B.Sc. (Honours) Examination 2024
Semester-III
Statistics
Course: CC-5A (Sampling Distribution (Theory))
Full Marks: 40 Time: 3 Hours

(Answer any four questions.)

1. Having a paired random sample of size n from a bivariate normal population with parameters $(\mu_1, \mu_2, \sigma_1, \sigma_2, \rho = 0)$, find the sampling distribution of the sample correlation coefficient. 10

2. (a) If $X \sim {\chi_n}^2$, find the moment generating function of X. Hence find the mean and variance of X.
   (b) Establish a recurrence relation between the central moments of X. 5+5

3. (a) If $X_i \sim Geometric(p_i); i = 1, 2,$independently, find the distribution of $\min(X_1, X_2)$ and $X_1 - X_2$. Prove that they are independent.
   (b) Let $X_1, X_2, \ldots X_n$ be a sample of size n from the $U(0, 1)$ distribution. Find the sampling distribution of the sample range. 6+4

4. (a) Find the mean and mode of a F distribution with degrees of freedom n_1 and n_2. Hence, comment on the skewness of the distribution. What is the relationship between F and t distribution?
   (b) State Chebyshev's WLLN. (4+2+2)+2

5. (a) Suppose $X \sim Beta(n_1, n_2), Y \sim Beta(n_1 + \frac{1}{2}, n_2)$ independently. Find the distribution of $U = \sqrt{XY}$.
   (b) Suppose X_1, X_2 are i.i.d. $U(0, 1)$ variables. How can you generate two i.i.d. $N(\mu, \sigma^2)$ variables using them? 5+5

6. (a) Define Pearsonian χ^2, Fisher's t and Paired t statistics with their applications.
   (b) Explain the concept of sampling distribution of a statistic with examples. 6+4

B.Sc. (Honours) Semester III Examination 2024
Subject: Statistics
Paper: CC 6A
Statistical Inference (Theory)

Full Marks: 40 Time: 3 hours.

Answer any four of the following six questions of equal marks.
(Notations carry usual meanings)

1. (a) Define null and alternative hypothesis in the context of testing of hypothesis.
   (b) A medical researcher is working on a new treatment for a certain type of cancer. The average survival time after diagnosis on the standard treatment is two years. In an early trial, she tries the new treatment on three subjects who have an average survival time after diagnosis of four years. But the results are statistically insignificant. Eventually, the new treatment does increase the mean survival time in the population of all patients with this particular type of cancer. What type of error is committed here? Justify your answer.
   (c) Explain what is meant by an *unbiased estimator*, and give a definition. Find an unbiased estimator of the parameter λ for the $Poisson(\lambda)$ distribution.

3+2+5

2. (a) For testing $H_0 : \theta = 2$ against $H_1 : \theta = 4$ when $X \sim f_\theta(x) = \frac{\theta}{x^2}$, where $0 < \theta \leq x < \infty$ the critical region is given as $X > 2.4$. Find out probability of type I error and power of the test.
   (b) Outline the general steps of deriving a *maximum likelihood estimator (MLE)* for a parametric family of distributions. Describe an application of this method to find an estimator of a parameter of a probability distribution.

5+5

3. (a) For testing $H_0 : \mu = \mu_0$ against $H_1 : \mu < \mu_0$ when $X_1, X_2, \cdots, X_n$ be a random sample of size n from $N(\mu, \sigma^2)$, construct the power function.
   (b) Define a *sufficient statistic* and state the Factorization Theorem. Illustrate with an example how to apply the Factorization Theorem to show that a statistic is sufficient for a given parameter.

5+5

4. (a) State and prove Neymann Pearson fundamental lemma.
   (b) State the Rao-Blackwell theorem. Explain how one can use it to derive a minimum-variance unbiased estimator (MVUE) from any unbiased estimator.

5+5

5. (a) For testing $H_0 : \sigma = \sigma_0$ under $N(\mu, \sigma^2)$ when μ unknown with n observations $X_1, X_2, \cdots, X_n$, write the test statistic. What is the distribution of this test statistic under H_0?

