After successful completion of three-year Bachelor's degree (Semesters - I to VI) in Mathematics.*

DETAILS OF SYLLABI

SEMESTER I (Level 400)

COMMON COURSES (CCs): MA401, MA402, MA403, MA404, and MA405.

MA401(Unit I: Introduction to Lebesgue Integration)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Introduction to Lebesgue Integration	MA401	2	10	40	50	2	0	0

Learning Objectives:

- Introduce students to measurable functions and integration theory.
- Provide understanding of different convergence concepts in measure theory.
- Distinguish between Riemann and Lebesgue integrals with examples.
- Equip students with powerful tools such as Fatou's Lemma and Dominated Convergence Theorem for advanced analysis.

Learning Outcomes:

- Define and analyze measurable functions and their properties.
- Understand and apply the concepts of convergence: almost everywhere, in measure, and almost uniform convergence.
- Compare and contrast Riemann and Lebesgue integrals with applications.
- Utilize major theorems such as Egoroff's, Lusin's, Fatou's Lemma, and the Dominated Convergence Theorem in analysis problems.

Syllabus:

Measurable functions and their properties, Equivalent functions. Simple functions, Measurable functions as limit of simple functions. Sequence of measurable functions.

Almost everywhere convergence, convergence in measure and almost uniform convergence. Riesz theorem, Egoroff's theorem, Lusin's theorem, Frechet's theorem.

Lebesgue's criterion for Riemann integrability. Lebesgue integral of bounded function and its properties. Comparison of Riemann and Lebesgue integrals.

Passage to the limit under the sign of integration, Bounded convergence theorem. Summable functions and their integrals. Fatou's Lemma, Monotone convergence theorem, Dominated convergence theorem.

Books Recommended:

1. H. L. Royden and P. Fitzpatrick, Real Analysis, Prentice Hall, 2010.
2. S.N.Mukhopadhyay & S. Ray, Measure and Integration: an Introduction, Springer, 2025

3. P. R. Halmos, Measure Theory, Prentice Hall, 1978.
4. P. K. Jain & V. P. Gupta, Lebesgue Measure and Integration, Wiley Eastern Ltd., 1986.
- 4 Z. Vulikh, A Brief Course in the Theory of Functions of a Real Variable, Mir Publishers, 1976.
- 5 P. Natanson, Theory of Functions of a Real Variable, Vol. 1, Frederick Ungar Pub. Co., 2016.
- 6 K. Rana, An Introduction to Measure and Integration, Narosa Publishing House, 2007.

MA401 (Unit II: Complex Analysis II)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Complex Analysis II	MA401	2	10	40	50	2	0	0

Learning Objectives:

The primary objectives of this course are to:

- compute definite and contour integral of complex valued functions of complex variables.
- gain the basic concept of entire and meromorphic functions.
- motivate the students to make easier at understanding the use of complex analysis in applied problems.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- integrate a complex valued function having certain types of singularities.
- classify the notion of different types of singularities.
- gain the knowledge about Maximum modulus, Minimum modulus theorems and Cauchy residue theorem.
- solve many applied problems using complex analysis.

Syllabus:

Simply connected region and primitives of analytic functions, Morera's theorem. Maximum and Minimum modulus theorem, Schwarz's lemma, Taylor's theorem.

Zeros of an analytic functions, Uniqueness of analytic functions, Singularities and classification of singularities, Laurent's theorem, Limit points of zeros and poles, Riemann's theorem on removable singularity, Casorati-Weierstrass theorem.

Cauchy's residue theorem, Evaluation of integration by residues theorem, Argument principle, Rouché's theorem, Open mapping theorem.

Behavior of a function at the point at infinity, Conformal mapping, Riemann mapping theorem (Statement only), Introduction to analytic continuation, Riemann Surface.

Books Recommended:

1. W. Rudin, *Real and Complex Analysis*, McGraw Hill, 1987.
2. R. V. Churchill and J. W. Brown, *Complex Variables and Applications*, McGraw Hill, 2013.
3. E. T. Copson, *Theory of functions of a complex variable*, Oxford University Press, 1970.
4. E. C. Titchmarsh, *The theory of functions*, Oxford University Press, 1976.
5. Shanti Narayan, *Theory of functions of a Complex variable*, S. Chand & Co., New Delhi, 2005.
6. John B. Conway, *Functions of one complex variable*, Springer International, 1995.
7. J. K. Lu, S. G. Zhong & S. Q. Liu, *Introduction to the theory of Complex Functions*, World Scientific Publishing Company, 2002.
8. A. P. Baisnab and M. Saha, *Complex Analysis*, Levant Books, 2025

MA402 (Unit I: Linear operators in inner product spaces)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Linear operators on inner product spaces	MA402	4	10	40	50	2	0	0

Learning objectives:

The objectives of this course are to:

- Study roles of metric properties like norm and angle in an inner product space to characterize linear operators on an inner product space.
- Introduce and develop the notion of adjoint of a linear transformation.
- Characterize normal, self-adjoint, orthogonal and unitary linear operators.
- Study Hermitian matrices with a focus to positive definite and semi-definite matrices.
- Develop polar decomposition of positive semi-definite matrices.
- Study bilinear and quadratic forms. Application of these forms to classify conic sections.

Learning outcomes:

After successful completion of this course the students will be able to:

- Realize orthogonality from abstract viewpoints which will help to identify governing intrinsic feature of orthogonality.
- Find an orthogonal basis of a finite dimensional inner product space.
- Classify normal linear operators.
- Appreciate the orthogonal diagonalizability of every real symmetric matrix.
- Characterize definite and semidefinite matrices.

- Find polar decomposition of semidefinite matrices.
- Apply results on bilinear and quadratic forms.

Syllabus:

Inner product spaces and norms. Gram-Schmidt orthogonalisation process, Orthogonal and orthonormal basis. Orthogonal complements and projection onto a subspace. Linear operators on inner product spaces, adjoint of a linear operator; matrix representation of the adjoint. Normal, self adjoint operators, unitary and orthogonal operators; and their matrices. Orthogonal projections, the spectral theorems and consequences.

Hermitian matrices, properties and characterizations of Hermitian matrices. Congruence and diagonalization. Positive definite and semi-definite matrices, properties and characterizations, Sylvester's criterion. The polar decomposition of positive semi-definite matrices.

Bilinear and quadratic forms, characterization and classification theorems. Definite and semidefinite real quadratic forms. Congruence and Sylvester's Law of Inertia. Classification of conic sections and surfaces.

Books recommended:

1. Sheldon Axler, Linear Algebra Done Right, 3rd Ed., Springer, 2014.
2. R. B. Bapat, Linear Algebra and Linear Models, 3rd Ed., Springer, 2012.
3. Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence, Linear Algebra, 5th Ed., Pearson, 2022.
4. Kenneth Hoffman, Ray Alden Kunze, Linear Algebra, 2nd Ed., Pearson, 2015.
5. Jin Ho Kwak and Sungpyo Hong, Linear Algebra, 2nd Ed., Birkhauser, 2004.
6. Vivek Sahai and Vikas Bist, Linear Algebra, 2nd Ed., Narosa, 2013.
7. Helene Shapiro, Linear Algebra and Matrices, American Mathematical Society, 2015.
8. Gilbert Strang, Introduction to Linear Algebra, 6th Ed., Wellesley-Cambridge Press, 2023.
9. Fuzhen Zhang, Matrix Theory : Basic Results and Techniques, 2nd Ed., Springer, 2011.

MA402 (Unit II: Functional Analysis)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Functional Analysis	MA402	2	10	40	50	2	0	0

Learning Objectives:

The primary objectives of this course are to:

- Understand the concept of norm, idea of convergent sequence, Cauchy sequence, etc.
- Recognize many important properties in finite dimensional normed linear spaces.
- Introduce the notion of bounded linear operator in normed linear spaces, find out relationships between boundedness and continuity.

- Present the concept of operator norm and establish different mathematical expressions of operator norm.
- Introduce the concept of Dual space and study some properties.
- Familiarize the students with the four fundamental theorems (Hahn-Banach theorem, Uniform Boundedness theorem, Open mapping theorem and Closed Graph theorem).
- Understand the concept of inner product spaces, their properties, how they relate to norm.
- Introduce the concept of Hilbert space and present some examples.
- Recognize the key role of Parallelogram law, Bessel's Inequality and Riesz representation theorem.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Gain knowledge about normed linear space, convergent sequence, Cauchy sequence etc.
- Learn various important properties of finite dimensional normed linear spaces.
- Relate the behavior of bounded linear operators in normed linear space and establish the relation between bounded and continuity.
- Develop the idea of operator norm and deduce different mathematical expressions of operator norms.
- Gain knowledge about Dual space.
- Understand the statement of four fundamental theorems (Hahn-Banach theorem, Uniform Boundedness theorem, Open mapping theorem and Closed Graph theorem) which play key role in functional analysis.
- Learn to give examples of inner product spaces and can study various properties.
- Gain knowledge about Hilbert spaces and their properties.
- Know the utility of Parallelogram law, Bessel's Inequality and Riesz representation theorem in Hilbert spaces.

Syllabus:

Normed Linear Space: Definition, Convergent sequence, Cauchy sequence, Continuity of norm function, Definition of Banach space, Examples of Banach spaces, Properties of finite dimensional Banach spaces, Equivalent norms, Riesz lemma.

Linear operators on normed linear spaces: Boundedness and Continuity of linear operators, Norm of a bounded linear operator and its formulae, Bounded linear functional, Dual space.

Four fundamental theorems (Statement only): Hahn-Banach theorem, Open mapping theorem, Closed graph theorem, Uniform boundedness principle theorem.

Inner product space: Definition, Cauchy-Schwartz inequality, Continuity of inner product function, Parallelogram law, Relation between inner product space and normed linear space. Definition of Hilbert space and some examples, Orthogonal and orthonormal set and their simple properties, Bessel's inequality, Riesz representation theorem.

Books Recommended:

- E. Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley & Sons, 1989.
- G. Bachman & L. Narici, *Functional Analysis*, Academic Press, 2003.
- C. Gottman & G. Pedrick, *First Course in Functional Analysis*, Prentice Hall of India, 2002.
- A. H. Siddiqui, *Functional Analysis with Applications*, Tata-McGraw Hill, 1986.

- G. F. Simmons, *Introduction to Topology and Modern Analysis*, McGraw Hill, 1963.
- B. K. Lahiri, *Elements of Functional Analysis*, The World Press, 1982.
- V. K. Krishnan, *Text Book of Functional Analysis*, Prentice Hall, 2004.
- B. V. Limaye, *Functional Analysis*, Wiley Eastern, 2004.

MA403 (Unit I: Advanced Ordinary Differential Equations)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Advanced Ordinary Differential Equations	MA403	2	10	40	50	2	0	0

Learning Objectives:

The primary objectives of this course are to:

- Understand Existence and uniqueness theorems for ODEs.
- Recognize Sturm-Liouville systems.
- Understand method of solving systems of linear ordinary differential equations.
- Comprehend Adjoint and self-adjoint linear differential equations.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Solve problems related to Existence and uniqueness theorems for ODEs.
- Solve problems related to Sturm-Liouville problems.
- Explain problems related to systems of linear ordinary differential equations.
- Solve problems related to self-adjoint linear differential equations.

Syllabus:

Existence and uniqueness: Continuity theorem, strong continuity theorem, existence and uniqueness theorem for 1st order and higher order differential equations, existence and uniqueness theorems for systems of differential equations.

Adjoint and self-adjoint linear differential equations: Abel's identity, oscillatory solutions. Nonoscillating solutions, estimation of distance between two successive zeros of solutions. Separation and comparison theorems.

Sturm-Liouville problems: Solution of Sturm-Liouville problems, Characteristic values and characteristic functions, Orthogonality of characteristic functions, expansion of a function in a series of orthogonal functions.

Green's function: Solutions of second order linear differential equations by Green's function approach for initial and boundary value problems.

Fundamental matrix. Exponential matrix function and their properties. Method of solving systems of linear ordinary differential equations by fundamental matrix and exponential matrix function.

Books Recommended:

1. Differential Equations, S. L. Ross, John Wiley and Sons, 2007.
2. Elementary Differential Equations and Boundary value problems, Boyce and DiPrima, Wiley, 2008.
3. A First Course in Ordinary Differential Equations, R. E. Langer, Wiley, 1954.
4. Differential Equations with Boundary-Value Problems, Zill and Wright, CENGAGE, 2017.
5. Ordinary Differential Equations, Ince, Dover, 2003.
6. Differential Equations, H. T. H. Piaggio, G. Bell and Sons, 1921.
7. Nonlinear Ordinary Differential Equations, Jordon and Smith, Cambridge, 2007.
8. Differential Inequalities, J. Szarski, PWN, 1965.

MA403 (Unit II: Advanced Partial Differential Equations)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Advanced Partial Differential Equations	MA403	2	10	40	50	2	0	0

Learning Objectives:

The primary objectives of this course are to:

- Initiate a comparative discussion among well-posed and ill-posed partial differential equations from real-world examples.
- Develop analytical skills for solving some basic types of partial differential equations.
- Categorize second order partial differential equations and simplify their solution methods by reducing them to canonical forms.
- Explore the theory and applications of higher-order classical partial differential equations depending on initial and boundary value conditions.
- Focus on the application of knowledge and skills as partial differential equations are increasingly important in machine learning (ML), artificial intelligence (AI), and networking, offering a powerful framework for modeling complex phenomena and enhancing algorithm design.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Classify higher-order partial differential equations and concerned canonical type of solutions.
- Understand the origin of diffusion, Laplace, wave equations and concerned analytical solution with modelling perspectives.
- To capture knowledge related to solving second order partial differential equations by choosing the most suitable method.
- Interpret the geometrical and physical significance of solutions, enlighten various real-world phenomena.

Syllabus:

Second order PDEs, The Cauchy-Kowalewskaya theorem. Idea of initial and boundary value problems. Non-homogeneous PDEs.

Classification of second order PDE. Elliptic, Hyperbolic and Parabolic PDEs. The Canonical forms for Hyperbolic, Parabolic and Elliptic equations.

Laplace equation: Harmonic function. Basic properties of harmonic functions, Laplace PDE. Boundary value problems, Uniqueness of solutions of Dirichlet and Neumann problems. Mean value theorem for harmonic functions. Maximum and minimum principles for harmonic functions. Uniqueness and stability of various BVPs.

Methods of solving PDEs: Separation of variables, Representation of solutions.

Boundary value problems of Dirichlet and Neumann: Dirichlet's principle, Dirichlet problem for a rectangle, The Neumann problem for a rectangle, Interior Dirichlet problem for a circle, Exterior Dirichlet problem for a circle, Interior Neumann problem for a circle, Green's function solution of Dirichlet's and Neumann's problem for sphere, formulation of Dirichlet's problems as a problem of integral equation, Poisson's integral solution.

Books Recommended:

1. Partial Differential Equations: An Introduction, W.A. Strauss, John Wiley & Sons Ltd., 2007.
2. An Elementary Course in Partial Differential Equations, T. Amarnath, Narosa, 2003.
3. Partial Differential Equations, F. Prasad and R. Ravindran, New Age International Publishers, 2022.
4. Elements of Partial Differential Equations, I. N. Sneddon, Tata McGraw-Hill, 2006.
5. Lawrence C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, Vol. 19, American Mathematical Society, 1998.
6. Kenneth S. Miller, Partial Differential Equations, Krieger Publishing Company, 2020.
7. F. John, Partial Differential Equations, Springer, 1981.
8. P. Prasad, R. Ravindran, Partial Differential Equations, John Wiley and Sons Ltd., 1985.
9. J. David Logan, Applied Partial Differential Equations, Springer, 2014.
10. E. DiBenedetto, Partial Differential Equations, Birkhäuser, 2009.
11. Linear Partial Differential Equations for Scientists and Engineers, TynMyint-U, L. Debnath Birkhäuser, 2017.
12. Nonlinear Partial Differential Equations for Scientists and Engineers, L. Debnath, Birkhäuser, 2012.

MA404 (Unit I: Statics)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Statics	MA404	2	10	40	50	2	0	0

Learning Objectives:

The primary objectives of this course are to:

- Analyse forces and their effects on objects at rest.
- Understand the concept of static equilibrium.
- Calculate reactions at supports, which are essential for understanding how structures are held in place.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Analyse the properties of a force and a system of forces in 2D and 3D.
- Solve the equilibrium problems in various types of structures using analytical models.
- Understand resolution of forces which is fundamental to Continuum Mechanics.

Syllabus:

Coplanar forces: Conditions of equilibrium; Astatic equilibrium and Astatic centre.

Principle of virtual work for a single particle and rigid body, Examples.

Forces in three dimensions: Moment of a force about a line; Axis of a couple; Resultant of any couples acting on a body; Conditions of equilibrium of a system of forces acting on a body; Poinsot's central axis; Wrench, Pitch, Intensity and Screw.

Books Recommended:

1. S. L. Loney-An Elementary Treatise on Statics, Radha Publishing House, Kolkata, 2003.
2. M. C. Ghosh-Analytical Statics, Sreedhar Prakashani, Kolkata, 2004
3. A. S. Ramsey, Dynamics (Vols. I & II), Cambridge University Press, 2009
4. H. Lamb-Statics and Hydrostatics.
5. M. Spiegel, Theory and Problems of Theoretical Mechanics, Schaum's Outline Series, 2021.
6. E. J. Routh, A Treatise on Analytical Statics with Numerous Examples, Cambridge Library Collection, 2013.

MA404 (Unit II: Hydrostatics)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Hydrostatics	MA404	2	10	40	50	2	0	0

Learning Objectives:

The primary objectives of this course are to:

- Introduce fundamental concepts and classify fluids based on their behavior and properties.
- Understand the concept of pressure in fluids at rest, the factors affecting it, and how to calculate it for various scenarios.
- Introduce the principles of fluid statics and applying them to analyze forces and pressure within fluids at rest.
- Understand the concept of center of pressure focus on both theoretical knowledge and practical application.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Identify different types of fluids and classify them based on their properties.
- Solve problems involving pressure in fluids at rest under the action of gravity only.
- Solve problems involving fluids in equilibrium under the action of external forces.
- Calculate the center of pressure for simple geometries (e.g., rectangular, circular) in both horizontal and inclined planes.

