4-Year Undergraduate Program (with Statistics Major) Examination 2024
Semester-I
Statistics
Course: MJST01T (Descriptive Statistics I (Theory))
Full Marks: 60 Time: 3 Hours

Group A

(Answer any three questions. Each question carries 5 marks.)

1. What is trimmed mean? Discuss a situation where it is used. Suggest any other measure for the same situation. 2+2+1

2. Define geometric mean (g) and harmonic mean (h) and show that $g \geq h$. When does the equality hold? 2+2+1

3. Three positive integers a, b and c are such that $a \leq b \leq c$. If their arithmetic mean and median are respectively 20 and $a + 11$, find the least possible value of c. 5

4. Suppose that a random variable takes values $x_1, x_2, \ldots x_n$ such that $a \leq x_i \leq b; \forall i = 1(1)n$. Show that $s^2 \leq (b - \overline{x})(\overline{x} - a)$. 5

5. Is the median of the logarithms of a set of positive real numbers equal to the logarithm of the median of the numbers? What will your answer be in the case of arithmetic mean? Justify your answers. 2+3

Group B

(Answer any three questions. Each question carries 15 marks.)

6. (a) Distinguish between
   i. Time series data and cross sectional data
   ii. Histogram and stem-leaf display.

   (b) Mention the advantages of tabular representation of data over the textual representation.

   (c) Suppose a train moves n equal distances each of b kms, say, with speeds $V_1, V_2, \ldots V_n$ km/hour. Is the arithmetic mean is a suitable average here to determine the average speed of the train?

   (d) Give an example in each of the cases where you recommend the use of (i) a multiple bar diagram, (ii) a pie diagram.

   (e) What are the merits and demerits of mode? (3+3)+3+2+2+2

7. (a) Let $\overline{x}$ and s be the mean and standard deviation of a data set $x_1, x_2, \ldots x_n$. If $N(k)$ denotes the number of x_i such that $\mid x_i - \overline{x} \mid \geq k.s$, for some $k > 0$, prove that $\dfrac{N(k)}{n} \leq \dfrac{1}{k^2}$.

(b) Now, assume that $x_1, x_2, \dots x_n$ denotes the lifetimes of n electric bulbs. Using the result stated in (a), determine the lower bound of the percentage of bulbs having lifetime between $\overline{x} - 2s$ and $\overline{x} + 2s$.

(c) Consider a set $S = \{2, 4, 6, 8, x, y\}$ with distinct elements. If x and y are both prime numbers and $0 < x, y < 40$, then examine, with proper justification, the validity of the following statements:

i. The maximum possible range of the set is greater than 33.

ii. If $y = 37$, the arithmetic mean of the set is greater than the median.

(d) Indicate how central tendency, dispersion, and skewness of a given data set can be examined with the help of a single diagram. Can you use the same diagram to detect some possible outliers in the data set? 4+2+(2+2)+(3+2)

8. (a) Prove that the mean deviation about median can not exceed the standard deviation.

(b) Two groups with n_1 and n_2 observations have AMs $\overline{x_1}$ and $\overline{x_2}$, SDs s_1 and s_2. Derive the formula for the combined standard deviation based on these quantities and discuss each of the following special cases:

i. $n_1 = n_2$

ii. $\overline{x_1} = \overline{x_2}$.

(c) Let (R) and (s) denote the range and the standard deviation of a set of n observations. Prove that $\frac{R^2}{2n} \leq s^2 \leq \frac{R^2}{4}$. Discuss the boundary cases. 3+6+6

9. (a) Define quartiles of a frequency distribution. How can you develop a measure of skewness based on them? Find limits of this measure.

(b) Develop another measure of skewness based on sample mean and median. Find the limits of this measure. You have to prove the necessary results.

(c) A variable assumes 3 equidistant values $x_0 - h, x_0, x_0 + h$ with relative frequencies $\frac{1-p}{2}, p$ and $\frac{1-p}{2}$ respectively. Calculate Pearson's b_2 measure and find its limits as $p \to 0$ and $p \to 1$. (2+1+2)+(1+4)+5

10. (a) Show that the mean deviation of a set of n observations about A, (MD_A) can be obtained by the formula

$$nMD_A = (S_2 - S_1) + A(n_1 - n_2)$$

where
S_1: Sum of the values which are less than A.
n_1: Number of values less than A.
S_2: Sum of the values which are greater than A.
n_2: Number of values greater than A.

(b) Hence or otherwise, prove that mean deviation about median is the least.