Syllabus:

Definition of fluid, Classification of fluids, Notion of stress.

Pressure at a point in a fluid. Pressure of heavy fluids in equilibrium under gravity.

Equilibrium of fluids in a given field of force: Pressure gradient; Surface of equipressure; Condition of equilibrium for a system of given forces.

Thrust on plane surfaces.

Centre of pressure; Depth of centre of pressure of a plane area under gravity; Effect of additional depth without rotation.

Books Recommended:

1. J. M. Kar, Hydrostatics, K. P. Basu Publishing Co., 2016
2. Md. M. Rahaman, Hydrostatics, New Central Book Agency Ltd., Kolkata, 2012.
3. B. Bhattacharyya, Hydrostatics, New Central Book Agency Ltd., Kolkata, 2013.

4. A. Mukherjee and N. K. Bej, Advanced Hydrostatics, ShreetaraPrakashani, 2012.

MA405: Project Work

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Dissertation	MA405	4	20	80	100	3	1	0

%%%%%%%%%%

SEMESTER II (Level 400)

COMMON COURSES (CCs): MA406, MA407, MA408, MA409, and MA410.

MA406(Unit I: Advanced Group Theory)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Advanced Group Theory	MA406	2	10	40	50	2	0	0

Learning Objectives:

The primary objectives of this course are to:

- State and prove the structure theorem of finite abelian groups.
- Introduce the notion of group actions and prove the three theorems of Sylow with the help of it.
- Discuss the ideas of solvable groups.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Identify groups of a given order up to isomorphism using Sylow's theorems.
- Prepare themselves to learn the main theorem of Galois theory which is a crowning achievement in algebra.

Syllabus:

Product of groups: Internal direct product of normal subgroups, necessary and sufficient conditions. Equivalence of internal and external direct products.

Structure of groups: Structure of finite abelian groups. The primary decomposition theorem and the fundamental theorem of finite abelian groups. Invariant factors and elementary divisors. Conjugation and the conjugacy class equation. Cauchy's Theorem for finite groups. The converse of the Lagrange's theorem for finite abelian groups. Center of a p-group is nontrivial.

Group action and Sylow's theorems: Group action, orbits and isotropy group. The class equation of a finite group. Burnside counting principle. Sylow's Theorems on finite groups. Applications of Sylow's theorems to test the simplicity of finite groups. If $1 \leq n < 60$ and n is not prime then no groups of order n is simple. Any simple group of order 60 is isomorphic to A_5 .

Solvability of groups: Solvable groups. Subgroups and homomorphic images of solvable groups. Commutator subgroups. Solvability of the symmetric groups S_n .

Books recommended:

1. Michael Artin, Algebra, 2nd Ed., Pearson, 2011.

2. David S. Dummit and Richard M. Foote, Abstract Algebra, 3rd Ed., Wiley, 2011.
3. John B. Fraleigh, A First Course in Abstract Algebra, 7th Ed., Pearson, 2013.
4. Joseph A. Gallian, Contemporary Abstract Algebra, 9th Ed., Cengage, 2020.
5. I. N. Herstein, Topics in Algebra, 2nd Ed., Wiley, 2006.
6. Charles C. Pinter, A Book of Abstract Algebra, 2nd Ed., Dover, 2010.
7. Vivek Sahai and Vikas Bist, Algebra, 4th Ed., Alpha Science, 2018.
8. M. K. Sen, Shamik Ghosh, Parthasarathi Mukhopadhyay and Sunil Kumar Maity, Topics in Abstract Algebra, 4th Ed., Universities Press, 2022.

MA406(Unit II: Classical Mechanics)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Classical Mechanics	MA406	2	10	40	50	2	0	0

Learning Objectives:

The primary objectives of this course are to:

- Understand basic postulates of classical mechanics.
- Understand D'Alembert's principle and Lagrange's equations.
- Understand Hamilton's principle and principle of least action.
- Understand canonical transformations and related problems.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Solve problems related to Lagrange's equations.
- Solve problems related to principle of least action.
- Solve problems related to canonical transformations.
- Understand different formalism for classical mechanics.

Syllabus:

Basic Postulates, Generalized coordinates, Constraints, D'Alembert's principle, Lagrange's equations, Hamilton's canonical equations, Hamilton's principle and principle of least action. Theory of small oscillations.

Canonical transformations, Generating functions, Poisson's bracket, Jacobi's identity, Poisson's theorem, Jacobi-Poisson theorem.

Books Recommended:

1. H. Goldstein, Classical Mechanics, Narosa Publishing House, New Delhi, 1980.
2. F. Gantmacher, Lectures in Analytical Mechanics, MIR Publishers, Moscow, 1975.
3. J.L. Synge and B.A. Griffith, Principles of Mechanics, McGraw-Hill, New York, 1970.
4. N.C. Rana and P.S. Joag, Classical Mechanics, Tata McGraw Hill Pub. Company Ltd., New Delhi, 1998.
5. N.H. Louis and Janet D. Finch, Analytical Mechanics, C.U.P., 1998.
6. E.T. Whittaker, A Treatise of Analytical Dynamics of Particle and Rigid Bodies, C.U.P. 1977.
7. S.W. McCusky, An Introduction to Advanced Dynamics, Addison-Wesley Publ. Co. Inc. Massachusetts, 1953.
8. A.S. Ramsey, Dynamics Part-II, C.U.P., 1972.
9. L. Elsgolts, Differential Equations and the Calculus of Variations, MIR Publishers. Moscow, 1973.
10. R.H. Dicke and J.P. Wittke, Introduction to Quantum Mechanics, Addison Wesley, 1960.
11. Rydrik, ABC of Quantum Mechanics, Peace Publisher, Moscow, 2023.
12. F. Chorlton, Textbook of Fluid Dynamics, CBS Publications, Delhi, 1985.
13. A.S. Ramsey, Newtonian Attractions, Cambridge, 2007.
14. O.D. Kellog, Foundations of Potential Theory, Dover, 1963.

MA407 (Unit I: Methods of Applied Mathematics)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Methods of Applied Mathematics	MA407	3	15	60	75	3	0	0

Learning Objectives:

The primary objectives of this course are to:

Integral Transforms:

- Understand the foundational concepts, existence criteria and properties of Laplace and Fourier transforms.
- Develop proficiency in finding transforms and inverse transforms.
- Apply Laplace and Fourier transforms to solve initial and boundary value problems in ordinary differential equations and partial differential equations.

Integral Equations:

- Understand the nature and classification of integral equations: Volterra, Fredholm, and singular types.
- Convert differential equations into integral equations and vice versa.

- Develop various methods to solve integral equations.
- Study Fredholm alternative theorem and Hilbert-Schmidt theorem.
- Determine the conditions ensuring existence and uniqueness.

Calculus of Variations:

- Understand the concept of functionals and variation, including problems with fixed and moving boundaries.
- Derive and apply Euler's equation and transversality conditions.
- Analyze sufficient conditions for extremum using Jacobi, Legendre, and Weierstrass criteria.
- Formulate and solve variational problems with subsidiary conditions.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

Integral Transforms:

- Compute Laplace and Fourier transforms and their inverses of different functions.
- Apply properties to simplify and solve problems.
- Solve ordinary and partial differential equations using Laplace and Fourier transforms.

Integral Equations:

- Classify and formulate Volterra, Fredholm and singular integral equations.
- Convert differential equations into integral equations and integral equations into differential equations.
- Find the resolvent kernel.
- Solve integral equations using methods such as:

Series solutions, Successive approximations, Successive substitutions

- Adomian and modified Adomian decomposition
- Laplace transforms.
- Analyze and solve Fredholm integral equations with separable kernels.
- Understand and apply Fredholm alternative theorem.
- Understand and apply Hilbert-Schmidt theorem for symmetric kernels.

Calculus of Variations:

- Formulate variational problems and apply Euler's equation to determine extrema.
- Solve problems involving higher-order derivatives and functionals with multiple independent variables.
- Determine sufficient conditions for extrema using advanced theoretical criteria.
- Address isoperimetric and constrained optimization problems using variational techniques.

Syllabus:

Integral Transforms:

Laplace transform: Convergence, continuity requirements, exponential order, existence and uniqueness, properties of Laplace transform, Heaviside's unit step function, convolution theorem, complex inversion formula, inverse Laplace transform, properties of inverse Laplace transform, solutions of ordinary and partial differential equations using Laplace transforms.

Fourier transform: Derivation of Fourier transform from Fourier series, properties of Fourier transform, convolution theorem, inverse Fourier transform, properties of inverse Fourier transform, Parseval's identity, solutions of ordinary and partial differential equations using Fourier transforms.

Applications of Laplace and Fourier transforms.

Integral Equations:

Basic introduction, conversion of ordinary differential equations into integral equations, conversion of integral equations into ordinary differential equations.

Linear integral equations of Volterra type: solution of Volterra integral equations by successive substitutions, successive approximations, series solution, Adomian decomposition, modified Adomian decomposition and Laplace transform methods, resolvent kernel.

Linear integral equations of Fredholm type: Solution of Fredholm integral equations by successive substitutions, successive approximations, series solution, Adomian decomposition and modified Adomian decomposition methods, resolvent kernel, Neumann series, solution of integral equations with separable kernels, Fredholm alternative theorem, Hilbert-Schmidt theorem of integral equations for symmetric kernels.

Singular integral equations: Solution of singular integral equations by Laplace transform method.

Existence and uniqueness of integral equations.

Applications of integral equations.

Calculus of Variations:

Variational problems with fixed boundaries: The concept of variation and its properties, Euler's equation, functionals dependent on higher order derivatives, functionals dependent on functions of several independent variables.

Variational problems with moving boundaries: Transversality conditions, orthogonality conditions, variational problem with a moving boundary for a functional dependent on two functions, one sided variations.

Sufficient conditions for extremum: Field of extremals, weak and strong extremum, Jacobi condition, Weierstrass function, Legendre condition.

Variational problems with subsidiary conditions: Isoperimetric problems.

Applications.

Books Recommended:

1. F. G. Tricomi, Integral Equations (Inter - Science).
2. A. M. Wazwaz, Linear and Nonlinear Integral Equations (Springer).
3. R. P. Kanwal, Linear Integral Equations (Academic Press).
4. P. P. G. Dyke, An Introduction to Laplace Transforms and Fourier Series (Springer).
5. M. G. Spiegel, Laplace Transforms (Schaum's Outlines series).
6. J. L. Schiff, The Laplace Transform (Springer).
7. S. L. Ross, Ordinary Differential Equations (Wiley).
8. R. L. Bracewell, The Fourier Transforms and Its Applications (McGraw-Hill).
9. I. M. Gelfand and S. V. Fomin, Calculus of Variations (Prentice Hall).
10. A. S. Gupta, Calculus of Variations with Applications (Prentice Hall).
11. R. Weinstock, Calculus of Variations (Dover Publications).

MA407 (Unit II: IKS III--Indian Astronomy)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
IKS III (Indian Astronomy)	MA407	1	5	20	25	1	0	0

Learning Objectives:

The primary objectives of this course are to:

- Introduce students to ancient records of the observations of motions of celestial bodies
- Acquire basic ideas of planetary models, and examine their scientific and scholastic implications in the modern milieu.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Acquire knowledge on the history and development of Indian astronomy.
- Be familiar with Indian calendar systems in different regions of the country, as well as the underlying mathematical models developed by Indian astronomers.

Syllabus:

Calendar and Panchanga: Elements of Indian calendar systems in different regions of India, Gregorian calendar, Hindu calendar, Islamic calendar, and Panchanga.

Motion of celestial bodies: Introduction; Equations of center for the Sun and Moon; Observations of the motions of Sun, Moon, Nakshatra, and Graha in the Vedic corpus.

Planetary model: Basic planetary model of Aryabhata and its revision by Nilakantha

Eclipse: Lunar and Solar Eclipses.

Books Recommended:

1. History of Hindu Mathematics: A Source Book, B. Datta and A. N. Singh, Asia Publishing House, 1962.
2. Indian astronomy: An introduction by S. Balachandra Rao, Universities Press (India) Ltd, Hyderabad, 2000.
3. Studies in Indian Mathematics and Astronomy: Selected Articles of Kripa Shankar Shukla, Kolachana et. al. (eds.), Culture and History of Mathematics 12, HBA, 2019.
4. Indian Astronomy: A source book (Based primarily on Sanskrit Texts), Compiled by B. V. Subbarayappa and K. V. Sarma, Nehru Center, Bombay, 1985.
5. History of Astronomy in India by S. N. Sen and K. S. Shukla, 2nd Edition, INSA, Delhi, 2001.

MA408 (Unit I: Linear Programming Problem)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Linear Programming Problem	MA408	2	10	40	50	2	0	0

Learning Objectives:

The primary objectives of this course are to:

- Impart knowledge and understanding of fundamentals in optimization problems.
- Provide mathematical modelling with a rigorous framework to analyze diverse linear systems in real life situation.
- Develop techniques in solving various linear programming problems using different methods.
- Explore transportation and assignment problems to acquire knowledge in the management system.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Formulate and classify general linear programming problem.
- Identify and explore analytical results with the characteristics of an optimal solution.
- Apply appropriate techniques to determine optimal solution and corresponding nature using the simplex method.
- Use methods to explain transportation and assignment problems with some fundamental solution techniques.

Syllabus:

Optimization-Definition, Techniques and Facts. Diverse optimal solutions structure with graphical method and nature of solutions.

Basic concepts: Basis, replacing a vector in a basis. Solution of a system of linear equations- Basic solution, Basic feasible solution. Convex set-Extreme points. Matrix formulation of LPP. Standard form of LPP. Relation among the optimal solution of a LPP, Basic feasible solution and extreme point of the convex set of all feasible solutions.

Fundamental theorem of LPP, Theory of simplex method, Reduction of a feasible solution to a BFS; improving a basic feasible solution; Optimality checking, Unboundedness; Existence of alternative optimal solution. Simplex algorithm and the simplex tableau, Slack, Surplus and Artificial variables-introduction and its subsequent removal, Inconsistency and redundancy.

Two-phase simplex method. Solving simultaneous equations or inequations.

Concept of duality: Its economic interpretation. Construction of dual. The relation between feasible solution of dual and primal problems. Fundamental properties of Dual Problems. Weak duality theorem, Strong duality theorem, Fundamental duality theorem. Computational aspects of simplex method and duality. Dual simplex method-The algorithm and difference between regular simplex method and dual simplex method.

Transportation problems: Mathematical formulation; finding an initial basic feasible solution; The transportation algorithm-checking for optimality, improving a BFS. Degeneracy in transportation problem. Resolution of degeneracy in the initial stage. Resolution of degeneracy during solution stage. Unbalanced transportation problems. Maximization in transportation problems.

Assignment problems: Mathematical formulation; Hungarian assignment method, Unbalanced assignment problem. Maximization in assignment problem. Travelling salesman problem.

Books Recommended:

1. Linear Programming and Game theory, Chakraborty & Ghosh; Moulik Library, Kolkata, 2021.
2. Operations Research: An introduction, Hamdy A. Taha, 10th Edition, Pearson, 2011.
3. Operations Research: Principles and Practice-Ravindran, Phillips, Solberg; Wiley India, 1987.
4. Operations Research-Richard Bronson, GovindasamiNaadimuthu, Tata McGraw-Hill, 1997.
5. Principles of Operations Research, Harvey M. Wagner, PHI, 1980.
6. Operations Research: Theory and Applications, J.K.Sharma, Macmillian India Ltd., 2012.
7. An introduction to Linear Programmig, Mukhopadhyay, Maity & Mazumdar, Kalimata Pustakalaya, Kolkata, 2016.
8. Linear Programming-Methods and Applications, S. I. Gass, Tata McGraw-Hill, New York, 1985.

MA408 (Unit II: Advanced Numerical Analysis)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Advanced Numerical Analysis	MA408	2	10	40	50	2	0	0

Learning Objectives:

The primary objectives of this course are to:

- Understand the concept of piecewise polynomials, constructing interpolating polynomials for data points, and assessing the smoothness of the resulting function.
- Develop proficiency in understanding how to represent complex functions with simpler ones.
- Grasp the core idea of Gaussian quadrature, which involves selecting integration points and weights to match a polynomial of a specific degree exactly.
- Familiar with numerical methods for approximating eigenvalues and eigenvectors and understand their limitations.
- Introduce various numerical methods for solving IVPs and BVPs.
- Understand fundamental concepts of finite difference methods for solving PDEs.

Learning Outcomes

Upon successful completion of this course, students will be able to:

- Apply piecewise interpolation techniques to model data, estimate values between known points.
- Choose suitable approximation methods and apply to solve practical problems in different domains, such as approximating value functions in reinforcement learning or fitting a curve to data points.
- Compute definite integrals with a higher degree of accuracy using Gaussian quadrature.
- Apply the learned concepts and techniques to solve problems involving eigenvalues and eigenvectors, such as finding the dominant eigenvalue and corresponding eigenvector of a matrix.
- Solve simple IVPs by multi-step methods and BVPs using the shooting method and finite difference methods.
- Use the finite difference method to solve real-world problems modeled by PDEs in their respective fields of study.

Syllabus:

Interpolation: Piecewise polynomial interpolation, Hermite interpolation formula, Error in Hermite interpolation. Cubic spline interpolation, Bivariate interpolation: Local matching methods, Global methods.

Approximation of Functions: Norms, Least-squares polynomial approximation, Orthogonal polynomials, Gram-Schmidt orthogonalization, Chebyshev polynomials, Uniform approximation in mini-max sense of error, Lanczos economization.

Numerical Integration: Errors in Newton-Cotes quadrature formula, Gauss-Legendre and Gauss-Chebyshev quadrature formulae, Romberg integration.

Matrix Eigenvalues and Eigenvectors: Power method, inverse power method and Deflation method for square matrices; Jacobi's method, Givens' method, Householder's method and The QR algorithm for symmetric matrices, Singular Value Decomposition.

Numerical Solution of Initial Value problems (IVP) and Boundary Value Problems (BVP) for ODE: First Order Equations: Multi-step Predictor-Corrector Methods: Adams-Bashforth method, Adams-Moulton method, Milne's method --- Convergence and stability. Two-Point Boundary Value problems for ODE: Shooting method, Finite difference method.

Numerical Solution of Partial Differential Equations (PDE) by Finite Difference Methods: Classification of second order PDE, Representation of derivatives by forward, backward and central differences, Thomas Algorithm, Parabolic Equation (Heat-Conduction Equation) (1D) by explicit finite difference and implicit Crank-Nicolson methods, consistency, stability and convergence, Lax equivalence theorem, Poisson Equation on a rectangular region, Hyperbolic Equation (Wave Equation) in one space dimension.

Books Recommended:

1. M. K. Jain, S. R. K. Iyengar, R. K. Jain, Numerical Methods for Scientific and Engineering Computation, New Age International Publisher, New Delhi (2007).
2. K. E. Atkinson, An Introduction to Numerical Analysis, John Wiley and Sons (1989).
3. A. Gupta and S. C. Bose, Introduction to Numerical Analysis, Academic Press (1989).
4. P. Niyogi, Numerical Analysis and Algorithms, Tata McGraw-Hill, New Delhi (2004).
5. E. Isaacson, H. B. Keller, Analysis of Numerical Methods, Dover Publications, Inc., New York (1966).
6. D. Kincaid, W. Cheney, Numerical Analysis: Mathematics of Scientific Computing, American Mathematical Society, Rhode Island (2002).
7. R. L. Burden, J. D. Faires, Numerical Analysis: Theory and Applications, Cengage Learning, New Delhi (2005).
8. S. D. Conte and Carl de Boor, Elementary Numerical Analysis - An Algorithmic Approach, 3rd Edition, McGraw Hill (1980).
9. P. Niyogi, S. K. Chakrabarty, M. K. Laha, Introduction to Computational Fluid Dynamics, Pearson, 2005.

10. G. D. Smith, Numerical Solutions to Partial Differential Equations, Oxford University Press, 3rd Edition, 1986
11. Joe D. Hoffman, Numerical methods for Engineers and Scientist, McGraw-Hill, 1993.

MA409 and MA410 (Project work on two different topics)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Dissertation	MA409	4	20	80	100	3	1	0
Dissertation	MA410	4	20	80	100	3	1	0

%%%%%%%%%%

SEMESTER II (Level 500)

COMMON COURSES (CCs): MA501, MA502, MA408 (Level 400), MA503, and MA504.

MA501(Unit I: Advanced Group Theory)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Advanced Group Theory	MA501	2	10	40	50	2	0	0

Learning Objectives:

The primary objectives of this course are to:

- State and prove the structure theorem of finite abelian groups.
- Introduce the notion of group actions and prove the three theorems of Sylow with the help of it.
- Discuss the ideas of solvable groups.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Identify groups of a given order up to isomorphism using Sylow's theorems.
- Prepare themselves to learn the main theorem of Galois theory which is a crowning achievement in algebra.

Syllabus:

Product of groups: Internal direct product of normal subgroups, necessary and sufficient conditions. Equivalence of internal and external direct products.

Finitely generated abelian groups: Structure of finite abelian groups. The primary decomposition theorem for finite abelian groups. Invariant factors and elementary divisors. The fundamental theorem of finitely generated abelian groups. Semidirect product

Group actions: Conjugation and the conjugacy class equation. Cauchy's Theorem for finite groups. The converse of the Lagrange's theorem for finite abelian groups. Center of a p-group is nontrivial. Group action, orbits and isotropy group. The class equation of a finite group. Burnside counting principle.

Sylow's theorems: Sylow's Theorems on finite groups. Applications of Sylow's theorems to test the simplicity of finite groups. Isomorphism classes of groups up to order 15. If $1 \leq n < 60$ and n is not prime then no groups of order n is simple. Any simple group of order 60 is isomorphic to A_5 . The simplicity of A_n .

Solvability of groups: Solvable groups. Subgroups and homomorphic images of solvable groups. Commutator subgroups. Solvability of the symmetric groups S_n .

Books recommended:

1. Michael Artin, Algebra, 2nd Ed., Pearson, 2011.
2. David S. Dummit and Richard M. Foote, Abstract Algebra, 3rd Ed., Wiley, 2011.
3. John B. Fraleigh, A First Course in Abstract Algebra, 7th Ed., Pearson, 2013.
4. Joseph A. Gallian, Contemporary Abstract Algebra, 9th Ed., Cengage, 2020.
5. I. N. Herstein, Topics in Algebra, 2nd Ed., Wiley, 2006.
6. Charles C. Pinter, A Book of Abstract Algebra, 2nd Ed., Dover, 2010.
7. Vivek Sahai and Vikas Bist, Algebra, 4th Ed., Alpha Science, 2018.
8. M. K. Sen, Shamik Ghosh, Parthasarathi Mukhopadhyay and Sunil Kumar Maity, Topics in Abstract Algebra, 4th Ed., Universities Press, 2022.

MA501(Unit II: Classical Mechanics)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Classical Mechanics	MA501	2	10	40	50	2	0	0

Learning Objectives:

The primary objectives of this course are to:

- Understand basic postulates of classical mechanics.
- Understand D'Alembert's principle and Lagrange's equations.
- Understand Hamilton's principle and principle of least action.
- Understand canonical transformations and related problems.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Solve problems related to Lagrange's equations.
- Solve problems related to principle of least action.
- Solve problems related to canonical transformations.
- Understand different formalism for classical mechanics.

Syllabus:

Basic Postulates, Generalized coordinates, Constraints, D'Alembert's principle, Lagrange's equations, Hamilton's canonical equations, Hamilton's principle and principle of least action. Theory of small oscillations.

Canonical transformations, Generating functions, Poisson's bracket, Jacobi's identity, Poisson's theorem, Jacobi-Poisson theorem.

Special Theory of Relativity: Inertial Frame. Galilean Transformation. Lorentz Transformation. Event. Space-time Coordinate of an Event. The Clock-paradox. K.E. in Relativistic Mechanics.

Books Recommended:

1. H. Goldstein, Classical Mechanics, Narosa Publishing House, New Delhi, 1980.
2. F. Gantmacher, Lectures in Analytical Mechanics, MIR Publishers, Moscow, 1975.
3. J.L. Synge and B.A. Griffith, Principles of Mechanics, McGraw-Hill, New York, 1970.
4. N.C. Rana and P.S. Joag, Classical Mechanics, Tata McGraw Hill Pub. Company Ltd., New Delhi, 1998.
5. N.H. Louis and Janet D. Finch, Analytical Mechanics, C.U.P., 1998.
6. E.T. Whittaker, A Treatise of Analytical Dynamics of Particle and Rigid Bodies, C.U.P. 1977.
7. S.W. McCusky, An Introduction to Advanced Dynamics, Addison-Wesley Publ. Co. Inc. Massachusetts, 1953.
8. A.S. Ramsey, Dynamics Part-II, C.U.P., 1972.
9. L. Elsgolts, Differential Equations and the Calculus of Variations, MIR Publishers. Moscow, 1973.
10. R.H. Dicke and J.P. Wittke, Introduction to Quantum Mechanics, Addison Wesley, 1960.
11. Rydrik, ABC of Quantum Mechanics, Peace Publisher, Moscow, 2023.
12. F. Chorlton, Textbook of Fluid Dynamics, CBS Publications, Delhi, 1985.
13. A.S. Ramsey, Newtonian Attractions, Cambridge, 2007.
14. O.D. Kellog, Foundations of Potential Theory, Dover, 1963.
15. Arthur Beiser, Perspectives of Modern Physics, McGraw-Hill, 1968.

MA502 (Unit I: Methods of Applied Mathematics)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Methods of Applied Mathematics	MA502	3	15	60	75	3	0	0

Learning Objectives:

The primary objectives of this course are to:

Integral Transforms:

- Understand the foundational concepts, existence criteria and properties of Laplace and Fourier transforms.
- Develop proficiency in finding transforms and inverse transforms.
- Apply Laplace and Fourier transforms to solve initial and boundary value problems in ordinary differential equations and partial differential equations.

Integral Equations:

- Understand the nature and classification of integral equations: Volterra, Fredholm, and singular types.
- Convert differential equations into integral equations and vice versa.

- Develop various methods to solve integral equations.
- Study Fredholm alternative theorem and Hilbert-Schmidt theorem.
- Determine the conditions ensuring existence and uniqueness.

Calculus of Variations:

- Understand the concept of functionals and variation, including problems with fixed and moving boundaries.
- Derive and apply Euler's equation and transversality conditions.
- Analyze sufficient conditions for extremum using Jacobi, Legendre, and Weierstrass criteria.
- Formulate and solve variational problems with subsidiary conditions.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

Integral Transforms:

- Compute Laplace and Fourier transforms and their inverses of different functions.
- Apply properties to simplify and solve problems.
- Solve ordinary and partial differential equations using Laplace and Fourier transforms.

Integral Equations:

- Classify and formulate Volterra, Fredholm and singular integral equations.
- Convert differential equations into integral equations and integral equations into differential equations.
- Find the resolvent kernel.
- Solve integral equations using methods such as:
Series solutions, Successive approximations, Successive substitutions
- Adomian and modified Adomian decomposition
- Laplace transforms.
- Analyze and solve Fredholm integral equations with separable kernels.
- Understand and apply Fredholm alternative theorem.
- Understand and apply Hilbert-Schmidt theorem for symmetric kernels.

Calculus of Variations:

- Formulate variational problems and apply Euler's equation to determine extrema.
- Solve problems involving higher-order derivatives and functionals with multiple independent variables.
- Determine sufficient conditions for extrema using advanced theoretical criteria.
- Address isoperimetric and constrained optimization problems using variational techniques.

Syllabus:

Integral Transforms:

Laplace transform: Convergence, continuity requirements, exponential order, existence and uniqueness, properties of Laplace transform, Heaviside's unit step function, convolution theorem, complex inversion formula, inverse Laplace transform, properties of inverse Laplace transform, solutions of ordinary and partial differential equations using Laplace transforms.

Fourier transform: Derivation of Fourier transform from Fourier series, properties of Fourier transform, convolution theorem, inverse Fourier transform, properties of inverse Fourier transform, Parseval's identity, solutions of ordinary and partial differential equations using Fourier transforms.

Applications of Laplace and Fourier transforms.

Integral Equations:

Basic introduction, conversion of ordinary differential equations into integral equations, conversion of integral equations into ordinary differential equations.

Linear integral equations of Volterra type: solution of Volterra integral equations by successive substitutions, successive approximations, series solution, Adomian decomposition, modified Adomian decomposition and Laplace transform methods, resolvent kernel.

Linear integral equations of Fredholm type: Solution of Fredholm integral equations by successive substitutions, successive approximations, series solution, Adomian decomposition and modified Adomian decomposition methods, resolvent kernel, Neumann series, solution of integral equations with separable kernels, Fredholm alternative theorem, Hilbert-Schmidt theorem of integral equations for symmetric kernels.

Singular integral equations: Solution of singular integral equations by Laplace transform method.

Existence and uniqueness of integral equations.

Applications of integral equations.

Calculus of Variations:

Variational problems with fixed boundaries: The concept of variation and its properties, Euler's equation, functionals dependent on higher order derivatives, functionals dependent on functions of several independent variables.

Variational problems with moving boundaries: Transversality conditions, orthogonality conditions, variational problem with a moving boundary for a functional dependent on two functions, one sided variations.

Sufficient conditions for extremum: Field of extremals, weak and strong extremum, Jacobi condition, Weierstrass function, Legendre condition.

Variational problems with subsidiary conditions: Isoperimetric problems.

Applications.

Books Recommended:

1. F. G. Tricomi, Integral Equations (Inter - Science).
2. A. M. Wazwaz, Linear and Nonlinear Integral Equations (Springer).
3. R. P. Kanwal, Linear Integral Equations (Academic Press).
4. P. P. G. Dyke, An introduction to Laplace Transforms and Fourier Series (Springer).
5. M. G. Spiegel, Laplace Transforms (Schaum's Outlines series).
6. J. L. Schiff, The Laplace Transform (Springer).
7. S. L. Ross, Ordinary Differential Equations (Wiley).
8. R. L. Bracewell, The Fourier Transforms and Its Applications (McGraw-Hill).
9. I. M. Gelfand and S. V. Fomin, Calculus of Variations (Prentice Hall).
10. A. S. Gupta, Calculus of Variations with Applications (Prentice Hall).
11. R. Weinstock, Calculus of Variations (Dover Publications).

MA502 (Unit II: IKS III--Indian Astronomy)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
IKS III (Indian Astronomy)	MA502	1	5	20	25	1	0	0

Learning Objectives:

The primary objectives of this course are to:

- Introduce students to ancient records of the observations of motions of celestial bodies
- Acquire basic ideas of planetary models, and examine their scientific and scholastic implications in the modern milieu.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Acquire knowledge on the history and development of Indian astronomy.
- Be familiar with Indian calendar systems in different regions of the country, as well as the underlying mathematical models developed by Indian astronomers.

Syllabus:

Calendar and Panchanga: Elements of Indian calendar systems in different regions of India, Gregorian calendar, Hindu calendar, Islamic calendar, and Panchanga.

Motion of celestial bodies: Introduction; Equations of center for the Sun and Moon; Observations of the motions of Sun, Moon, Nakshatra, and Graha in the Vedic corpus.

Planetary model: Basic planetary model of Aryabhata and its revision by Nilakantha

Eclipse: Lunar and Solar Eclipses.

Books Recommended:

1. History of Hindu Mathematics: A Source Book, B. Datta and A. N. Singh, Asia Publishing House, 1962.
2. Indian astronomy: An introduction by S. Balachandra Rao, Universities Press (India) Ltd, Hyderabad, 2000.
3. Studies in Indian Mathematics and Astronomy: Selected Articles of Kripa Shankar Shukla, Kolachana et. al. (eds.), Culture and History of Mathematics 12, HBA, 2019.
4. Indian Astronomy: A source book (Based primarily on Sanskrit Texts), Compiled by B. V. Subbarayappa and K. V. Sarma, Nehru Center, Bombay, 1985.
5. History of Astronomy in India by S. N. Sen and K. S. Shukla, 2nd Edition, INSA, Delhi, 2001.

MA408 (Unit I: Linear Programming Problem)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Linear Programming Problem	MA408	2	10	40	50	2	0	0

Learning Objectives:

The primary objectives of this course are to:

- Impart knowledge and understanding of fundamentals in optimization problems.
- Provide mathematical modelling with a rigorous framework to analyze diverse linear systems in real life situation.
- Develop techniques in solving various linear programming problems using different methods.
- Explore transportation and assignment problems to acquire knowledge in the management system.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Formulate and classify general linear programming problem.
- Identify and explore analytical results with the characteristics of an optimal solution.
- Apply appropriate techniques to determine optimal solution and corresponding nature using the simplex method.
- Use methods to explain transportation and assignment problems with some fundamental solution techniques.

Syllabus:

Optimization-Definition, Techniques and Facts. Diverse optimal solutions structure with graphical method and nature of solutions.

Basic concepts: Basis, replacing a vector in a basis. Solution of a system of linear equations- Basic solution, Basic feasible solution. Convex set-Extreme points. Matrix formulation of LPP. Standard form of LPP. Relation among the optimal solution of a LPP, Basic feasible solution and extreme point of the convex set of all feasible solutions.

Fundamental theorem of LPP, Theory of simplex method, Reduction of a feasible solution to a BFS; improving a basic feasible solution; Optimality checking, Unboundedness; Existence of alternative optimal solution. Simplex algorithm and the simplex tableau, Slack, Surplus and Artificial variables-introduction and its subsequent removal, Inconsistency and redundancy.

Two-phase simplex method. Solving simultaneous equations or inequations.

Concept of duality: Its economic interpretation. Construction of dual. The relation between feasible solution of dual and primal problems. Fundamental properties of Dual Problems. Weak duality theorem, Strong duality

theorem, Fundamental duality theorem. Computational aspects of simplex method and duality. Dual simplex method-The algorithm and difference between regular simplex method and dual simplex method.

Transportation problems: Mathematical formulation; finding an initial basic feasible solution; The transportation algorithm-checking for optimality, improving a BFS. Degeneracy in transportation problem. Resolution of degeneracy in the initial stage. Resolution of degeneracy during solution stage. Unbalanced transportation problems. Maximization in transportation problems.

Assignment problems: Mathematical formulation; Hungarian assignment method, Unbalanced assignment problem. Maximization in assignment problem. Travelling salesman problem.

Books Recommended:

1. Linear Programming and Game theory, Chakraborty & Ghosh; Moulik Library, Kolkata, 2021.
2. Operations Research: An introduction, Hamdy A. Taha, 10th Edition, Pearson, 2011.
3. Operations Research: Principles and Practice-Ravindran, Phillips, Solberg; Wiley India, 1987.
4. Operations Research-Richard Bronson, GovindasamiNaadimuthu, Tata McGraw-Hill, 1997.
5. Principles of Operations Research, Harvey M. Wagner, PHI, 1980.
6. Operations Research: Theory and Applications, J.K.Sharma, Macmillian India Ltd., 2012.
7. An introduction to Linear Programmig, Mukhopadhyay, Maity & Mazumdar, Kalimata Pustakalaya, Kolkata, 2016.
8. Linear Programming-Methods and Applications, S. I. Gass, Tata McGraw-Hill, New York, 1985.

MA408 (Unit II: Advanced Numerical Analysis)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Advanced Numerical Analysis	MA408	2	10	40	50	2	0	0

Learning Objectives:

The primary objectives of this course are to:

- Understand the concept of piecewise polynomials, constructing interpolating polynomials for data points, and assessing the smoothness of the resulting function.
- Develop proficiency in understanding how to represent complex functions with simpler ones.
- Grasp the core idea of Gaussian quadrature, which involves selecting integration points and weights to match a polynomial of a specific degree exactly.
- Familiar with numerical methods for approximating eigenvalues and eigenvectors and understand their limitations.
- Introduce various numerical methods for solving IVPs and BVPs.

- Understand fundamental concepts of finite difference methods for solving PDEs.