(c) Suppose that the variable assumes positive values only and that the deviations $x_i - \overline{x}$ are small compared to $\overline{x}$. Show that in such a case,

$$\overline{x}_h \simeq \overline{x}\left(1 - \frac{s^2}{\overline{x}^2}\right)$$

(d) You have to compare the dispersions of two sets of 50 recordings on the minimum daily temperatures recorded in each of Kolkata and Shimla during 50 consecutive winter days. Suggest a sutiable measure you would like to use, with justification. 2+5+5+3

11. (a) Express the rth order central moment in terms of the rth and lower order raw moments.

(b) Why Sheppard's correction for moments are required? State two conditions that are necessary for these corrections to be valid.

(c) Prove that

i. $b_2 \geq 1$

ii. $b_2 - b_1 - 1 \geq 0$,

where the symbols have their usual meanings. Discuss in detail the cases of equality. —

3+4+8

12. (a) How would you design and administer a questionnaire?

(b) Discuss the merits and demerits of the questionnaire method of data collection.

(c) Write a short note on Scrutiny of data. 5+4+6

Undergraduate Program with Statistics Major Examination, 2024
Semester I
Statistics (Practical)
Course: MJST01P (Descriptive Statistics I (Practical))
Full Marks :20 Time: 2 Hours

(1) For the two frequency distributions given in the adjoining table, the mean calculated from the first was 25.4 and that from the second was 32.5.

Class	Distribution I Frequency	Distribution II Frequency
10-20	20	4
20-30	15	8
30-40	10	4
40-50	x	$2x$
50-60	y	y

(a) Find the values of x and y.
(b) Evaluate the medians and modes of these two frequency distributions.
(c) Evaluate any measure of skewness for each of these two frequency distributions and comment on the nature of these two frequency distributions. 4+4+4

(2) From a frequency distribution of 10 individuals, the 1st moment about 2 is 2.8, the 2nd moment about 4 is 7.0, the 3rd moment about 3 is 43.2 and the 4th moment about 5 is 72.4. Calculate the **first four central moments**. 5

(3) Practical Note Book and Viva-voce. 3

Four Year Undergraduate Programme
Sem-I Examination 2024

Subject: Statistics Major
Paper: MJST02T
Probability and Probability Distributions I (Theory)

Answer any **five** out of the following eight questions of equal marks.
Nonations are of usual meanings.

Full Marks: 80 Time: 3 hours

1. (a) If A and B are events in a sample space S with $P(A) = 0.4$ and $P(B) = 0.2$, and A and B are mutually exclusive, then $P(A \cup B)$ is:
    i. 0.6
    ii. 0.8
    iii. 0.2
    iv. 0.4

   (b) Given events A and B with $P(A) = 0.5$, $P(B) = 0.5$, and $P(A \cap B) = 0.25$, what can be said about their independence?
    i. A and B are independent
    ii. A and B are mutually exclusive
    iii. $P(A \cup B) = 0.5$
    iv. None of the above

   (c) A random variable X takes values $1, 2$, and 3 with probabilities $0.2, 0.5$, and 0.3, respectively. The expected value $E[X]$ is:
    i. 1.8
    ii. 2.1
    iii. 2.4
    iv. 1.5

   (d) Let X and Y be random variables. If $E[X] = 2$, $E[Y] = 3$, and X and Y are independent, then $E[XY]$ is:
    i. 6
    ii. 5
    iii. 2
    iv. 3

   (e) The moment generating function (mgf) of a random variable X is defined by:

i. $M_X(t) = E[e^{-tX}]$
ii. $M_X(t) = E[tX]$
iii. $M_X(t) = E[e^{tX}]$
iv. $M_X(t) = e^{E[tX]}$

(f) If $P(A \mid B) = P(A)$, then we say:

i. A and B are conditional
ii. A and B are dependent
iii. A and B are independent
iv. B is a subset of A

(g) If X has a probability generating function (pgf) $G_X(s) = E[s^X]$, then $G'_X(1) =$

i. $E[X]$
ii. $E[X^2]$
iii. 1
iv. 0

(h) Classical definition of probability for an event A (when all outcomes are equally likely) is:

i. $\frac{\text{Number of favorable outcomes}}{\text{Sample space size}}$
ii. $\frac{\text{Sample space size}}{\text{Number of favorable outcomes}}$
iii. $\frac{\text{Sample space size}-\text{Number of favorable outcomes}}{\text{Sample space size}}$
iv. $1 - P(A^c)$

2. (a) Let a die be rolled twice and let A be the event that the first roll is 2 or 5, and B be the event that the second roll is 4 or 6. Using the classical definition of probability, find $P(A)$, $P(B)$, and $P(A \cap B)$. Are A and B independent events?