Learning Outcomes

Upon successful completion of this course, students will be able to:

- Apply piecewise interpolation techniques to model data, estimate values between known points.
- Choose suitable approximation methods and apply to solve practical problems in different domains, such as approximating value functions in reinforcement learning or fitting a curve to data points.
- Compute definite integrals with a higher degree of accuracy using Gaussian quadrature.
- Apply the learned concepts and techniques to solve problems involving eigenvalues and eigenvectors, such as finding the dominant eigenvalue and corresponding eigenvector of a matrix.
- Solve simple IVPs by multi-step methods and BVPs using the shooting method and finite difference methods.
- Use the finite difference method to solve real-world problems modeled by PDEs in their respective fields of study.

Syllabus:

Interpolation: Piecewise polynomial interpolation, Hermite interpolation formula, Error in Hermite interpolation. Cubic spline interpolation, Bivariate interpolation: Local matching methods, Global methods.

Approximation of Functions: Norms, Least-squares polynomial approximation, Orthogonal polynomials, Gram-Schmidt orthogonalization, Chebyshev polynomials, Uniform approximation in mini-max sense of error, Lanczos economization.

Numerical Integration: Errors in Newton-Cotes quadrature formula, Gauss-Legendre and Gauss-Chebyshev quadrature formulae, Romberg integration.

Matrix Eigenvalues and Eigenvectors: Power method, inverse power method and Deflation method for square matrices; Jacobi's method, Givens' method, Householder's method and The QR algorithm for symmetric matrices, Singular Value Decomposition.

Numerical Solution of Initial Value problems (IVP) and Boundary Value Problems (BVP) for ODE: First Order Equations: Multi-step Predictor-Corrector Methods: Adams-Bashforth method, Adams-Moulton method, Milne's method --- Convergence and stability. Two-Point Boundary Value problems for ODE: Shooting method, Finite difference method.

Numerical Solution of Partial Differential Equations (PDE) by Finite Difference Methods: Classification of second order PDE, Representation of derivatives by forward, backward and central differences, Thomas Algorithm, Parabolic Equation (Heat-Conduction Equation) (1D) by explicit finite difference and implicit Crank-Nicolson methods, consistency, stability and convergence, Lax equivalence theorem, Poisson Equation on a rectangular region, Hyperbolic Equation (Wave Equation) in one space dimension.

Books Recommended:

12. M. K. Jain, S. R. K. Iyengar, R. K. Jain, Numerical Methods for Scientific and Engineering Computation, New Age International Publisher, New Delhi (2007).
13. K. E. Atkinson, An Introduction to Numerical Analysis, John Wiley and Sons (1989).
14. A. Gupta and S. C. Bose, Introduction to Numerical Analysis, Academic Press (1989).
15. P. Niyogi, Numerical Analysis and Algorithms, Tata McGraw-Hill, New Delhi (2004).
16. E. Isaacson, H. B. Keller, Analysis of Numerical Methods, Dover Publications, Inc., New York (1966).
17. D. Kincaid, W. Cheney, Numerical Analysis: Mathematics of Scientific Computing, American Mathematical Society, Rhode Island (2002).

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Dissertation	MA503	4	20	80	100	3	1	0
Dissertation	MA504	4	20	80	100	3	1	0

[illegible]

SEMESTER III (Level 500)**COMMON COURSES (CCs): MA505, MA506, MA507, MA508A, and MA508B****OPTIONAL COURSEs: MA509 (A1-A12; P1-P7)****Course Title: Advanced Analysis****Course Code: MA505****(Unit-I: Topological Foundations and Multivariate Analysis)**

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Unit I: Topological Foundations and Multivariate Analysis	MA505	03	15	60	75	3	0	0

Learning Objectives:

The primary objectives of this course are to:

- understand the basic notions of general topology (bases/sub-bases, neighbourhoods, continuity) used in analysis.
- study countability and separation axioms, and apply compactness/connectedness (including path connectedness) in standard examples.
- apply topological ideas to multivariable calculus: continuity, differentiability, Taylor expansion, inverse/implicit function theorems, and constrained optimization.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- define and use topological notions (bases/sub-bases, neighbourhoods, subspace and product topology) and verify continuity/homeomorphism in standard examples.
- apply countability and separation axioms to analyse spaces and understand their consequences (e.g., uniqueness of limits in Hausdorff spaces).
- use compactness, connectedness and path connectedness; interpret key results using nets/filters (basic characterizations).
- compute limits, Jacobians and higher derivatives of functions of several variables and determine differentiability via total derivative.

- use Taylor's theorem in several variables to obtain local approximations and apply it in standard problems.
- apply the Inverse and Implicit Function Theorems and solve constrained optimization problems using Lagrange multipliers.

Syllabus:

Basic review:

Topological spaces; open and closed sets; basis and sub-basis; neighbourhoods; continuous functions, homeomorphism; countability axioms.

Advanced topology :

Separation axioms, Compactness , connectedness, path connectedness, path components; nets and filters, their convergence and cluster points.

Multivariate analysis:

Limits and continuity in $\mathbb{R}^n$; differentiability and total derivative; Jacobian matrix and chain rule; higher order derivatives; Taylor's theorem in several variables; Inverse Function Theorem and Implicit Function Theorem; extrema, Lagrange multipliers, and constrained optimization.

References:

- P. Mikusinski and M. D. Taylor, An Introduction to Multivariable Analysis, Birkhauser, 2002.
- Sean Dineen, Multivariate Calculus and Geometry, 3rd Edition, Springer, 2014.
- T. M. Apostol, Calculus, Vol. 2 (Multivariable Calculus and Linear Algebra), Wiley.
- J. R. Munkres, Topology, 2nd Edition, Prentice Hall.
- S. Willard, General Topology, Addison-Wesley (for nets and filters).

(Unit-II: Complex Analysis III)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Unit II: Complex Analysis III	MA505	01	05	20	25	1	0	0

Learning Objectives:

The primary objectives of this course are to:

- gain the basic idea of maximum and minimum principle (extrema occur only on boundaries)
- achieve a powerful tool for analyzing many deep problems in number theory as well as other areas of mathematics

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- gain the clear knowledge about harmonic functions and doubly periodic meromorphic functions.
- solve many applied problems using harmonic functions theory.

Syllabus:

Harmonic functions, examples and properties, characterization of harmonic functions by mean value property, Poisson's integral formula, Dirichlet problem for a disc and the uniqueness of its solution.

Doubly periodic functions, elliptic functions, irreducible set of poles, cell, Weierstrass' elliptic function, examples and properties.

References:

- A. I. Markushevich, *Theory of functions of a complex variable*, Prentice Hall Inc.
- R. V. Churchill and J. W. Brown, *Complex Variables and Applications*, McGraw-Hill, New York.
- E. T. Copson, *An introduction to the theory of functions of a Complex Variable*, Oxford University Press.
- E. C. Titchmarsh, *The Theory of functions*, Oxford University Press.
- W. Kaplan, *An introduction to analytic functions*, Addison-Wesley Pub. Co.

Course Title: Algebra and Bifurcation Theory

Course Code: MA506

(Unit I: Algebra)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Unit I: Algebra	MA506	03	15	60	75	3	0	0

Learning Objectives:

The primary objectives of this course are to:

- Introduce the notion of extension of fields and prime fields, degree of a field extension, algebraic and transcendental elements, minimal polynomials.

- Prove the Tower theorem, isomorphism of several fields using minimal polynomials and introduce the concepts of multiple roots, separability and splitting fields.
- Construct and classify finite fields and to prove the impossibility of certain ruler and compass constructions.
- Introduce the definition of modules over a commutative ring with unity as a generalization of vector spaces over fields and define sub-modules, quotient modules together with examples.
- Define module homomorphisms and prove isomorphism theorems and introduce direct sum and product of modules, free modules and finitely generated modules.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Understand abstract notions of algebra more clearly.
- Relate some questions arising from elementary number theory to algebraic structures.
- Develop intuition to understand some results learned as a formula in high school.

Syllabus:

Definition and examples of fields, Prime fields, homomorphism from the ring of integers to a field, definition of the degree of a field extension, finite and infinite extensions, Tower theorem and its applications, algebraic and transcendental elements, quadratic extensions of rationals (upto isomorphism), symbolic adjunction of roots, multiple roots and separability, constructible numbers, ruler and compass construction, finite fields and their classification, definition and examples of modules, homomorphisms of modules, kernel and image of a module homomorphism, isomorphism of modules, annihilators of modules, direct sums of modules, direct products of modules, free and finitely generated modules.

References:

- Algebra by M. Artin.
- Abstract Algebra by R. Dummit & D. Foote.
- Fields and Galois Theory by P. Morandi.
- Module Theory by T. S. Blyth.
- Introduction to Commutative Algebra by M. F. Atiyah and I. G. Macdonald.

(Unit II: Bifurcation Theory)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Unit II: Bifurcation Theory	MA506	01	05	20	25	1	0	0

Learning Objectives:

The objectives of this course are to:

- Introduce definitions and basic concepts of bifurcation.
- Study different types of bifurcation.
- Understand the definitions and concept of center manifold theorem.
- Derivation of normal forms with examples.

Learning Outcomes:

After successful completion of the course, a student will be able to

- investigate local and global behavior of bifurcations;
- develop the concepts of bifurcations associated to co-dimensions;
- derives the techniques to compute the center manifold theorem;
- basic concepts of normal form theory.

Syllabus:

Definitions of bifurcation. Local and Global bifurcation

Bifurcation of Dimensional 1: Transcritical, Saddle-Node, Pitchfork

Bifurcation of Dimensional 2: Saddle-Node Bifurcation, Pitchfork Bifurcation, Pitchfork Bifurcation, Transcritical Bifurcation, Hopf-Bifurcation, Homoclinic and Heteroclinic bifurcation.

Center Manifold: Notations & Definitions; Local Center Manifold, Examples.

Parameter-dependent center manifold. Normal Forms.

References:

- S Strogatz - Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, And Engineering, Hachette (2014).
- G C Layek - An Introduction to Dynamical Systems, Springer (2015).
- S Wiggins - Introduction to Applied Nonlinear Dynamical Systems and Chaos, Springer (1996).
- L Perko, Differential Equations and Dynamical systems, springer, 2001.

Course Title: Solid Mechanics and Fluid Mechanics**Course Code: MA507****(Unit I: Solid Mechanics)**

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	L	T	P
Unit I: Solid Mechanics	MA507	02	10	40	50	2	0	0

Learning Objectives:

The objectives of this course are to:

- Equip students with a deep and comprehensive understanding of the principles and methods of solid mechanics.
- Develop a rigorous understanding of the fundamental concepts of strain, stress, constitutive behaviour and the fundamental principles of solids.

Learning Outcomes:

Upon successful completion of the course, students will be able to

- Explain the concepts of continuum hypothesis, fundamental equations, strain, stress, and material behaviour in both solids and fluids.

Syllabus:

Notion of a Continuum and of Deformable Bodies. Linear Elastic Solid (or Hookean Solid).

Analysis of Stress: Body and surface forces. Vector stress (or stress vector) and Notation for its Components. Specification of stress at a point. Stress Tensor. Equations of Equilibrium. Symmetry of Stress Tensor. Surface Boundary Conditions. Rule of Transformation of Stress Components. Principal Stresses and Stress Invariants. Stress quadric of Cauchy. Maximum normal and Shearing Stresses. Mohr's Diagram. Problems

Analysis of Strain: Strain Tensor (Lagrangian and Eulerian). Finite Strain Components. Infinitesimal Strain and Rotation Components. Geometrical Interpretation of Infinitesimal Strain Components. Transformation of Infinitesimal Strain Components. The Strain Quadric. Principal Strains and Principal Axis of Strain. Rate of Deformation Tensor. Problems.

Motion of Deformable Bodies: Lagrangian and Eulerian Descriptions. Path line and Stream line. Material Derivative, Condition on a Boundary Surface. Conservation of Mass. The Continuity Equation. Momentum Principles. Equation of Motion. Energy Balance. Laws of Thermodynamics, Equation of State. Generalised Hooke's law. Isotropy. Elastic Moduli.

References:

- R. N. Chatterjee, Mathematical Theory of Continuum Mechanics (Narosa Publishing House, 1999).
- S. Sokolnikoff, Mathematical Theory of Elasticity (McGraw Hill, 1956).
- T. J. Chung, Continuum Mechanics (Prentice Hall, 1988).
- S. C. Hunter, Mechanics of Continuous Media (Ellis Horwood Ltd., England, 1983).
- A. J. M. Spencer, Continuum Mechanics (Longman, 1980).

(Unit-II: Fluid Mechanics)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Unit-II: Fluid Mechanics	MA507	02	10	40	50	2	0	0

Learning Objectives:

The objectives of this course are to:

- Introduce fundamentals of fluid mechanics.
- To train students in basic problem-solving skills in fluid mechanics, and make students aware of role of fluid mechanics in various disciplines of science and engineering.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Define the physical properties of a fluid and to explain the consequence of such properties on fluid flow.
- Apply Lagrangian and Eulerian methods for describing fluid motion.
- Use the concept of stream and potential functions to describe a flow field.
- Analyse stresses in fluid motion and relation between stress and rate of strain.
- Derive the fundamental equations of fluid dynamics.

Syllabus:

Fluids: Ideal Frictionless Fluid. Linearly Viscous Fluid. Stokes' Condition. Different types of flows (Laminar and Turbulent, Steady and Unsteady, Uniform and Non-uniform, Rotational and Irrotational etc.) .

Kinematics: Lagrangian and Eulerian descriptions. Velocity and acceleration of a fluid particle. Equation of continuity or conservation of mass. Boundary surfaces. Stream lines. Path lines and streak lines. Velocity potential. Vortex lines. Irrotational and rotational motions. Simple problems.

Equations of Motion of Inviscid Incompressible Fluid: Lagrange's and Euler's equations of motion. Integrals of Equation of Motion, Kelvin's circulation theorem. Helmholtz's vorticity equation. Principle of permanence of Irrotational Motion. Equations referred to moving axes. Impulsive actions. Stream function. Simple problems.

Irrotational motion in two dimensions. Complex velocity potentials. Sources, sinks, doublets and their images. Conformal mapping. Milne-Thomson circle theorem. 2D irrotational motion produced by motion of circular, co-axial and elliptic cylinders in an infinite mass of liquid. Kinetic energy of liquid. Theorem of Blasius.

General theory of stress and rate of strain: Newton's law of viscosity. Newtonian and non-Newtonian fluids. Stress vector and components of stress tensor. Plane stress. Normal and shearing strain. Constitutive equations for a compressible Newtonian fluid. Navier-Stokes equations of motion of a viscous incompressible fluid. Some exact solutions. Steady motion between parallel plates. Dissipation of energy. Simple problems.

References:

- A. S. Ramsey, A Treatise of Hydrodynamics. Part-II, G. Bell & Sons Ltd. (1957).
- G.K. Batchelor, Introduction to Fluid Dynamics, Cambridge University Press (1967).
- F. Chorlton, Text Book of Fluid Dynamics, Van Norstrand (1967).
- Y. A. Cengel, J. M. Cimbala, Fluid Mechanics: Fundamentals and Applications, McGraw Hill (2014).
- F. M. White, Fluid Mechanics, McGraw Hill, Seventh Edition (2011).
- M. C. Potter, D. C. Wiggert, B. Ramadan, Tom I-P. Shih, Mechanics of Fluids, Cengage Learning (2012).
- S. K. Som, G. Biswas, S. Chakraborty, Introduction to Fluid Mechanics and Fluid Machines, McGraw Hill (2012).
- A. I. Ruban, J. S. B. Gajjar, Fluid Dynamics- Part 1: Classical Fluid Dynamics, Oxford University Press (2014).

Course Title: Nonlinear Ordinary Differential Equations and Operations Research

Course Code: MA508A

(Unit I: Nonlinear Ordinary Differential Equations)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Unit I: Nonlinear Ordinary Differential Equations	MA508A	01	05	20	25	1	0	0

Learning Objectives:

The primary objectives of this course are to:

- How are physical rules used to model a real-world problem?
- How a physical model is converted into mathematical models?
- The importance of differential equations in math modelling
- To understand the origin of ODEs via mathematical modelling

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Classification of mathematical models
- Handle various Deterministic and Stochastic models
- Understand Limitations of Mathematical Modelling
- Obtain the mathematical modelling from real world problems

Syllabus:

1st order nonlinear ODEs, Riccati Equation, Transformations.

Perturbation Techniques: Simple pendulum, Mathieu Equation, van-dar Pol Oscillator, KBJ Approximation.

Second order nonlinear IVPs and BVPs.

Series solution of nonlinear ODEs.

References:

- Nonlinear Ordinary Differential Equations: An introduction to Dynamical Systems, D.W. Jordan, P. Smith, Oxford University Press (1999).
- Differential Equations and Dynamical Systems, L. Perko, Springer (2001).
- Nonlinear Ordinary Differential Equations and Dynamical Systems, Ferdinand Verhulst, Springer (1996).
- Nonlinear Differential Equations, Raimond A. Struble, Dover Publications (2018).

(Unit II: Operations Research)

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Unit II: Operations Research	MA508A	01	05	20	25	1	0	0

Learning Objectives:

The primary objectives of this course are to:

- Develop a rigorous mathematical foundation for understanding modelling phenomena through the framework of optimization.
- Introduce qualitative and quantitative analysis of integer and nonlinear programming problems, with direct relevance to management / industrial systems.
- Bridge mathematical theory with real-life situations interpretation, enabling students to translate industrial assumptions into mathematical formulations and interpret analytical and numerical results.
- Foster economic thinking and research readiness, preparing students for doctoral research and applications in advanced optimization techniques.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Formulate and analyze mathematical models describing various optimization problems.
- Perform qualitative analysis of integer and nonlinear programming problems and understand the applicable industrial domains.
- Understand and characterize inventory and queuing theory in the applicable domain of management systems and relate economic situations to human-life processes.
- Incorporate various optimization techniques into different mathematical models and assess their effects on market stability for a long span of time.
- Demonstrate readiness for advanced research or optimal applications in industry economics.

Syllabus:

Operations Research-An overview.

Revised Simplex Method (RSM): Maximization and minimization problems.

Sensitivity Analysis: Changes in the objective function, Variations in the requirement vector, Addition of a variable, Addition of a constraint.

Integer Linear Programming Problem (ILPP): Branch and Bound algorithm, Cutting plane algorithm.

Non-linear Programming Problem (NLPP): Formulation of non-linear programming problem, Graphical method of solution. Kuhn-Tucker conditions for constrained optimization. Convex programming. Quadratic programming problems by (i) Wolfe's method and (ii) Beale's method.

Inventory: Introduction, Features of inventory system, Inventory model building. Deterministic models with (i) No Shortage, (ii) Shortage. Multi item inventory models with constraints. Probabilistic models-Single period probabilistic models (i) without set up cost, (ii) with set up cost.

Queuing Theory: Introduction. Essential features of queuing system. Probability distribution in queuing models. Classification of queue models. Solution of queuing models

- (i) $\{(M/M/1): (\text{infinity}/\text{FCFS})\}$, (ii) $\{(M/M/1): (n/\text{FCFS})\}$,
- (iii) $\{(M/M/s): (\text{infinity}/\text{FCFS})\}$, (iv) $\{(M/M/s): (n/\text{FCFS})\}$.

References:

- H.A. Taha, Operations Research-An Introduction, Macmillan Publishing Company.
- G. Hadley, Nonlinear and Dynamic Programming, Addison-Wesley Publishing Company.
- S.S. Rao, Optimization Theory and Application, Wiley Eastern.
- K Sarup, P.K. Gupta and Man Mohan, Operation Research, Sultan Chand & Sons.
- J.K. Sharma, Operation Research, Macmillan Publishing Company.
- S.D. Sharma, Operation Research, Kedarnath& Ramnath, Meerat.
- O.L. Mangasarian, Non linear Programming, McGraw Hill.
- Peressini, Sullivan and Uhl, The Mathematics of Nonlinear Programming, Springer.
- Rabindran, Don T. Phillips, James J. Solberg, Operation Research, John Wiley & Sons.

Course Title: Numerical Computer Laboratory Course Code: MA508B

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Numerical Computer Laboratory	MA508B	02	10	40	50	0	0	2

Course Description: This practical course is designed to develop computational proficiency in numerical methods through systematic laboratory practice. Emphasis is placed on algorithmic implementation, accuracy analysis, and visualization of results using modern scientific programming languages.

Learning Objectives:

The primary objectives of this course are to:

- Understand the theoretical foundations and practical significance of numerical methods in scientific computing.
- Develop proficiency in implementing root-finding, interpolation, and approximation techniques.
- Apply numerical integration methods for accurate evaluation of definite integrals.
- Compute eigenvalues, eigenvectors, and matrix inverses using efficient numerical algorithms.
- Solve ordinary differential equations arising in scientific and engineering problems.
- Apply numerical techniques to solve boundary value problems and partial differential equations.
- Analyze the convergence, stability, and accuracy of numerical algorithms.
- Use computational tools to visualize and interpret numerical solutions.
- Integrate mathematical modeling with computational implementation.

- Enhance problem-solving and analytical skills through hands-on programming practice.

Learning Outcomes:

After successful completion of this course, students will be able to:

- Implement and compare various root-finding methods to solve nonlinear equations.
- Perform data interpolation and function approximation using appropriate numerical techniques.
- Apply least squares methods for curve fitting and model estimation.
- Evaluate definite integrals using advanced quadrature techniques.
- Compute eigenvalues, eigenvectors, and matrix inverses for large systems.
- Solve initial value problems using single-step and multi-step numerical methods.
- Apply shooting and finite difference/finite volume methods for boundary value problems.
- Numerically solve heat, wave, and Laplace equations using suitable discretization schemes.
- Assess numerical errors, convergence rates, and stability properties of algorithms.
- Develop efficient, well-documented programs for scientific and engineering applications.
- Interpret computational results in the context of physical, biological, and engineering systems.
- Demonstrate independent problem-solving ability in numerical analysis and scientific computing.

Syllabus:

Root-Finding Methods: Bisection method. Regula falsi method. Newton–Raphson method. Secant method. Fixed-point iteration method.

Interpolation: Lagrange’s interpolation, Newton’s divided difference interpolation. Cubic spline interpolation.

Approximation: Least squares approximation. Polynomial approximation. Trigonometric approximation.

Numerical Integration: Composite trapezoidal, Simpson’s, and Weddle’s rules. Romberg integration. Gaussian quadrature.

Eigenvalues and Eigenvectors: Power method. Jacobi’s method, QR decomposition, Singular value decomposition.

Matrix Inversion: Gauss-Jordan elimination method. LU decomposition method.

Iterative Methods for System of Linear Equations: Gauss-Jacobi method. Gauss-Seidel method. SOR method.

Solution of Initial Value Problems: Euler method. Modified Euler method. Runge–Kutta method. Adams-Bashforth method. Adams-Moulton method. Milne-Simpson method.

Solution of Two-Point Boundary Value Problems: Shooting method.

Solution of Partial Differential Equations: Finite difference method. Numerical solution of heat, wave, and Laplace equations.

N. B. This course component is primarily practice-oriented and shall be conducted through hands-on laboratory sessions. No purely theoretical questions shall be included in the assessment of this component. Programming shall be taught and practiced in the laboratory environment. Only essential concepts, algorithms, and program structures will be discussed in class and subsequently implemented on the computer. Students are required to learn, implement, and analyze the following numerical methods using Python/ C/ MATLAB.

References:

- Numerical Analysis Richard L. Burden & J. Douglas Faires, Cengage Learning, Boston, USA.

- Numerical Methods for Engineers, Steven C. Chapra & Raymond P. Canale, McGraw-Hill Education, New York, USA.
- Applied Numerical Methods with MATLAB, Steven C. Chapra, McGraw-Hill Education, New York, USA.
- Numerical Methods in Engineering with Python, Jaan Kiusalaas, Cambridge University Press, Cambridge, UK.
- An Introduction to Numerical Analysis, Josef Stoer & Roland Bulirsch, Springer, Berlin–Heidelberg, Germany.
- A First Course in Numerical Methods, Uri Ascher & Chen Greif, SIAM, Philadelphia, USA.
- Numerical Methods E. Balagurusamy, McGraw-Hill Education, New Delhi, India.
- Numerical Methods for Scientific and Engineering Computation, M.K. Jain, S.R.K. Iyengar & R.K. Jain, New Age International Publishers, New Delhi, India.

OPTIONAL COURSES (OCs): MA509 (A1-A12; P1-P7)

Optional Courses: A1-A12

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Atmospheric Modeling with Physics Informed Neural Networks (PINNs)	MA509 (A1)	04	20	80	100	3	1	0

Learning Objectives:

The primary objectives of this course are to:

- Introduce basic concepts and principles of atmospheric dynamics.
- Demonstrate various atmospheric flows, waves, and instabilities.
- Give knowledge in circulation and vorticity.
- Introduce basic concepts of Machine learning and Physics-informed neural networks (PINNs), and numerical modeling.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Apply the equations of motion to the description of various atmospheric wave motions.
- Obtain dispersion relations of waves and study their characteristics and instabilities.
- Demonstrate various aspects of large-scale atmospheric flows, including circulation and vorticity.
- Develop a PINN to solve two-dimensional/three-dimensional atmospheric flows.

Syllabus:

Atmospheric fluid dynamics: Fundamentals of geophysical fluid dynamics, Conservation laws, Hydrodynamic equations of motion in rotating frame; Scale analysis; Atmospheric stability of stratified fluids; Circulation and vorticity; Boussinesq approximation; Thermal expansion; Acoustic gravity and Internal waves; Rossby and Kelvin waves.

Numerical modeling: Finite difference methods (FDM), spectral methods, stability analysis (CFL criterion); Numerical solutions of Navier-Stokes equations.

Machine learning and Physics-informed neural networks (PINNs): Architecture of neural networks, Activation functions, Automatic differentiation, Residual functions, Back propagation and optimization techniques; Neural network models (FFNNs, ANNs, DNNs, PINNs); Solutions of PDEs (e.g., Navier-Stokes equations) using PINNs.

References:

- J. R. Holton, An Introduction to Dynamic Meteorology, Fourth Edition, Elsevier Press, 2004.
- H. Lynch and J. J. Cassano, Atmospheric Dynamics, Wiley, 2006.
- Gill, Dynamics of Ocean and Atmosphere, Academic Press.
- P. Joseph, Waves in the Ocean and Atmosphere: Introduction to Wave Dynamics, Springer.
- G. F. Vallis, Atmospheric and Oceanic Fluid Dynamics: Fundamentals and Large-Scale Circulation, Second Edition, Cambridge University Press, 2017.
- Numerical Fluid Dynamics, T. K. Bose, Narosa Publishing House, 2001.
- Fundamentals of Computational Fluid Dynamics, T. K. Sengupta, Universities Press, 2004.
- Neural Networks and Learning Machines, Simon Haykin, Third Edition, Pearson Education India, 2016.
- Neural Networks: A Classroom Approach, Satish Kumar, Second Edition, McGraw Hill Education, 2017.

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Computational Fluid Dynamics	MA509 (A2)	04	20	80	100	3	1	0

Learning Objectives:

The primary objectives of this course are to:

- Equip students with the knowledge base essential for application of computational fluid dynamics to engineering flow problems.
- Provide the essential numerical background for solving the partial differential equations governing the fluid flow
- Prepare students for advanced research in computational fluid dynamics.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Analyse the results obtained from the CFD techniques.
- Solve advanced multi-disciplinary problems involving fluid mechanics, heat transfer and related transport process phenomena using appropriate CFD techniques.

Syllabus:

Finite difference method: Treatment of model equations of parabolic, hyperbolic, elliptic types. Explicit and implicit schemes. Truncation error, consistency, convergence, stability (Von Neumann stability analysis only) of model equations with appropriate initial and boundary conditions. Thomas algorithm. ADI method for 2-D heat conduction problem. First- order wave equation. Upwind scheme, consistency, CFL stability condition. Reduction of a boundary value problem to algebraic equations. Illustrative example, like solution of Dirichlet problem for 2-D Laplace equation by finite volume / difference method.

Conservation principles of fluid dynamics. Basic equations of viscous and inviscid flow. Basic equations in conservation form. Associated typical boundary conditions for Euler and N-S equations. Lax-Wendroff and McCormack schemes for 2-D unsteady Euler equations. Grid generation using elliptic partial differential equations. Incompressible viscous flow field computation: Primitive variable approach (MAC method in staggered grid) and Stream- function vorticity approach. High-Order Compact Finite Difference Methods with Applications to Viscous Flows.

References:

- P. Niyogi, S. K. Chakraborty and M. K. Laha- Introduction to Computational Fluid Dynamics, Pearson education, Delhi 2005.
- C. A. J. Fletcher- Computational Techniques for Fluid Dynamics, Vol-I and Vol-II, Springer 1988.
- R. Peyret and T. D. Taylor – Computational Methods for Fluid Flow, Springer 1983.
- J. F. Thompson, Z. U. A. Warsi and C.W. Martin- Numerical Grid Generation, Foundations and Applications, North Holland 1985.
- L. D. Landau and E. M. Lifshitz- Fluid Mechanics, Trans, Pergamon Press 1989.
- H. Schlichting and K. Gersten- Boundary – Layer Theory, 8th Ed., Springer 2000.
- W. F. Spitz and G. F. Carey – High-Order Compact Finite Difference Methods with Applications to Viscous Flows, Texas Institute for Computational and Applied Mathematics, University of Texas, Austin, 1994.
- Anderson J. – Computational Fluid Dynamics, McGraw-Hill, 1995.

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Differential and Difference Equations in Ecology	MA509 (A3)	04	20	80	100	3	1	0

Learning Objectives:

The primary objectives of this course are to:

- Introduce analytical and numerical studies of ecological systems using both continuous and discrete-time approaches with emphasis on stability, bifurcation, and chaos.
- Acquire a solid foundation in continuous-time and discrete-time dynamical systems.
- Analyze equilibrium behavior and stability of ecological models.
- Understand and classify bifurcation phenomena and nonlinear dynamics.
- Develop mathematical models for single-species and multi-species interactions.
- Examine functional responses and their role in population dynamics.
- Apply epidemic and time-delay models in biological systems.
- Incorporate stochastic effects in population modeling.
- Use computational tools for simulation and visualization of ecological dynamics.
- Interpret mathematical results in ecological and environmental contexts.
- Develop analytical and computational skills for advanced research in mathematical ecology.

Learning Outcomes:

After successful completion of this course, students will be able to:

- Formulate ecological and epidemiological problems using differential and difference equations.
- Determine equilibrium points and analyze their local and global stability.
- Identify and classify bifurcations and chaotic behavior in nonlinear systems.
- Study and analyze the fundamental components of continuous-time and discrete-time ecological models.
- Analyze species interactions using appropriate functional responses.
- Apply time-delay and stochastic models to realistic population systems.
- Perform numerical simulations using MATLAB and Python.
- Validate theoretical results through computational experiments.
- Apply mathematical modeling techniques to fisheries management, conservation biology, pest control, and disease management.

Syllabus:**Concepts of Dynamical Systems:**

Continuous-time and discrete-time dynamical systems. Phase space and trajectories. Jacobian matrix and eigenvalues. Routh–Hurwitz criteria. Jury’s condition. Global stability. Centre manifold theorem. Limit cycles. Saddle-node bifurcation. Transcritical bifurcation. Pitchfork bifurcation. Hopf bifurcation. Neimark–Sacker bifurcation. Flip bifurcation. Homoclinic bifurcation. Heteroclinic bifurcation. Quasiperiodicity. Chaos. Routes to chaos. Lyapunov exponents.

Single-Species Models:

Malthus model. Logistic model. Harvesting model. Insect outbreak model.

Functional Responses:

Holling type I, type II, type III, and type IV responses. Ratio-dependent response. Beddington–DeAngelis response. Crowley–Martin response. Ivlev response. Trigonometric response. Cosner response.

Multi-Species Models:

Predator–prey models. Competition models. Food chain models. Models with generalist predators. One-prey–two-predator models. Two-prey–one-predator models. Two-prey–two-predator models. Metapopulation models.

Models with Different Ecological Phenomena:

Allee effect. Prey refuge. Migration. Fear effect. Mutualism. Commensalism. Parasitism. Vigilance. Hunting cooperation. Predator switching. Cross predation.

Epidemic Models:

SI, SIS, SIR, SIRS, and SEIR models. Threshold conditions. Endemic and disease-free equilibria. Stability and control strategies.

Time Delay Models:

Single and double delays. Derivation of a critical delay for stability of a system. Periodic solutions.

Stochastic Models:

Stochastic birth, death, immigration, and emigration processes.

Numerical Simulations:

Exploration of model dynamics using MATLAB and Python.

Applications:

Fisheries management. Pest control. Conservation biology. Environmental management. Disease control.

N.B. All models will be studied for continuous-time systems using differential equations and for discrete-time systems using difference equations.

References:

- Bazykin, A. D. Nonlinear Dynamics of Interacting Populations. World Scientific Publishing, Singapore.
- Murray, J. D. Mathematical Biology, Vol. I. Springer-Verlag, Berlin.
- Strogatz, S. H. Nonlinear Dynamics and Chaos. CRC Press, New York.
- Layek, G. C. An Introduction to Dynamical Systems and Chaos. Springer, New Delhi.
- Kuznetsov, Y. A. Elements of Applied Bifurcation Theory. Springer-Verlag, New York.
- Wiggins, S. Introduction to Applied Nonlinear Dynamical Systems and Chaos. Springer, New York.
- Martcheva, M. An Introduction to Mathematical Epidemiology. Springer, New York.
- Thieme, H. R. Mathematics in Population Biology. Princeton University Press, Princeton.
- Elaydi, S. An Introduction to Difference Equations. Springer, New York.

- Guckenheimer, J., & Holmes, P. *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*. Springer-Verlag, New York.
- Kot, M. *Elements of Mathematical Ecology*. Cambridge University Press, Cambridge.
- Pratap, R. *Getting Started with MATLAB: A Quick Introduction for Scientists and Engineers*. Oxford University Press, Oxford.
- Lynch, S. *Dynamical Systems with Applications Using Python*. Springer, New York.
- Lynch, S. *Dynamical Systems with Applications Using MATLAB*. Springer, New York.
- Brauer, F., & Castillo-Chávez, C. *Mathematical Models in Population Biology and Epidemiology*. Springer-Verlag, New York.

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Differential Equations with Artificial Neural Networks (ANNs)	MA509 (A4)	04	20	80	100	3	1	0

Learning Objectives:

The primary objectives of this course are to:

Frame DEs as minimization problems, constructing solution forms that satisfy boundary/initial conditions, training networks using back-propagation, and achieving differentiable, compact, accurate solutions for complex systems.

Grasp the Physics-Informed Neural Network (PINN) framework, implement training and evaluating generalization.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Obtain solutions that are analytical, differentiable functions, not just discrete points, allowing for easy calculation of derivatives or further analysis.
- Develop models with fewer parameters than traditional numerical methods, leading to efficient storage and computation.
- Achieve high accuracy and demonstrate good generalization to unseen data points.
- Create single, unified network models capable of solving diverse ODEs and PDEs, including complex, non-linear systems.

Syllabus:

Overview of Differential Equations: Classification and types of differential equations, Differential equations related to physical problems in science and engineering, General introduction of numerical techniques for solving differential equations. Advantages of neural networks over standard numerical methods for solving differential equations.

Artificial Neural Networks: Brief history of neural networks, Biological Neural Network, Artificial Neural Network (ANN), Mathematical model of ANN, Activation function: Linear, Sign, Sigmoid and Step activation function, Architecture of ANN: Feed Forward, Recurrent, Hopfield Networks, Paradigms of learning in ANN: Supervised, Unsupervised, Reinforcement and Competitive Learning, Multilayer ANN, Regression-Based ANN, Single-Layer Functional Link ANN, Single-Layer Functional Link ANN with Regression-Based weights.