(b) A fair coin is tossed three times. Define events:

$$E = \{\text{Exactly two heads appear}\}, \quad F = \{\text{At least one head appears}\}.$$

Find $P(E \mid F)$ and $P(F \mid E)$. Are these events independent?

3. (a) Suppose X is a discrete random variable taking values in $\{0, 1, 2, 3\}$ with probabilities: $P(X = 0) = 0.1$, $P(X = 1) = 0.3$, $P(X = 2) = 0.4$, $P(X = 3) = 0.2$. Compute the expectation $E[X]$ and the variance $\text{Var}(X)$.

(b) There are 3 coins, identical in appearance, one of which is ideal and the other two biased with probabilities $\frac{1}{3}$ and $\frac{2}{3}$ respectively for a head. One coin is taken at random and tossed twice. If a head appears both times, what is the probability that the ideal coin was chosen?

(c) Let X be a continuous random variable with cumulative distribution function (CDF)

$$F_X(x) = \begin{cases} 0, & x < 0, \\ \frac{x}{4}, & 0 \leq x \leq 4, \\ 1, & x > 4. \end{cases}$$

Find the pdf $f_X(x)$ and compute $P(1 \leq X \leq 3)$.

4. (a) Each of the n urns contains a white and b black balls. One ball is transferred from the first urn into the second, then one ball from the latter into the third, and so on. Finally, one ball is taken from the last urn. What is the probability of it being white?

   (b) A sample space consists of equally likely outcomes $\{(a, b) \mid a \in \{1, 2, 3\}, b \in \{4, 5, 6\}\}$. Event A is defined as $\{(1, 4), (1, 6), (2, 5), (3, 4)\}$ and event B is defined as $\{(1, 4), (2, 5), (3, 6)\}$. Find $P(A)$, $P(B)$, $P(A \cap B)$ and $P(A \cup B)$.

5. (a) Let X be a discrete random variable with probability mass function (pmf) given by

$$p_X(k) = \frac{c}{2^k}, \quad k = 1, 2, 3, \ldots$$

   where c is a normalizing constant. Determine the value of c, and then find the expectation $E[X]$.

   (b) Let X be a random variable whose moment generating function (mgf) is given by

$$M_X(t) = \frac{1}{(1 - \alpha t)^3}, \quad \text{for } t < \frac{1}{\alpha},$$

   where α is a positive constant. Find the first and second moments of X, i.e., $E[X]$ and $E[X^2]$.

   (c) Let the random variable X has the pdf

$$f_X(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

   Find the pdf of $Y = X^2$ and $E(Y)$

6. (a) A man with n keys wants to open his door and tries the keys independently and at random. Find the expected number of trials required to open the door, (i) if unsuccessful keys are not eliminated from further selection, and (ii) if they are.

   (b) Let Y be a random variable that takes positive values only. Prove that

$$E[Y] = \int_0^\infty P(Y > x)\, dx.$$

7. (a) Let X be a random variable with probability generating function (pgf)

$$G_X(s) = E[s^X].$$

Show how $E[X]$ and $\text{Var}(X)$ can be found using the first and second derivatives of $G_X(s)$ evaluated at $s = 1$. Clearly state the formulas.

(b) Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables (not necessarily with a named distribution). Define $S_n = X_1 + X_2 + \cdots + X_n$. Show that

$$M_{S_n}(t) = \big(M_{X_1}(t)\big)^n,$$

where $M_{S_n}(t)$ and $M_{X_1}(t)$ are the mgfs of S_n and X_1, respectively.

(c) Find the pgf of the random variable with pmf

$$f_X(x) = \begin{cases} p^x(1-p)^{1-x} & x = 0, 1 \\ 0 & \text{otherwise} \end{cases}$$

Hence or otherwise find the pgf of $Y = \sum_{i=1}^{n} X_i$ where X_i's are independent.

8. (a) State and Prove Markov's inequality. Hence or otherwise, prove Chebyshev's inequality.

(b) Using probability inequality prove that $AM \geq GM$.

Undergraduate Program (with Statistics Major) Examination 2023
Semester-I
Statistics
Course: MJST01T (Descriptive Statistics I (Theory))
Full Marks: 60 Time: 3 Hours

Group A

(Answer any three questions. Each question carries 5 marks.)

1. What is mode? Derive its working formula for a continuous frequency distribution. 1+3

2. What are ogives? Show that the median of a continuous frequency distribution is the abscissa of the point of intersection of the two ogives. 2+3

3. Let X be variable taking values $1, 2, \ldots m$ with frequencies $f_i, i = 1, 2, \ldots m$ and $\sum_{i=1}^{m} f_i = n$. Let F_i denote the corresponding greater than type cumulative frequencies. Obtain the mean and variance of X as a function of F_i s. 5

4. Comment why the arithmetic mean is considered as an inappropriate measure of central tendency in the following cases: 5

   (a) To find the average of teacher-student ratio of n different schools.
   (b) To find the average speed of a sprinter who runs his first 100m at a speed of v_1m/s and the next 100m at a speed of v_2m/s.

5. Is the median of the logarithms of a set of positive real numbers equal to the logarithm of the median of the numbers? What will be your answer in case of arithmetic mean? Justify your answers. 5

Group B

(Answer any three questions. Each question carries 15 marks.)

6. Distinguish between any five of the following: 3×5

   (a) Primary data and secondary data
   (b) Time series data and cross sectional data
   (c) Variable and attribute
   (d) Histogram and stem-leaf display
   (e) Bar diagram and pie diagram
   (f) Questionnaire and schedule.

7. (a) Define quartiles of a frequency distribution. How can you develop a measure of skewness based on them? Find limits of this measure.

   (b) Develop another measure of skewness based on sample mean and median. Find the limits of this measure. You have to prove the necessary results.

   (c) A variable assumes 3 equidistant values x_0-h, x_0, x_0+h with relative frequencies $p, 1-2p$ and p respectively. Calculate Pearson's b_2 measure and find its limits as $p \to 0$ and $p \to \infty$. (2+1+2)+(1+4)+5

8. (a) Prove that the variance of a set of observations may be regarded as the mean square of all possible pairs of differences of the observations.

   (b) Show that the mean deviation of a set of n observations about A, (MD_A) can be obtained by the formula

$$nMD_A = (S_2 - S_1) + A(n_1 - n_2)$$

   where
   S_1: Sum of the values which are less than A.
   n_1: Number of values less than A.
   S_2: Sum of the values which are greater than A.
   n_2: Number of values greater than A.

   (c) Hence or otherwise, prove that mean deviation about median is the least.

   (d) What do you mean by relative measures of dispersion? Define one such measure and explain its uses. 4+2+5+(1+3)

9. (a) Prove that

      i. $b_2 \geq 1$
      ii. $b_2 - b_1 - 1 \geq 0$,

   where the symbols have their usual meanings. Discuss in detail the cases of equality.

   (b) Let $S = \sum_{i=1}^{k} \mid x_i - A \mid f_i$ be the sum of absolute deviations about the mid-point A of the class in which our chosen average m lies. If $d = m - A$, show that the mean deviation about m will be

$$\frac{1}{n}\left[S + (n_1 - n_3)d + n_2\left(\frac{h}{4} + \frac{d^2}{h}\right)\right],$$

   where
   n_2 =frequency of the class containing m,
   h =width of the class interval.
   n_1 =total frequency in all lower classes,
   n_3 =total frequency in all higher classes,
   provided we assume a uniform distribution of frequency in each class.

(c) In a set of 50 observations, the maximum and minimum values are 85 and 25 respectively. Find the maximum possible value of the standard deviation of these observations.

7+6+2

10. Write short notes on any three of the following: 5×3

(a) Box-plots and their uses

(b) Interview method of data collection

(c) Tabulation and its advantages

(d) Sheppard's correction for moments

(e) Scrutiny of data.

Undergraduate Program with Statistics Major Examination, 2023
Semester I
Statistics (Practical)
Course: MJST01P (Descriptive Statistics I (Practical))
Full Marks :20   Time: 2 Hours

(1) For a frequency distribution of a variable X, several class intervals of constant width were formed and the corresponding frequencies were tabulated. In the following table, Y denotes the deviation of midpoints of the above mentioned classes divided by the length of class intervals, and f, the corresponding class frequencies.

Y	-4	-3	-2	-1	0	1	2	3
f	3	15	45	57	50	36	25	9

It is also given that the mean and standard deviation of the variable X is 40.6042 and 7.9221 respectively.

(a) Determine the actual class boundaries.
(b) Find the median and the mode of the actual frequency distribution.
(c) Calculate any measure of skewness and comment. 4+4+2

(2) In a batch of 10 children, the I.Q. of a dull boy is 36 below the average I.Q. of the other children. Find a lower bound of the standard deviation for all children. If this standard deviation is actually 11.4, determine what the standard deviation will be when the dull boy is left out. 4

(3) Find the **consistency** of the following moments: m_2=12.72, m_3=8.5531636, m_4=164.5497. 3

(4) Practical Note Book and Viva-voce. 3

Undergraduate Programme with Statistics Major Semester I Examination 2023

Paper: MJST02

(Probability and Probability Distribution I)

Full Marks: 80 Time: 3 hours

Answer any five of the following eight questions of equal marks.