ANN for Ordinary Differential Equations: Method of Multilayer Perceptron Neural Network (MLPNN) for solving ODE—Gradient Computation with respect to Network Inputs and Parameters, Method of Radial Basis Function Neural Networks (RBFNN), Method of Multiquadric Radial Basis Function (MQRBF) Neural Network, Method of Cellular Networks. Applications.

ANN for Partial Differential Equations: Direct Radial Basis Function Network (DRBFN) method for solving partial differential equations (PDE), Indirect Radial Basis Function Network (IRBFN technique), Method of Finite Element Neural Networks, The Deep Double Ritz Method, and Method of Wavelet Neural Networks. Applications. Physics Informed Deep Learning: Data-driven solutions of nonlinear PDE. Applications.

References:

- W.E. Boyce and R.C. DiPrima. Elementary Differential Equations and Boundary Value Problems. John Wiley & Sons, Inc., New York (2001).
- D.G. Zill and M.R. Cullen. Differential Equations with Boundary Value Problems. Brooks/Cole, Cengage Learning, Belmont, CA (2008).
- Y. Pinchover and J. Rubinstein. Introduction to Partial Differential Equations. Cambridge University Press, Cambridge, England, (2005).
- S. Haykin. Neural Networks: A Comprehensive Foundation. Pearson Education, Singapore (2002).
- J.A. Anderson. An Introduction to Neural Networks. MIT Press, Cambridge, London, England, U.K. (1995).
- J. M. Zurada. Introduction to Artificial Neural Systems. Jaico Publishing House, St. Paul (2001).
- D. Kincaid, W. Cheney. Numerical Analysis: Mathematics of Scientific Computing. 3rdedn. American Mathematical Society, Providence (2010).
- R.B. Bhat and S. Chakraverty. Numerical Analysis in Engineering. Alpha ScienceInt., Ltd., Oxford, U.K. (2004).
- N. Yadav, A. Yadav and M. Kumar. An Introduction to Neural Network Methods for Differential Equations. Springer (2015).
- S. Chakraverty, S. Mall. Artificial Neural Networks For Engineers and Scientists. CRC Press (2017).
- K.S. Mcfall. An artificial neural network method for solving boundary value problems with arbitrary irregular boundaries. Ph.D. thesis, Georgia Institute of Technology (2006).
- C. Uriarte. Solving Partial Differential Equations Using Artificial Neural Networks. PhD Thesis, The University of Basque Country (UPV/EHU) (2023).

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical

Differential Equations and Dynamical Systems	MA509 (A5)	04	20	80	100	3	1	0
--	-------------------	----	----	----	-----	---	---	---

Learning Objectives:

The objectives of this course are to:

- Introduce definitions and basic concepts of dynamical systems.
- Gain an idea about single species and two species population.
- Concepts about epidemic models and find their equilibrium points and their stability analysis.
- Understand the concept of difference equations and their stability analysis.
- Develop basic concept about optimal control strategy.

Learning Outcomes:

After successful completion of this course, a student will be able to:

- Formulate mathematical models of various biological and physical systems (interaction of species, epidemiology, environmental modeling etc.) in terms of differential & difference equations.
- Develop mathematical models by using ODEs and PDEs.
- Investigate systematically mathematical models of biological systems locally as well as globally.
- Learn the techniques of optimal control strategy.
- Apply the knowledge of mathematics in selected real life situations.

Syllabus:

Continuous growth functions: Malthus growth, logistic growth, Gompertz growth, Holling type growth. One species models: Different growth models for single species, harvesting of species. Two species models: Equilibria and their stability analysis.

Epidemic Models: Deterministic model of simple epidemic, Infection through vertical and horizontal transmission, General epidemic-Karmac-Mackendric Threshold Theorem, SI, SIR, SIRS models.

Discrete system: Overview of difference equations, steady state solution and linear stability analysis, Introduction to Discrete Models, Linear Models, Growth models, Decay models, Discrete Prey-Predator model and Epidemic model.

Difference Equation and its Application: Difference Calculus, Linear first – order difference equations, Nonlinear difference equations, Higher order linear difference equations, Systems of difference equations, Stability Theory, Applications.

An introduction to partial differential equations and diffusion in biology: Functions of several variables: a review; Random motion and diffusion equation; Diffusion equations and some of its consequences.

Optimal Control Problems:

Preliminaries: The Basic Problem and Necessary Conditions; Pontryagin's Maximum Principle

Existence and Uniqueness Results; Interpretation of Adjoint; Principle of Optimality.

The Hamiltonian and Autonomous Problems: Necessary Conditions; Linear Quadratic Regulator Problems;

Higher Order Differential Equations; Isoperimetric Constraints;

Examples: HIV treatment, Bear populations, Glucose model, Predator-prey model

References:

- S. Lenhart, J.T. Workman.: Optimal Control Applied to Biological Models: (Chapman & Hall / CRC Mathematical and Computational Biology), 2007.
- L. Perko, Differential Equations and Dynamical Systems, Springer Verlag, 2001.

- F. Verhulst, Nonlinear Differential Equations and Dynamical Systems, Springer.
- F. Brauer, C. Castillo-Chavez. Mathematical Models in Population Biology and Epidemiology
- J. D. Murray, Mathematical Biology, Springer and Verlag.
- Leach Edelstein-Keshet.: Mathematical Models in Biology, Birkhauser.

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Fundamentals of Magnetohydrodynamics	MA509 (A6)	04	20	80	100	3	1	0

Learning Objectives:

The primary objectives of this course are to:

- Introduce the basic concepts of electromagnetic theory.
- Understand basics of plasmas and their dynamics.
- Understand fundamental principles of MHD.
- Learn mathematical modeling and derivation of MHD equations.
- Study waves and instabilities in plasmas.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Analyze MHD waves, including Alfvén waves and Magnetosonic waves.
- Understand physical significance of magnetic Reynolds number, magnetic pressure, etc.
- Apply MHD principles to understand physical processes in space and astrophysical environments.
- Analyze how magnetic fields affect fluid flows, such as inducing currents and generating Lorentz forces.

Syllabus:

Basics of Electromagnetic (EM) Theory: *Electrostatics*-- Coulomb's law, Electric field and potential, Gauss' law; *Magnetostatics*--Lorentz force, Equation of continuity, Magnetic vector potential, Ampere's circuit law; *Electrodynamics*--Faraday's law, Displacement current, Maxwell's equations, Electromagnetic potential, Electromagnetic energy.

Plasma Dynamics Preliminaries: Basic concepts of plasmas, Elements of plasma dynamics, Conservation of energy; Fluid theory of plasmas--Equations of continuity, Equation of motion, etc., Electrostatic and Electromagnetic waves in plasmas; Kinetic plasma theory--Vlasov and Boltzmann equations, Fluid equations from Boltzmann equation, Landau damping.

Basics of Magnetohydrodynamics (MHD): MHD validity and assumptions; MHD equations--Single fluid and Two-fluid approaches, Generalized Ohm's law, Conservation of energy; Reduced MHD equations; Induction equation--Magnetic viscosity and Magnetic Reynold's number, Frozen-in magnetic fields; MHD waves--Alfvén waves, Magnetosonic waves, etc.; MHD instabilities—Rayleigh-Taylor instability, Kelvin-Helmholtz instability, etc.

References:

- Introduction to Electrodynamics- David J. Griffiths, Prentice Hall, 1999.
- Classical Electrodynamics- John David Jackson, John Wiley and Sons, Third Edition, 1999.
- Electricity and magnetism- D. Chattopadhyay and P. C. Rakshit, New Central Book Agency, 2011.
- Fundamentals of Plasma Physics, J. A. Bittencourt, Springer-Verlag New York, 2008.
- Introduction to Plasma Physics and Controlled Fusion, F. F. Chen, Plenum Press, New York, Second Edition, Vol. 1, 1984.
- Introduction to Plasma Physics, R. J. Goldston and P. H. Rutherford, IOP Publishing, Bristol & Philadelphia, 1995.
- Magnetohydrodynamics, T. G. Cowling, Interscience Publishers, New York, 1957.
- A Text Book of Magnetohydrodynamics, J. A. Shercliff, Pergamon Press, Oxford, 1965.
- S. I. Pai, Magnetohydrodynamics and Plasma Dynamics, Springer Verlag, New York, 1962.

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Fundamentals of Vortex Dynamics	MA509 (A7)	04	20	80	100	3	1	0

Learning Objectives:

The course aims to focus on understanding fundamentals of rotational fluid motion, the identification, transport and interaction of vortical structures and their applications in science and engineering.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Define vorticity and to explain its significance in fluid flow.
- Analyse vortex behavior in fluid flow.
- Apply vortex theory to understand vortex-shedding, vortex-induced vibration, oceanic phenomenon etc.
- Evaluate different modeling techniques such as LES, DNS to simulate vortex flows.
- Calculate induced velocity fields from vorticity distributions.

Syllabus:

Fundamental Processes in Fluid Motion: Descriptions and visualizations of fluid motion, Deformation kinematics: Vorticity and dilatation. Fundamental equations of Newtonian fluid motion, intrinsic decomposition of vector fields: Functionally orthogonal decomposition, Monge-Clebsch decomposition, Helical-Wave decomposition, Splitting and coupling of fundamental processes: Triple decomposition of strain rate and velocity gradient, triple decomposition of stress tensor and dissipation.

Vorticity Dynamics: Physical interpretation of vorticity, Vorticity integrals and far-field asymptotics, Lamb vector and Helicity, Vorticity evolution, Circulation-Preserving flows, Vorticity diffusion vector, Vorticity field at small Reynolds Numbers, Vorticity dynamics in boundary layers, Vortex sheet dynamics, Vorticity-based formulation of viscous flow problem. Vorticity dynamics in flow separation: Flow separation and boundary-layer separation, three-dimensional steady flow separation, unsteady flow separation.

Vortex Dynamics: Typical vortex solutions—Exact strained vortex solutions, asymptotic strained vortex solutions, Objective definition of vortex: Existing criteria, analytical comparison of the criteria, Applications. Separated vortex flows—Topological theory of separated flows, Steady separated bubble flows, Steady free vortex-layer separated flow, unsteady bluff-body separated flow. Vortex formation and core structure, Vortex filament, Vortex-boundary interactions, Vortical flow instability and vortex breakdown, Vortical structures in transitional shear flows—Coherent structures.

References:

- F. M. White, Fluid Mechanics, McGraw Hill, Seventh Edition (2011).
- J.-Z. Wu, H.-Y. Ma, M.-D. Zhou. Vortical Flows, Springer (2015).
- J.-Z. Wu, H.-Y. Ma, M.-D. Zhou. Vorticity and Vortex Dynamics, Springer (2006).
- K. E. Gustafson, J. A. Sethian. Vortex Methods and Vortex Motion, SIAM, Philadelphia (1991).
- A. J. Majda, A. L. Bertozzi. Vorticity and Incompressible Flow, Cambridge University Press (2002).
- P. G. Saffman. Vortex Dynamics, Cambridge University Press (1992).
- J. Détery. Three-dimensional Separated Flow Topology, Wiley (2013).
- V. Holmen. Methods for Vortex Identification, PhD Thesis, Lunds University (2012).
- G. Haller. Transport Barriers and Coherent Structures in Flow Data, Cambridge University Press (2023).
- T. Kambe. Geometrical Theory of Dynamical Systems and Fluid Flows, World Scientific, Singapore (2004).

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Mathematical Biology	MA509 (A8)	04	20	80	100	3	1	0

Learning Objectives:

The primary objectives of this course are to:

- **Develop a rigorous mathematical foundation** for understanding biological phenomena through deterministic, stochastic, and spatially extended dynamical systems.
- **Introduce qualitative and quantitative analysis of nonlinear dynamical systems**, including phase plane analysis, stability theory, bifurcation, chaos, and pattern formation, with direct relevance to biological systems.
- **Equip students with modelling skills** for population dynamics, epidemiology, eco-epidemiology, and food web interactions using ordinary and partial differential equations.
- **Bridge mathematical theory with biological interpretation**, enabling students to translate biological assumptions into mathematical formulations and interpret analytical and numerical results biologically.
- **Expose students to advanced topics** such as chaos in three-dimensional systems, reaction-diffusion mechanisms, spatiotemporal pattern formation, and invasion dynamics in ecological systems.
- **Train students in analytical and computational techniques**, including local and global stability analysis, bifurcation analysis, and numerical simulations using software tools such as MATLAB and MAPLE.
- **Foster interdisciplinary thinking and research readiness**, preparing students for doctoral research and applications in mathematical biology, ecology, epidemiology, and biomedical sciences.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- **Formulate and analyze mathematical models** describing single-species, multi-species, predator-prey, competitive, symbiotic, and eco-epidemic interactions using systems of differential equations.
- **Perform qualitative analysis of dynamical systems**, including phase portrait construction, linearization, application of Hartman's theorem, and identification of stable, unstable, and center manifolds.
- **Analyze local and global stability of equilibria** in biological models and interpret their implications for population persistence, extinction, coexistence, and disease dynamics.
- **Understand and characterize chaotic dynamics** in three-dimensional autonomous systems such as the Lorenz and Rössler systems, and relate chaos to biological and chemical processes.
- **Incorporate various ecological implications** into population models and assess their effects on system stability, oscillations, and long-term behavior.
- **Apply reaction-diffusion and advection-diffusion frameworks** to study spatial spread, invasion processes, travelling waves, and pattern formation in ecological and epidemiological contexts.
- **Explain diffusion-driven instabilities and Turing patterns**, including mode selection, dispersion relations, and ecological pattern formation in real-world systems such as semiarid vegetation.
- **Conduct numerical simulations and interpret computational results**, validating analytical findings and exploring complex system behavior beyond closed-form solutions.

- **Critically evaluate mathematical biology literature**, drawing connections between theoretical models and empirical biological phenomena.
- **Demonstrate readiness for advanced research or interdisciplinary applications**, particularly in mathematical biology, ecology, epidemiology, systems biology, and applied nonlinear science.

Syllabus:

Temporal dynamics

1. **Introduction:** Modelling biological systems, Relevance of modelling environmental dynamics, Mathematics and biology as an interdisciplinary subject, Dynamical systems, etc.
2. **Phase Portraits for planar systems:** Canonical forms, Eigenvectors defining stable and unstable manifolds, Phase portraits of linear systems in the plane, Linearization and Hartman-Grobman's theorem, Constructing phase plane diagrams.
3. **Classical single species population models:** Population growth-A non-age structured model.
4. **Two-dimensional models for interacting populations**
 - (i) Classical Lotka-Volterra model, Two species predator-prey population model, Competing populations, Symbiotic populations, Gause-type interacting species model, Leslie-Gower model, Classical Holling-Tanner model, Various population growth models with their modifications, Migration / Harvesting models and optimal control theory, Model with prey switching / generalist predator / Allee effect / Refuge phenomena / hunting co-operation, etc.
 - (ii) Local and global stability
 - (iii) Local and global bifurcations (Co-dimension 1 and 2)
 - (iii) Stochastic processes and need of stochastic models. Pure birth process, Pure death process, Birth and death process. Linear birth-death-immigration-emigration processes. Effects of both immigration and emigration on the dynamics of population.
 - (iv) Food web
5. **Three-dimensional autonomous systems and chaos:** Linear systems and canonical forms, Nonlinear systems and stability, The Rössler system and chaos, The Lorenz equations, The Belousov-Zhabotinski reaction, The Briggs-Rauscher reaction, etc.
6. **Three species population models:** Competitive food chain model, Mathematical model formulation with various ecological implications, Stability and bifurcation analysis, System behaviour around boundary and co-existence equilibria, Numerical computer simulation
7. **Epidemic models:** Basic epidemic models, Kermack-McKendrick threshold theorem, Age- and stage-dependent epidemic system, Case study: various diseases, Analysis of a disease with two states
8. **Eco-epidemic systems:** Prey-diseased-predator interactions, Predator-diseased-prey interactions, Diseased competing species models, Eco-epidemics models of symbiotic communities (Disease effects on the symbiotic system, Disease control by use of a symbiotic species)

Spatiotemporal dynamics and pattern formation

1. **Spatial aspect:** Diffusion as a paradigm, Fick's laws of diffusion, One-dimensional diffusion model and its solution, Some solutions of two-dimensional diffusion equation, Various modifications of diffusion equation to diffusion-reaction models arising in population biology or ecology.
2. **Diffusion processes:** Diffusion and random motion, Nonlinear diffusion models, Mathematical aspects of reaction-diffusion (Fisher's equation, Traveling wave solutions, Perturbation solution, Stability of

travelling waves, Nagumo's equation), and advection-diffusion (Burgers' equation, Travelling wave solutions, Initial value problem) mechanism, Biological reaction-diffusion systems.

3. **Instabilities and dissipative structures:** Reaction-diffusion (Turing) mechanisms, Turing patterns through diffusion-driven instability, Turing space, Mode selection and the dispersion relation, Differential flow instability, Ecological example (semiarid vegetation patterns etc.), Biological relevance of complex spatiotemporal patterns.

References:

- J.D. Murray, Mathematical Biology (Volumes I, II), Springer, 1993.
- R.M. May, Stability and Complexity in Model Ecosystems, Princeton University Press, 2001.
- A. Okubo, Diffusion and Ecological Problems: Mathematical Models, Springer, 1980.
- A. Okubo, S.A. Levin, Diffusion and Ecological Problems: Modern Perspective, Springer, 2001.
- M. Kot, Elements of Mathematical Ecology, Cambridge University Press, 2003.
- D.W. Jordan, P. Smith, Nonlinear Ordinary Differential Equations: An introduction to Dynamical Systems, Oxford University Press, 1999.
- L. Perko, Differential Equations and Dynamical Systems, Springer, 2001.
- S.H. Strogatz, Nonlinear Dynamics and Chaos with Applications to Physics, Biology, Chemistry, and Engineering, Persues Books Publishing, 1994.
- D.S. Jones, M. Plank, B.D. Sleeman, Differential Equations and Mathematical Biology, Chapman and Hall (CRC), 2009.
- A.D. Bazykin, A.I. Khibnik, B. Krauskopf, Nonlinear Dynamics of Interacting Populations, World Scientific Publishing Company Incorporated, 1998.
- J. Burdon, Diseases and Plant Population Biology, Cambridge University Press, 1987.
- J.N. Kapur, Mathematical Models in Biology and Medicine, East-West Press Pvt. Ltd., 1985.
- F.R. Giordano, M.D. Weir, W.P. Fox, A First Course in Mathematical Modelling, China Machine Press, 2013.
- B.D. Hassard, N.D. Kazarinoff, Y.H. Wan, Theory and Application of Hopf-bifurcation, Cambridge University Press, 1981.
- R.S. Cantrell, C. Cosner, Spatial Ecology via Reaction-Diffusion Equations, Wiley, 2003.
- H. Meinhardt, Models of Biological Pattern Formation, Academic Press, 1982.
- Horst Malchow, Sergei V. Petrovskii, Ezio Venturino, Spatiotemporal Patterns in Ecology and Epidemiology: Theory, Models, Simulations. CRC Mathematical and Computational Biology Series. CRC Press, 2008.