(Notations carry usual meanings)

1. (a) What is the probability of rolling a sum of 7 with two fair six-sided dice?
   i. $\frac{1}{6}$
   ii. $\frac{1}{12}$
   iii. $\frac{1}{9}$
   iv. $\frac{1}{36}$

   (b) If $P(A) = 0.4$ and $P(B) = 0.5$, what is the probability of A given that B has occurred, i.e., $P(A|B)$? A and B are independent.
   i. 0.4
   ii. 0.5
   iii. 0.8
   iv. 0.2

   (c) Bayes' theorem is used to:
   i. Calculate the probability of an event based on prior knowledge of conditions that might be related to the event.
   ii. Estimate the number of successes in a fixed number of independent Bernoulli trials.
   iii. Find the expected value of a random variable.
   iv. Determine the probability density function of a continuous random variable.

   (d) If events A and B are independent, what is $P(A \cap B)$?
   i. $P(A) + P(B)$
   ii. $P(A) \times P(B)$
   iii. $P(A) - P(B)$
   iv. $\frac{P(A)}{P(B)}$

   (e) Which of the following is not a property of a probability density function (pdf)?
   i. Always non-negative
   ii. Always less than or equal to 1
   iii. Integrates to 1 over its entire support
   iv. Can take negative values

(f) Consider two events A and B. If $P(A) = 0.3$, $P(B) = 0.4$, and $P(A \cup B) = 0.6$, what is the probability of A and B both occurring?

i. 0.3
ii. 0.1
iii. 0.2
iv. 0.5

(g) Which of the following functions represents a valid probability density function (pdf) for a continuous random variable?

i. $f(x) = 2x^2 - 3x + 1, 0 < x < 1.$
ii. $f(x) = \frac{1}{x}$, $1 \leq x \leq 2$
iii. $f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, -\infty < x < \infty$
iv. $f(x) = \frac{1}{x^2+1}, -\infty < x < \infty$

(h) Which of the following is not true for expectation of a random variable?

i. it always exists
ii. it can be obtained from the mgf of the random variable
iii. it provides mean of the probability distribution of the random variable
iv. it is a raw moment

2. (a) Suppose you are a doctor and you have a patient who just tested positive for a rare disease. The test is known to be 99% accurate, meaning if a person has the disease, the test will correctly identify it 99% of the time. However, it also has a false positive rate of 5%, meaning that if a person doesn't have the disease, the test will incorrectly identify them as having it 5% of the time. Given that the disease is quite rare, affecting only 0.1% of the population, what is the probability that a person who tests positive actually has the disease?

(b) Let A and B are two independent events. Prove that A^C and B^C are also independent. Comment on independence of the events A^C and B with reasons.

3. (a) Let X be a continuous random variable with probability density function (pdf) given by:

$$f_X(x) = \begin{cases} Kx, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

where K is a constant.
Find the value of K. Also calculate the expected value (mean) and variance of X.

(b) Define the probability generating function (pgf) of a discrete random variable Y. If the discret random variable Y has the pgf $G_Y(t) = \frac{t}{5}(2 + 3t^2)$, find the probability mass function (pmf) of Y.

4. (a) Let Z be a random variable with MGF $M_Z(t) = \frac{1}{4}(e^{2t}+3e^t)$. Determine the first four moments (mean, variance, skewness, and kurtosis) of Z.

   (b) Consider a continuous random variable W with CDF given by:

$$F_W(w) = \begin{cases} 0, & w < 0 \\ \frac{1}{4}w^2, & 0 \leq w < 2 \\ 1, & w \geq 2 \end{cases}$$

   Examine the continuity of the distribution function at $x = 0$ and $x = 2$. Also calculate the probability that W lies between 1 and 3.

5. (a) Let V be a random variable with mean $E(V) = 5$ and variance $Var(V) = 9$. Find the expectation and variance of $(3V + 2)$.

   (b) Let X be a continuous random variable representing the weight (in kilograms) of a randomly selected fruit, following a distribution with pdf

$$f_X(x) = \begin{cases} \frac{1}{3}, & 2 < x < 5 \\ 0, & otherwise \end{cases}$$

   Let $Y = 3X^2 - 4X + 2$. Find the mean of Y.

6. (a) Suppose X is a non-negative random variable, then show that for any $a > 0$

$$P(X \geq a) \leq \frac{E(X)}{a}$$

   Hence or otherwise prove that for any random variable X

$$P(|X - E(X)| \geq b) \leq \frac{Var(X)}{b^2}$$

   (b) Suppose a random variable X represents the number of defective items produced in a manufacturing process in a day. The probability function of X is given by:

$$P(X = k) = \begin{cases} \frac{1}{3^k}, & k = 0, 1, 2, 3 \ldots \\ 0, & otherwise \end{cases}$$

   Determine whether this function satisfies the properties of a probability mass function (PMF) or not. If it does not then modify the function to a pmf and then find the probability that at least one defective item is produced in a day.

7. (a) Let U and V be independent random variables with MGFs $M_U(t) = e^t$ and $M_V(t) = e^{2t}$. Find the MGF of their sum, $M_{U+V}(t)$, and hence or otherwise find the expectation and variance of $U + V$.

(b) Discuss whether the conditional probability satisfies the probability axioms or not.

8. (a) Let $T = 2X + 3$ be a linear transformation of a random variable X with pdf

$$f_X(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & otherwise \end{cases}$$

Find the pdf of T.

(b) A box contains 3 red balls and 2 green balls. Two balls are drawn without replacement. What is the probability that the second ball drawn is red, given that the first ball drawn was green?

B. Sc. Examination 2023
Semester: I
Statistics
Paper: CC2(Calculus)

Time: 3 Hours **Full Marks: 60**

Questions are of value as indicated in the margin

Group A

Answer any <u>five</u> questions **5X2**

1. Using formal definition of limit, find $\lim_{x \to x_0} k$, where k is a constant.

2. Explain why the following limit does not exist

$$lim_{x\to 0} \frac{x}{|x|}$$

3. Plot the curve $1 + |1 + |1 + x||$

4. Find $\int_1^5 [x-1] dx$

5. If $y = x^8$ find $\frac{d^5y}{dx^5}$

6. For any real number x, is $|x| = x$ always true**?**

7. Give an example of a function which does not have any global maxima or minima**.**

Group B

Answer any <u>five</u> questions **5X6**

1. State and prove the mean value theorem (MVT). Hence or otherwise show that functions with zero derivatives are constant.
2. Find the area of the region between x-axis and the graph of
$$f(x) = x^3 - x^2 - 2x - 1 \qquad -1 \le x \le 2$$
3. Fins the curve whose slope at the point (x, y) is $3x^2$if the curve is required to pass through the point $(-1,1)$.
4. Find the general solution of : $\frac{d^2y}{dx^2} + 4y = 0$l
5. Solve the ordinary differential equation
$$\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 16y = 0$$
6. Using Lagrange's multiplier method, for $x, y, z \ge 0$, show that

$$\frac{x+y+z}{3} \geq \sqrt[3]{xyz}$$

Group C

Answer any two questions **2X10**

1. (a). Using formal definition of limits, **p**rove that $\lim_{x\to 2} f(x) = 4 \quad if$

$$f(x) = \begin{cases} x^2 \; if \; x \neq 2 \\ 1 \; if \; x = 2 \end{cases}$$

(b). Give an example to show that $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ do not exist but $\lim_{x\to a}[f(x)+g(x)]$ exists.

(c) Find

$$\lim_{x\to 0}\left[\frac{e^x+\sin x-1}{\ln(1+x)}\right]$$

5+2+3

2. a. State and prove Leibnitz theorem for successive derivation.

b. Find n-th order derivative for the following functions:

i. $\sin ax \cos bx$ ii. $x^{n-1}\log x$ 6+4

3. (a) State and prove second fundamental theorem of calculus.

(b) For $s > 0$, let

$$\Gamma(s) = \int_0^\infty e^{-t}\, t^{s-1} dt$$

Show that $\Gamma(s+1) = s\Gamma(s)$ and $\Gamma(n+1) = n!$ for any positive integer n.

(c) Solve $\frac{dy}{dx} = \frac{x}{y}$ 4+3+3

B.Sc. Examination, 2023
Semester-I
Statistics
Course: SECST01
(Statistical Data Analysis with Excel)
Time: 3 Hours **Full Marks: 60**

Questions are of value as indicated in the margin
Notations have their usual meanings

Group A

(Select the current alternative) 10X2

1. What does the function VLOOKUP in Excel stand for?
   a. Vertical Lookup
   b. Variable Lookup
   c. Value Lookup
   d. View Lookup
2. In Excel, what does the function COUNTIF do?
   a. Counts the total number of cells in a range
   b. Counts the number of cells that meet a specified condition
   c. Adds up all the numbers in a range
   d. Multiplies numbers in a range
3. In Excel, what is the purpose of the IF function?
   a. To insert a new worksheet
   b. To perform logical tests and return values based on the results
   c. To calculate the average of a range of cells
   d. To sort data in ascending order
4. _____ logical function indicates TRUE only if all arguments are assessed as TRUE, and FALSE otherwise.
   a. OR
   b. AND
   c. IF
   d. ANDIF
5. The Greater Than sign (>) exemplifies a/an ______ operator.
   a. Arithmetic
   b. Conditional
   c. Logical
   d. None of the above