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Mathematical Modeling and Dynamical Systems with Machine Learning	MA509 (A9)	04	20	80	100	3	1	0

Learning Objectives:

The primary objectives of this course are to:

- Introduce the theory and applications of nonlinear dynamical systems.
- Develop a rigorous understanding of bifurcation and chaos.
- Integrate mathematical modeling with modern machine learning techniques.
- Promote data-driven analysis of complex systems.
- Provide hands-on experience in numerical simulation and computational modeling.
- Prepare students for interdisciplinary research and professional applications.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Formulate mathematical models for real-world systems.
- Analyze stability, bifurcation, and chaotic behavior.
- Apply analytical and numerical techniques to continuous and discrete systems.
- Use machine learning for prediction of system dynamics.
- Develop hybrid physics-based and data-driven models.
- Perform simulations and interpret results using Python/Contemporary Language Platform.
- Apply theoretical tools to practical problems across various disciplines.

Syllabus:

Fundamentals of Dynamical Systems:

Continuous-time and discrete-time dynamical systems. Phase space and trajectories. Fixed points and stability. Linearization. Jacobian matrix and eigenvalues. Routh–Hurwitz criteria. Jury’s condition. Saddle-node, transcritical, pitchfork, Hopf, flip, and Neimark–Sacker bifurcations. Homoclinic and heteroclinic bifurcations. Fractals. Sensitive dependence on initial conditions. Chaos. Routes to chaos: period-doubling, quasiperiodicity, intermittency, and homoclinic route. Lyapunov exponents.

Differential and Difference Equation Models:

Nonlinear ordinary differential equations. Nonlinear difference equations. Ecological models. Epidemiological models. Chemical models. Physical models. Discrete maps and iterated systems. Chaos in continuous-time systems. Chaos in discrete-time systems. Model scaling and non-dimensionalization.

Time Delay and Stochastic Systems:

Delay differential equations. Stability and bifurcation with delay. Stochastic differential equations. Noise-induced chaos and noise-induced transitions.

Fundamentals of Machine Learning:

Introduction to artificial intelligence and machine learning. Supervised, unsupervised, and semi-supervised learning. Generative and discriminative models. Classification and regression problems. Feature selection. Clustering techniques. Training, testing, and cross-validation. Bias–variance trade-off. Overfitting and regularization. Model selection and performance evaluation. Explainable AI.

Dimensionality reduction and Neural Networks:

Dimensionality reduction. Principal Component Analysis. Independent Component Analysis. Neural Networks: Single-Layer Networks & Multi-Layer Networks. Activation Functions. Autoencoders.

Data-Driven Dynamical Systems:

Dynamic Mode Decomposition. Sparse identification of nonlinear dynamics. Koopman operator theory. Data-driven Koopman analysis.

Machine Learning for Bifurcation and Chaos:

Time-series prediction models. Recurrent neural networks. Long Short-Term Memory networks. Chaos detection and regime classification. Bifurcation prediction using machine learning. Parameter estimation and inverse modeling.

Hybrid and Physics-Informed Models:

Neural ordinary differential equations. Physics-informed neural networks. Equation discovery methods. Model reduction techniques. Reduced-order and surrogate modeling. Interpretability and uncertainty quantification.

Applications of data-driven modeling in real-world problems.

N. B. Numerical simulations for machine learning-based system identification, prediction, and analysis will be performed using Python/Contemporary language platform.

References:

- Nonlinear Dynamics and Chaos – Steven H. Strogatz, Westview Press, Boulder, CO, USA.
- Elements of Applied Bifurcation Theory – Yuri A. Kuznetsov, Springer, New York, USA.
- Mathematical Biology – J. D. Murray, Springer, New York, USA.
- Mathematical Ecology – Mark Kot, Cambridge University Press, Cambridge, UK.
- An Introduction to Mathematical Epidemiology – Maia Martcheva, Springer, New York, USA.
- Pattern Recognition and Machine Learning – Christopher M. Bishop, Springer, New York, USA.
- Pattern Classification - R. O. Duda, P. E. Hart and D. G. Stork, Wiley, India.
- Hands-On Machine Learning with Scikit-Learn, Keras, and TensorFlow – AurélienGéron O'Reilly Media, Sebastopol, CA, USA.

- Data-Driven Science and Engineering – Steven L. Brunton & J. N. Kutz, Cambridge University Press, Cambridge, UK.
- Neural Networks and Learning Machines - S. O. Haykin, Pearson Education, India.
- The Elements of Statistical Learning – Trevor Hastie, Robert Tibshirani & Jerome Friedman Springer, New York, USA.
- Machine Learning: A Probabilistic Perspective – Kevin P. Murphy, MIT Press, Cambridge, MA, USA.
- Machine Learning – Tom M. Mitchell, McGraw-Hill, New York, USA.
- Introduction to Machine Learning with Python – Andreas C. Müller & Sarah Guido O'Reilly Media, Sebastopol, CA, USA.
- Machine Learning and Dynamical Systems – Brunton, Noack & Koumoutsakos Cambridge University Press, Cambridge, UK.
- Nonlinear System Identification – Stephen A. Billings, Wiley, Chichester, UK.

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Mathematical Neuroscience	MA509 (A10)	04	20	80	100	3	1	0

Learning Objectives:

The primary objectives of this course are to:

- **Provide a solid mathematical foundation for neuroscience**, enabling students to understand neuronal structure, neural signalling, and information processing through differential equation-based models.
- **Introduce core mathematical tools**-ordinary differential equations, numerical methods, and dynamical systems theory for the formulation and analysis of neuronal and network models.
- **Develop conceptual and analytical understanding of biophysical neuron models**, including the Hodgkin–Huxley, Morris–Lecar, and FitzHugh–Nagumo equations.
- **Train students in qualitative analysis of neural dynamics**, such as phase plane analysis, stability, bifurcation theory, bursting phenomena, and chaotic behaviour.
- **Expose students to spatial and spatiotemporal neural dynamics**, including cable theory, propagating action potentials, travelling waves, and reaction-diffusion models.
- **Prepare students for advanced research in mathematical and computational neuroscience**, emphasising the connection between mathematical theory, numerical simulation, and biological interpretation.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- **Formulate mathematical models of neuronal excitability and signalling**, translating biological assumptions into systems of differential equations.
- **Analyse the dynamics of neuron models** using linearization, phase plane methods, Lyapunov stability, bifurcation analysis, and numerical simulation techniques.
- **Explain the mechanisms underlying action potentials, bursting, and oscillatory behaviour**, and classify different firing patterns observed in neurons.

- **Apply cable theory and compartmental modelling** to study dendritic processing, signal propagation, and spatial aspects of neuronal dynamics.
- **Investigate neural oscillators and coupled neuronal systems**, including weak coupling, synchronisation, phase resetting, and network-level dynamics.
- **Interpret spatiotemporal pattern formation in neural systems**, including diffusion-driven instabilities and wave propagation, supported by computational simulations.

Syllabus:

1. Introduction

(a) Neurons

- What is a spike?
- Where is the threshold?
- Why are neurons different and why do we care?
- An historical sketch of nerves and their mathematics
- Neural coding
- The central nervous system
- Simple model formulation

(b) Mathematical Tools: ODEs, Numerical Methods and Dynamical Systems

- Linearization and solving linear systems of ODEs - Stability of solutions of ODEs, Lyapunov functions
- Numerical methods - Euler's method, The second-order Euler-Heun method, The Runge-Kutta method, Newton's method for finding zeros
- Geometric theory of dynamical systems - Stable and unstable manifolds, Planar systems of ODEs, Center manifolds and bifurcations, Periodic orbits and Poincare maps
- Concept of neural networks as nonlinear ODEs
- Neurocomputational properties

2. The Hodgkin-Huxley Equations

- The resting potential
- The Nernst equation
- The Goldman-Hodgkin-Katz equation
- Equivalent circuits: the electrical analogue
- The membrane time constant
- The cable equation
- The squid action potential
- Voltage-gated channels
- Hodgkin-Huxley (HH) model

3. Dendrites

- Multiple compartments
- The cable equation
- Linear cables with constant diameter
- Finite and semi-infinite cables.
- The membrane time constant
- Branching and equivalent cylinders.
- An isolated junction
- Dendrites with active processes

4. Model Dynamics

- The Morris-Lecar model
- The phase-plane
- Bifurcation analysis

- Bifurcation analysis of the Hodgkin-Huxley equations
- Reduction of the HH model to a two-variable model
- Fitzhugh-Nagumo equations

5. Variety of Channels

- Sodium channels
- Calcium channels
- Voltage-gated potassium channels
- M-current
- The inward rectifier
- Sag
- Currents and ionic concentrations
- Calcium-dependent channel

6. Bursting

- Introduction to Bursting
- Square-wave Bursters
- Elliptic Bursting
- Parabolic Bursting
- Classification of Bursters
- Chaotic Dynamics

7. Propagating Action Potentials

- Traveling waves and homoclinic orbits
- Scalar bistable equations
- Singular construction of wave – Wave trains
- Dispersion relations
- Morris-Lecar revisited and Shilnikov dynamics
- Stability of the wave
- Myelinated axons and discrete diffusion

8. Neural Oscillators: Weak Coupling

- Neural oscillators, phase, and isochrones - Phase resetting and adjoints, Bifurcations and adjoints, Spike-time response curve
- Relationship of the adjoint and the response to inputs, Forced oscillators, Coupled oscillators, Other map models, Weak coupling, Synaptic coupling near bifurcations, Small central pattern generators, Linear arrays of cells, Two-dimensional arrays, All-to-all coupling
- Pulse-coupled networks: solitary waves - Integrate-and-fire model, Stability

9. Networks

- Mathematical Models for Neuronal Networks
- Examples of Firing Patterns
- Singular Construction of the Action Potential
- Excitatory Synapses
- Post-Inhibitory Rebound - Two Mutually Coupled Cells, Clustering, Dynamic Clustering
- Thin Spikes - Existence of Antiphase Oscillations, Stability of Antiphase Oscillations
- Almost-Synchronous Solutions - Almost-Synchrony in a Network of Two-Mutually Coupled Cells
 - Coupled Cells
- Slow Inhibitory Synapses - Fast/Slow Decomposition, Antiphase and Suppressed Solution, Synchronous Oscillations with Slow Inhibitory Synapses

- Propagating waves

10. Spatiotemporal Dynamics of a Reaction-Diffusion Wilson Neuron Model

Neurodynamic behaviour, Diffusion-driven instability, Spatiotemporal pattern formation, Numerical computer simulation.

References:

- E.M. Izhikevich, Dynamical Systems in Neuroscience, MIT Press, 2007
- B. Ermentrout and D.H. Terman, Mathematical Foundations of Neuroscience, Springer, 2010
- A.Scott, Neuroscience: A mathematical Primer, Springer, 2002
- Mathematical Neuroscience, S. Brzykczyn and R.R. Poznanski, Academic Press, 2013
- M. Jacobson, Foundations of Neuroscience, Springer, 1993.
- B.Ermentrout, Neuroscience: A mathematical Primer, JSTOR, 2003
- T. Trappenberg, Fundamentals of Computational Neuroscience, Oxford University Press, 2009
- E.L. Schwartz, Computational Neuroscience, MIT Press, 1993.
- E. Rolls and G. Deco, Computational Neuroscience of Vision, Oxford University Press, 2001.

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Mathematical Pharmacology	MA509 (A11)	04	20	80	100	3	1	0

Learning Objectives:

The primary objectives of this course are to:

- To use of Mathematics to understand, predict, and optimize how drugs work in the body.
- To develop modelling skill.
- To understand ligand-receptor interaction.
- To optimize dosage regimens for better efficacy.
- To predict transient drug concentration for various scenarios, like compartmental and non-compartmental models, endovascular and transdermal transport.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- quantitatively analyze and predict drug behavior to improve therapy and drug development.
- predict drug concentration–time profiles in the body

- design improved medical devices.
- develop translational aspect of their learning.
- integrate mathematics with pharmacology and biology.

Syllabus:

Cell Surface Receptor/Ligand Binding Fundamentals:

Rate theory and occupancy theory; simple monovalent binding-constant ligand concentration, ligand depletion effect; cooperativity; multiple receptor states-simple one-step binding; non-interconverting receptor classes; interconverting receptor classes and their applications; basic ternary complex model and applications.

The Endocytic Cycle:

Whole-cell kinetic model and applications; kinetic model of sorting that predicts ligand effects-monovalent receptor-monovalent ligand, monovalent receptor-bivalent ligand.

Diffusivity and the Mechanism of Mass Transport:

Fick's law of binary diffusion (molecular mass transport); mass and molar concentration; diffusive mass and molar fluxes; convective mass and molar fluxes. Solution of diffusion equation with/without elimination or generation-in rectangular and curvilinear coordinates and applications.

Pharmacokinetics of Drug Distribution:

Compartmental models-one-compartment models; two-compartment models; two-compartment models with absorption; physiological models-flow-limited compartments; drug binding and metabolism in compartmental models; application of physiological models such as microvascular mixing in tissue compartments. Concepts of free and bound drug; irreversible, reversible, saturable and non-saturable binding of drug; effective diffusion coefficient and tortuosity.

Controlled Drug Delivery System:

Reservoir and transdermal delivery system-diffusion through planar membranes, diffusion through cylindrical membranes; matrix delivery system; diffusion through viable skin (microneedle).

Case studies: design of a controlled delivery system for T-20, design of a controlled delivery system from implanted devices-matrix type delivery device, drug-eluting stent.

References:

- Lauffenburger, D.A. and Linderman, J., 1993. Receptors: models for binding, trafficking, and signaling. Oxford University Press.
- Ciechanover, A., Schwartz, A.L. and Lodish, H.L., 1985. Sorting and Recycling of Cell Surface Receptors and Endocytosed Ligands. In Mechanisms of Receptor Regulation (pp. 225-253). Springer US.
- Evans, E.A., Skalak, R. and Weinbaum, S., 1980. Mechanics and thermodynamics of biomembranes.
- Ramkrishna, D. and Amundson, N.R., 1985. Linear operator methods in chemical engineering with applications to transport and chemical reaction systems. Prentice Hall.
- Segel, L.A., 1980. Mathematical models in molecular cellular biology. Cambridge University Press.
- Wells, J.W. and Hulme, E.C., 1992. Receptor-Ligand Interactions: A Practical Approach. Oxford, IRL Press.
- Limbird, L.E., 2012. Cell Surface Receptors: A Short Course on Theory and Methods. Springer Science & Business Media.
- Linderman, J.J. and Lauffenburger, D.A., 2013. Receptor/ligand sorting along the endocytic pathway (Vol. 78). Springer Science & Business Media.

- Pastan, I. and Willingham, M.C., 1985. The pathway of endocytosis. In Endocytosis (pp. 1-44). Springer US.
- Deen, W.M., 1998. Analysis of Transport Phenomena. Indian Edition. Oxford University Press.
- Bird, R.B., Stewart, W.E. and Lightfoot, E.N., 2007. Transport phenomena. John Wiley & Sons.
- Saltzman, W.M., 2001. Drug delivery: engineering principles for drug therapy. Oxford University Press.
- Murray J D., 2003. Mathematical Biology (vol.-II), Springer.

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Nonlinear Differential Equations	MA509 (A12)	04	20	80	100	3	1	0

Learning Objectives:

The primary objectives of this course are to understand:

- How a problem is converted into a nonlinear differential equation?
- How a physical model is connected to a nonlinear differential equations?
- The importance of nonlinear differential equations in mathematics.
- How to solve nonlinear ODEs, PDEs by different techniques?

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Understand different mathematical techniques those can handle nonlinear differential equations.
- Understand the differences between complete and partial integrability.

Syllabus:

First-order nonlinear PDEs.

Travelling wave Solutions: Tanh, tanh-coth, sine-cosine, Exponential function methods, etc.

Complete and Partial integrability. Painleve technique and series solutions of some nonlinear partial differential equations.

Transformations in PDEs: Cole-hopf transformation, Muira transformation, Backlund Transformation etc and solutions of PDEs.

Hirota's Bilinear Method: Introduction, D operator, Bilinear form of KdV, KP like evolution equations, Hirota's perturbation and multi soliton solutions of some evolution equations.

Lax Pair, Daurbox Transformation and solutions of some standard nonlinear differential equations.

Approximate analytical solutions for non-autonomous systems.

Wronskian technique for solving nonlinear PDEs.

References:

- Nonlinear Partial Differential Equations: Techniques and recent developments, P. Chatterjee, K. Roy and L.K. Mandy (World Scientific, 2025).
- Perturbation Methods (Cambridge Texts in Applied Mathematics) by E J Hinch (Cambridge University Press, 1991)
- Partial Differential Equations and Solitary Waves Theory by Abdul-Majid Wazwaz (Springer Science & Business Media, 2010)
- Nonlinear waves, solitons and chaos by E. Infeld & G. Rowlands (Cambridge University Press, 1990).

Optional Courses: P1-P7

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Advanced Complex Analysis	MA509 (P1)	04	20	80	100	3	1	0

Learning Objectives:

The primary objectives of this course are to:

- study growth properties of entire and meromorphic functions closely;
- motivate the students to make easier at understanding the use of complex analysis in research field.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- gain clear knowledge about Maximum modulus functions, Nevanlinna's characteristic functions and their several properties;
- solve many problems on value sharing and value distribution of entire and meromorphic functions.