6. What do Excel formulas start with?
   a. /
   b. f
   c. =
   d. –
7. What is an Excel feature that displays only the data in column (s) according to specified criteria?
   a. Filtering
   b. Sorting
   c. Formula
   d. Pivot
8. ____ is not a function in Excel.
   a. SUM
   b. MIN
   c. SUBTRACT
   d. MAX
9. _____ Excel function returns TRUE or FALSE based on two or more conditions
   a. =AVERAGEIFS
   b. =CONCAT
   c. =COUNTA
   d. =AND
10. ____ happens when you select a cell in MS Excel and type "=B25"
    a. The selected cell will show an error message
    b. Selected cells will show "=B24"
    c. Selected cells will show the same value as in B25
    d. Selected cells will be blank

Group B

(Answer any 5 questions) 5X3

1. What is a cell address in Excel?
2. How do you freeze panes in Excel?
3. How can you restrict someone from copying a cell from your worksheet?
4. Mention the order of operations used in Excel while evaluating formulas.
5. How will you write the formula for the following? - Multiply the value in cell A1 by 10, add the result by 5, and divide it by 2.

6. What is the difference between count, counta, and countblank?

Group C

(Answer any 3 questions) 3X5

1. How does the VLOOKUP function work in Excel?
2. How does the IF() function in Excel work? Give an example.
3. What is the purpose of the "Freeze Panes" feature in Excel, and how is it used?
4. What does the "Wrap Text" feature in Excel do, and how can it be applied to cells?

Group D

(Answer any 1 question.

[Copy the Code and the output in your answer sheet] 10X1

1. a. Given below is a student table. Write a function to add pass/fail to the results column based on the following criteria.

   If student marks > 60 and attendance > 75%, then pass else the student fails

Student	Marks	Attendance
Sam	50	80
Danny	90	89
Mark	55	60
Mia	69	85
Suzane	75	72
Sophia	65	78

b. Consider the following data

Row Labels	Sum of Amount
Apple	191257
Banana	340295
Beans	57281
Broccoli	142439
Carrots	136945
Mango	57079
Orange	104438
Grand Total	**1029734**

Create two groups as follows:

Group 1: Apple and Banana

Group 2: Others

Using a pivotal table, show that Apple and Banana (Group1) have a higher total than all the other products (Group2) together. 4+6

2. Consider the following data

A	B	C	D	E
23	56	78	34	90
45	12	67	89	21
76	32	54	87	10
11	68	90	43	25
34	78	21	56	89
55	23	45	67	12
88	45	32	10	76
12	90	68	25	43
67	21	56	89	34
22	77	44	58	91

a. How many values are above the average in each row?

b. Highlight values above the average in each column.

c. What are the values of the correlation coefficients between A and C, A and E?

d. How many values are above average in each row?

e. Highlight the maximum value in each column. 2+2+2+2+2

B.Sc. Semester I Examination 2022
Subject: Statistics
Paper: CC 1A
(Descriptive Statistics)

Full Marks: 40 Time: 3 hours

Answer any four of the following six questions of equal marks.
(Notations carry usual meanings)

1. (a) What do you mean by attribute and consistency of data in this regard? Define independence of attributes.

   (b) Prove that mean deviation is least when measured about median.

   5+5

2. (a) Find the mean deviation about mean and standard deviation of the following values

$$x, x+y, x+2y, \ldots, x+2ny, (y>0)$$

   (b) If the geometric mean of n_1 observations of a variable be g_1 and that of another n_2 observations be g_2 then find the geomteric mean of the combined data in terms of g_1 and g_2.

   6+4

3. (a) Define multiple and partial correlations. Show that

$$1 - R^2_{1.23} = (1 - r^2_{12})(1 - r^2_{13.2})$$

   (b) Indicate how would you fit an exponential curve to to a bivariate data set.

   7+3

4. (a) Prove that $\beta_2 \geq 1$ and $\beta_2 \geq \beta_1 + 1$

   (b) If the lines of regression of Y on X and X on Y are respectively $a_1X + b_1Y + c_1 = 0$ and $a_2X + b_2Y + c_2 = 0$, then prove that $a_1b_2 = a_2b_1$

   6+4

5. (a) Derive the rank correlation coefficient formula for untied cases.

   (b) Show that Marshal-Edgeworth price index number is weighted arithmetic mean of price relatives.

   5+5

6. (a) What do you mean by price index number? Answer with an example. Also mention some of its uses.

   (b) If $u = x\ cos\theta + y\ sin\theta$, $v = y\ cos\theta$ - $x\ sin\theta$ and the variables u and v are uncorrelated, then prove that

$$tan2\theta = \frac{2r_{xy}s_xs_y}{s_x^2 - s_y^2}$$

   .

   5+5

BSc (Honours) Semester -I Examination 2022

Subject- Statistics

Paper- CC1B (Descriptive Statistics-Practical)

Full Marks: 20 Time: 2 hours

Answer **all** questions:

(Notations have usual meanings)

1. The numbers 3.2, 5.8, 7.9 and 4.5 have frequencies x, (x+2), (x-3) and (x+6) respectively. If their arithmetic mean is 4.876, find the value of x. 3

2. Fit a regression equation using the non-linear form $Y = a + b_1 x + b_2 x^2$ to the following data. Also provide the value of Y for x = 9

x:	1	1	2	2	3	3	4	6	7	8
y:	3	6	11	15	9	16	12	18	14	18

6

3. In a partial record of a data analysis on two variables the following results are known: Variance of X = 9. Regression equations are: 8x - 10y + 66 = 0, 40x - 18y = 214.
What are: i) Means of X and Y,
ii) Correlation coefficient between X and Y and
iii) Standard deviation of Y? 4

4. From the following data, construct a price index number of the group of four commodities by using Fisher's formula: 4

Commodity	Base Year		Current Year	
	Price per unit	Expenditure (Rs.)	Price per unit	Expenditure (Rs.)
A	5	45	7	91
B	6	24	10	50
C	2	12	3	27
D	6	30	12	84

5. Practical Note-book and Viva-voce. 3

B. Sc. Examination 2022
Semester: I
Statistics
Paper: C-102 (Calculus)

Time: 3 Hours **Full Marks: 60**

Questions are of value as indicated in the margin

Group A

Answer any five questions **5X2**

1. Using formal definition of limit, find $\lim_{x \to x_0} x^2$.

2. Explain why the following limit does not exist

$$lim_{x\to 0} \frac{x}{|x|}$$

3. Plot the curve $1 - |1 - |1 - x||$

4. Find $\int_1^5 [x]dx$

5. If $y = x^4$ find $\frac{d^5y}{dx^5}$

6. For any real number x, is $|x| = x$ always true**?**

7. Give an example of a function which does not have any global maxima or minima**.**

8. Write down sandwich theorem for limits.

Group B

Answer any five questions **5X6**

1. Show that if a function is derivable at any point, then the function is continuous at that point. Is the converse true?

2. State and prove the mean value theorem (MVT). Hence or otherwise show that functions with zero derivatives are constant.

3. Find the area of the region between x-axis and the graph of

$$f(x) = x^3 - x^2 - 2x - 1 \qquad -1 \leq x \leq 2$$

4. Fins the curve whose slope at the point (x, y) is $3x^2$if the curve is required to pass through the point $(-1,1)$.

5. Find the general solution of : $y" + y = \tan x$

6. Solve the ordinary differential equation

$$x^2\frac{d^2x}{dx^2} - 2x\frac{dy}{dx} + 3y = 0$$

7. Using Lagrange's multiplier method, for $x, y, z \geq 0$, show that

$$\frac{x+y+z}{3} \geq \sqrt[3]{xyz}$$

Group C

Answer any two questions **2X10**

1. (a). Using formal definition of limits, **p**rove that $\lim_{x\to 2} f(x) = 4 \quad if$

$$f(x) = \begin{cases} x^2 \; if \; x \neq 2 \\ 1 \; if \; x = 2 \end{cases}$$

(b). Give an example to show that $\lim_{x\to a} f(x)$ and $\lim\limits_{x\to a} g(x)$ do not exist but $\lim_{x\to a}[f(x) + g(x)]$ exists.

(c) Find

$$\lim_{x\to 0}\left[\frac{e^x + \sin x - 1}{\ln(1+x)}\right]$$

5+2+3

2. a. State and prove Leibnitz theorem for successive derivation.

b. Find n-th order derivative for the following functions:

i. $\frac{1}{1-5x+6x^2}$ ii. $x^{n-1}\log x$ 6+4

3. (a) State and prove second fundamental theorem of calculus.

(b) For $s > 0$, let

$\Gamma(s) = \int_0^\infty e^{-t}\, t^{s-1} dt$

Show that $\Gamma(s+1) = s\Gamma(s)$ and $\Gamma(n+1) = n!$ for any positive integer n.

(c) Solve $\frac{dy}{dx} = \frac{x}{y}$ 4+3+3