Syllabus:

Maximum Modulus theorem (Statement only), Properties of $M(r)$ and $A(r)$, Connection of the degree of a polynomial in relation to $M(r)$ and $A(r)$, Hadamard's three circles theorem, Theorem of Borel and Caratheodory.

Entire functions, Weierstrass factorization theorem, Order and type of an entire function, The function $n(r)$, Jensen's formula, Zeros of entire functions, Exponent of convergence, Canonical product, Genus, Borel's first theorem, Borel's second theorem (Statement only), Picard's theorem, Hadamard's factorization theorem, Order and type of an entire function in terms of its Taylor's coefficients.

Meromorphic functions, Mittag-Leffler's theorem, Poisson-Jensen formula, Definitions of functions $m(r, a)$, $N(r, a)$ and $T(r, f)$, Nevanlinna's first fundamental theorem, Cartan's identity, Convexity theorem, Orders of growth, Order of a meromorphic function, Comparative growth of $T(r, f)$ and $\log M(r, f)$. Nevanlinna's second fundamental theorem (Statement only), Estimation of $S(r, f)$ (Statement only), Deficient values, Upper bound of the sum of deficiencies, Deficient functions, Nevanlinna's theorem on deficient functions (Statement only), Idea of value sharing. Nevanlinna's five-point uniqueness theorem, Milloux's theorem, Milloux's basic results.

References:

- E. C. Titchmarsh, The theory of functions, 2nd ed. Oxford University Press.
- G. Valiron, Lectures on the general theory of integral functions, Chelsea Pub. Co.
- B. Ja Levin, Distribution of zeros of entire functions, American Mathematical Society Providence.
- L.V. Ahlfors, Complex Analysis, Mc GRAW.HILL INTERNATIONAL EDITIONS.
- R. P. Boas, Invitation to Complex Analysis, The Mathematical Association of America.
- Markusevich, Theory of Functions of a Complex Variables, Vol. I & II, AMS Chelsea reprint, 2005.
- W. K. Hayman, Meromorphic Functions, Oxford University Press.
- C. Yang and H. X. Yi, Uniqueness theory of meromorphic functions, Kluwer Academic Press.

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Advanced Complex Function Theory	MA509 (P2)	04	20	80	100	3	1	0

Learning Objectives:

The primary objectives of this course are to:

- Introduce the spaces of analytic and meromorphic functions respectively and get skilled in solving related problems.
- Familiarize the students with the range of analytic functions and two famous theorems of Picard.
- Provide the notion of Nevanlinna theory regarding entire and meromorphic functions and study some famous results and their applications.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Understand the structure and nature of the spaces of analytic and meromorphic functions respectively.
- Develop the skill to solve problems based on Little Picard theorem and Great Picard theorem.
- Get the real essence of the Nevanlinna theory and also its strength through the improvements of some renowned results from general Complex Analysis.
- Use a few new techniques to solve problems and also enhance the ability of critical thinking for the core theoretical developments.

Syllabus:

Spaces of analytic functions, spaces of meromorphic functions, Range of an analytic function, Little Picard theorem, Great Picard theorem.

Simple connectedness, Runge's theorem, Mittag-Leffler's theorem.

Entire functions, $M(r, f)$ and its properties (statements only), Meromorphic functions and their expansions, Definition of the functions $m(r, a)$, $N(r, a)$ and $T(r, f)$.

Nevanlinna's first fundamental theorem, Cartan's identity and convexity theorems, Orders of growth, Order of a meromorphic function, Comparative growth of $\log M(r)$ and $T(r)$, Nevanlinna's second fundamental theorem, Estimation of $S(r)$ (Statement only), Deficiency of a value, Nevanlinna's theorem on deficient functions, Nevanlinna's five-point uniqueness theorem, Basic Milloux Theory.

References:

- A. I. Markushevich, *Theory of Functions of a Complex Variable*, (Vol. I, II, III), Prentice Hall Inc.
- E. T. Copson, *An Introduction to the Theory of Functions of a Complex Variable*, Oxford University Press.
- J. B. Conway, *Functions of One Complex Variable*, Springer-Verlag.
- E. C. Titchmarsh, *The Theory of Functions*, Oxford University Press.
- W. K. Hayman, *Meromorphic Functions*, The Clarendon Press, Oxford.
- L. Yang, *Value Distribution Theory*, Science Press, Springer-Verlag.

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Advanced Functional Analysis	MA509 (P3)	04	20	80	100	3	1	0

Learning Objectives:

The primary objectives of this course are to:

- Understand the concept of topological vector space (tvs).
- Recognize many important properties of tvs.
- Introduce the notion of continuity and bounded linear operator in tvs, find out relationships between them.
- Present the idea of semi-norm and Minkowski Functional and establish their basic properties.
- Familiarize the students with the fundamental theorems (Banach-Steinhaus theorem, Hahn-Banach theorem, Open mapping theorem, Closed Graph theorem) in TVS.
- Introduce the concept of Complete orthonormal set, Total orthonormal set in Hilbert space and establish some basic properties.
- Familiarize the students with Reflexive Hilbert space.
- Introduce the notion of different types of operators viz. Adjoint operator, Self-adjoint operator, Normal operator, Projection operator etc. and establish their several properties.
- Recognize spectral theorems for Bounded Linear operators in normed linear spaces and spectral theorems for Bounded self-adjoint operator in Hilbert spaces and also spectral theory in Banach algebra.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Gain knowledge about tvs, separation properties etc.
- Relate the behavior of continuous linear operator and bounded linear operator in tvs.
- Gain knowledge about semi-norm and Minkowski functional.
- Understand the role of fundamental theorems (Banach-Steinhaus theorem, Hahn-Banach theorem, Open mapping theorem, Closed Graph theorem) in TVS.
- Develop the idea of Complete orthonormal set, Total orthonormal set and Reflexive Hilbert space.
- Learn properties of different types of operators and solve problems.
- Develop results on spectral theorems for Bounded linear operators in normed linear spaces and for Bounded self-adjoint operators in Hilbert spaces.
- Gain knowledge about spectral theorems in Banach algebra.

Syllabus:

Topological Vector Spaces (TVS):

Definition and basic properties of TVS, Separation properties, Linear mapping, Locally compact space and finite dimensionality property, Metrization of TVS, Boundedness and Continuity of linear operator and their properties in TVS. Semi-norm, Minkowski Functional, Normability of TVS.

Fundamental Theorems in TVS:

Banach-Steinhaus theorem, Hahn-Banach theorem and its applications, Open mapping theorem, Closed Graph theorem.

Hilbert Space:

Best approximation theorem, Orthogonal complement, Direct sum, Projection theorem, Complete orthonormal set, Total orthonormal set, Reflexivity of Hilbert space.

Operator Theory:

Sesquilinear form, Adjoint operator, Self-adjoint operator, Unitary operator, Normal operator, Projection operator, Positive operator and study their fundamental properties.

Spectral Theory:

Eigen values of a linear operator, The Spectrum of a bounded linear operator, Spectral properties of bounded linear operator in normed linear spaces, Spectral properties of Bounded self-adjoint operator in Hilbert spaces, Spectral theory in Banach algebra.

References:

- E. Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley & Sons, 1989.
- G. Bachman & L. Narici, *Functional Analysis*, Academic Press, 2003.
- C. Gottman & G. Pedrick, *First Course in Functional Analysis*, Prentice Hall of India, 2002.
- A. H. Siddiqui, *Functional Analysis with Applications*, Tata-McGraw Hill, 1986.
- G. F. Simmons, *Introduction to Topology and Modern Analysis*, McGraw Hill, 1963.
- B. K. Lahiri, *Elements of Functional Analysis*, The World Press, 1982.
- V. K. Krishnan, *Text Book of Functional Analysis*, Prentice Hall, 2004.
- B. V. Limaye, *Functional Analysis*, Wiley Eastern, 2004.
- W. Rudin, *Functional Analysis*, Tata-McGraw Hill

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Advanced Real Analysis	MA509 (P4)	04	20	80	100	3	1	0

Learning Objectives:

The primary objectives of this course are to:

develop a rigorous understanding of sigma-algebras, Borel sets and measures, including outer measure constructions and completion.

study measurable functions and the Lebesgue integral, with emphasis on approximation by simple functions and properties of integrable functions.

understand $L_p[a,b]$ spaces, Holder and Minkowski inequalities, completeness, and density of standard subspaces.

learn differentiation theory of real functions using Dini derivatives, monotonicity theorems and Vitali covering, and relate it to Lebesgue integration.

explore advanced classes of real functions : absolutely continuous functions.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

construct measures from pre-measures using outer measure methods and work with Borel and Lebesgue-Stieltjes measures, including completion.

Verify measurability and compute Lebesgue integrals of simple and measurable functions, using standard properties and convergence principles (basic).

Apply Holder and Minkowski inequalities in L_p spaces, prove completeness, and identify dense subspaces such as step/simple/continuous functions.

Use Dini derivatives, monotonicity results and Vitali covering to explain differentiability a.e. of monotone functions and related theorems.

Characterize absolutely continuous functions and connect them with indefinite Lebesgue integrals and the Fundamental Theorem of Lebesgue Integral.

Syllabus:

Dini derivatives; differentiation of real functions; monotonicity theorems; Vitali covering theorem; differentiability of monotone functions.

Absolutely continuous functions and singular functions; characterization of absolutely continuous functions; indefinite Lebesgue integral; Fundamental Theorem of Lebesgue Integral.

$L_p[a,b]$ spaces: definition and examples; Holder and Minkowski inequalities; completeness and other basic properties; dense subspaces of $L_p[a,b]$ spaces (step/simple/continuous functions).

Algebra and sigma-algebra of sets; Borel sets; F_σ and G_δ sets; countably additive set functions.

Measures on sigma-algebras; pre-measure; measurable cover; construction of outer measure (Method I and Method II only) and Caratheodory measurability; complete measure and completion; regular and metric outer measures; Lebesgue-Stieltjes measure.

References:

- H. L. Royden and P. M. Fitzpatrick, Real Analysis, 4th Edition, Pearson.
- S.N.Mukhopadhyay & S. Ray, Measure and Integration : an Introduction, Springer 2025
- G. B. Folland, Real Analysis: Modern Techniques and Their Applications, 2nd Edition, Wiley.
- D. L. Cohn, Measure Theory, 2nd Edition, Birkhauser.
- W. Rudin, Real and Complex Analysis, 3rd Edition, McGraw-Hill.
- Zygmund, Trigonometric Series (selected topics for real-variable methods).
- D. Aliprantis and O. Burkinshaw, Principles of Real Analysis, 3rd Edition, Academic Press.
- G. F. Simmons, Introduction to Topology and Modern Analysis .

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Algebraic Coding Theory	MA509 (P5)	04	20	80	100	3	1	0

Learning Objectives:

The primary objectives of this course are to:

- Introduce the basic concepts of block codes and linear codes, including repetition codes and decoding techniques such as nearest neighbor decoding and syndrome decoding.
- Develop the necessary probabilistic background and explain Shannon's theorem (without proof) and its implications in coding theory.
- Study important families of binary codes such as Hamming codes, simplex codes, Reed–Muller codes, Golay codes, and their algebraic and combinatorial structures.
- Introduce bounds on codes including Plotkin bound, Gilbert–Varshamov bound, and other classical bounds.
- Develop the theory of cyclic codes, generator and parity check polynomials, BCH codes, Reed–Solomon codes, and quadratic residue codes.
- Study advanced codes such as quasi-cyclic codes, generalized Reed–Muller codes, and Goppa codes including their decoding and asymptotic behavior.
- Explore applications and connections of coding theory with combinatorial designs such as Steiner systems and Hadamard matrices.

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Understand the fundamental structure of linear and cyclic codes and apply encoding and decoding methods.
- Use syndrome decoding and nearest neighbor decoding methods to detect and correct errors.
- Construct classical families of binary codes and analyze their parameters.
- Apply standard bounds to estimate the performance and limitations of codes.
- Use algebraic tools over finite fields of characteristic two for constructing and decoding Reed–Muller and cyclic codes.
- Work with generator polynomials, check polynomials, and zeros of cyclic codes for constructing BCH and Reed–Solomon codes.
- Understand the minimum distance properties of Goppa codes and apply decoding methods for these codes.
- Relate coding theory to combinatorial structures such as Hadamard matrices and Steiner systems.

Syllabus:

Block codes and linear codes, dual of a code, repetition codes, generator and parity check matrix of a linear code, relation between hamming distance and parity check matrix of a linear code, decoding a linear code using quotient space, Shannon's theorem (without proof).

Binary Hamming codes, constructing codes by various operations, simplex codes, Hadamard matrices and codes constructed from Hadamard and conference matrices, Plotkin bound and various other bounds, Gilbert–Varshamov bound.

First order Reed–Muller codes, Reed–Muller code of order r , encoding and decoding using the algebra of finite fields with characteristic two.

weight enumerators, Krawtchouk polynomials, Lloyd’s theorem, binary Golay codes, connections with Steiner systems.

Automorphisms of field extensions, Galois extensions, Fundamental theorem of Galois theory, Galois group of a polynomial, cyclotomic polynomials

Cyclic codes, generator polynomial and check polynomial, zeros of a cyclic code, idempotent generators, BCH codes, Reed–Solomon codes, quadratic residue codes, generalized Reed–Muller codes.

Minimum distance of Goppa codes, generalized BCH codes, decoding of Goppa codes and their asymptotic behavior.

References:

- F. J. MacWilliams and N. J. A. Sloane, The Theory of Error-Correcting Codes.
- S. Lin and D. J. Costello, Error Control Coding.
- R. E. Blahut, Theory and Practice of Error Control Codes.
- W. C. Huffman and V. Pless, Fundamentals of Error-Correcting Codes.
- T. W. Hungerford, Algebra (for finite field background).

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Number Theory	MA509 (P6)	04	20	80	100	3	1	0

Learning Objectives:

The primary objectives of this course are to:

- Introduce the notion of algebraic numbers and algebraic integers.
- Introduce algebraic number fields, discriminants and integral bases.
- Classify quadratic fields and compute the number theoretic invariants associated to that.
- Introduce Dedekind domains and prove associated properties.
- Introduce ideal class groups and prove the finiteness of it.

- State and prove the law of Quadratic Reciprocity.
- State and prove Dirichlet's unit theorem.
- Prove Fermat's last theorem for regular primes (if time permits).

Learning Outcomes:

Upon successful completion of this course, students will be able to:

- Understand abstract notions of number theory more clearly.
- Relate some questions arising from elementary number theory to algebraic number theory.
- Develop intuition to solve certain Diophantine equations using algebraic techniques.

Syllabus:

Definition and examples of number fields, countability of algebraic numbers, ring of integers, discriminants, conjugates, traces and norms, quadratic fields, cyclotomic fields, Dedekind domain and the structure of its ideals, ideal class groups and their finiteness, Quadratic Reciprocity, Dirichlet's unit theorem, Fermat's Last Theorem for regular primes.

References:

- Algebraic Number Theory and Fermat's Last Theorem by I. Stewart and D. Tall
- Problems in Algebraic Number Theory by J. Esmonde and M. R. Murty.
- Algebraic Number Theory by R. Ash
- A Classical Introduction to Modern Number Theory by K. Ireland and M. Rosen

Course Name	Course Code	Credit	Marks			Weekly Distribution of Class Hours		
			Internal Marks	End Semester Marks	Total Marks	Lecture	Tutorial	Practical
Rings and Modules	MA509 (P7)	04	20	80	100	3	1	0

Learning Objectives:

- Understand modules as generalizations of vector spaces, including submodules, quotient modules, direct sums, free modules.
- Study Noetherian and Artinian rings and modules to analyze finiteness conditions.
- Understand the structure theorems for semisimple modules and rings.
- Introduction to the basics of homological classification of rings: study of tensor product, injective and projective modules.
- Demonstrate applications of modules to characterize rings: regular rings, hereditary rings, semisimple and J-semisimple rings.

Learning Outcomes:

After completion of this course, the students will be able to:

- Understand the structure and theory of rings and modules for advanced mathematical exploration.
- Pursue more advanced courses like Representation theory, Algebraic number theory and Homological Algebra. Course Contents:

Syllabus:

A quick review on modules, submodules and homomorphisms of modules. Quotient modules, direct sum of submodules.

Products and coproducts of rings and modules, their existence and uniqueness. External direct sum and internal direct sum of modules. Subdirect product of rings.

Groups of morphisms, Exact sequences, short and split exact sequences, short five lemma. Groups of morphisms and products, coproducts.

Free modules, universal mapping property and existence of free modules. Rings with invariant basis number, infinite dimensional vector spaces.

Modules over a principal ideal domain. Free modules over a PID. Finitely generated modules over a PID. Applications to the structure of finitely generated abelian groups.

Chain conditions on rings and modules. Artinian and Noetherian rings, Artinian and Noetherian modules, basic properties. The Jordan Holder Theorem, The Hilbert Basis Theorem.

Semisimple rings and modules, Simple Rings and Modules, Semisimple modules. The Artin-Wedderburn Theory.

Jacobson radical of a ring with unity, internal and external characterizations. Primitive rings and the structure of the Jacobson semisimple rings. Chain conditions and Jacobson semisimplicity. Jacobson radical of a module, Nakayama's Lemma. Structure of Jacobson semisimple modules.

Tensor products of modules. Uniqueness and existence. Further properties of the tensor product. Tensor product of free modules. Flat modules and regular rings.

Injective modules. Their products. Baer's Criterion for injective modules. Divisible abelian groups.

Projective modules. Their sums. Hereditary and semihereditary rings.

References:

- Bland, P. E., Rings and their modules, De Guyter, 2011.
- Blyth, T. S., Module Theory: An approach to linear algebra, Clarendon Press, 1977.
- Kasch, F., Modules and rings, Academic Press, 1982
- Lam, T. Y., A First Course in Non-Commutative Rings, Springer Verlag, 2001.
- Mac Lane, S., Categories for the Working Mathematician, Springer, 1998.
- Wisbauer. R., Foundations of Modules and Rings, Gordon and Breach Science Publishers, New York, 1991.

[illegible]