   (b) State the Cramér–Rao lower bound for an unbiased estimator $\hat{\theta}$ of a parameter θ. Give an example of an estimator which attains the CRLB.

5+5

6. (a) Define Fisher's t statistic. In which test do you use this statistic?

   (b) Explain how the posterior distribution is used to form point estimates such as the *posterior mean* (Bayes estimator under quadratic loss) or the *posterior median* (Bayes estimator under absolute loss).

5+5

4-Year Undergraduate Program (with Statistics Major) Examination 2024
Semester-III
Statistics
Course: MJST05 (Sampling Distribution)
Full Marks: 80 Time: 3 Hours

Group A

(Answer any four questions. Each question carries 5 marks.)

1. State and prove Cochran's theorem regarding the distribution of the sum of squares of i.i.d. normal variables subject to a number of linear constraints. 5

2. Let X_1, X_2, X_3 be three independent and identically distributed random variables having $U(0,1)$ distribution. Then find the expected value of $\left(\dfrac{\ln X_1}{\ln(X_1X_2X_3)}\right)^2$. 5

3. Suppose $X \sim F_{6,2}$ and $Y \sim F_{2,6}$. If $P(X \leq 2) = \dfrac{216}{343}$ and $P\left(Y \leq \dfrac{1}{2}\right) = \alpha$, then find the value of 686α. 5

4. Let $X_1, X_2, \ldots$ be a sequence of i.i.d. random variables such that

$$P(X_1 = 0) = P(X_1 = 1) = P(X_1 = 2) = \frac{1}{3}$$

Let $S_n = \dfrac{1}{n}\sum_{i=1}^{n} X_i;\ T_n = \dfrac{1}{n}\sum_{i=1}^{n} X_i^2;\ n = 1, 2, \ldots$. Suppose that

$$\alpha_1 = \lim_{n\to\infty} P\left(\mid S_n - \frac{1}{2} \mid < \frac{3}{4}\right);\ \alpha_2 = \lim_{n\to\infty} P\left(\mid S_n - \frac{1}{3} \mid < 1\right)$$

$$\alpha_3 = \lim_{n\to\infty} P\left(\mid T_n - \frac{1}{3} \mid < \frac{3}{2}\right);\ \alpha_4 = \lim_{n\to\infty} P\left(\mid T_n - \frac{2}{3} \mid < \frac{1}{2}\right).$$

Find the value of $\alpha_1 + 2\alpha_2 + 3\alpha_3 + 4\alpha_4$. 5

5. Let X_1 and X_2 be independent random variables, each distributed as a χ^2 variable with 2 degrees of freedom. Find the distribution of:

$$U = \frac{a_1X_1 + a_2X_2}{X_1 + X_2},$$

where a_1 and a_2 are constants with $a_1 > a_2$. 5

6. Define parameter, statistic and sampling distribution of a statistic with examples. 5

Group B

(Answer any four questions. Each question carries 15 marks.)

7. (a) State Lindeberg-Levy's central limit theorem and Chebyshev's WLLN.

   (b) Let f_n denote the number of successes in the first n trials for an infinite series of Bernoulli trials with probability of success p per trial. Show that the WLLN holds for

   $$X_n = \frac{f_n - np}{(np(1-p))^{\alpha}}; n = 1, 2, \ldots$$

   if $\alpha > \frac{1}{2}$.

   (c) Let X_n be a random variable having a Poisson distribution with expectation λ_n. Show that $\frac{X_n - \lambda_n}{\sqrt{\lambda_n}}$ converges in law to a $N(0,1)$ variable as $\lambda_n \to \infty$. (2+2)+6+5

8. (a) What do you mean by Jacobian of a transformation? Find the Jacobian of a 3-dimensional polar transformation. What do you mean by an orthogonal transformation? Prove that for an orthogonal transformation, the Jacobian equals unity.

   (b) If X_1, X_2 are i.i.d. $U(0,1)$ variables, find the distribution of $U = X + Y$ and $V = X - Y$. (2+3+2+2)+6

9. (a) For a χ^2 distribution with d.f. $2(n+1)$, prove that

   $$P(\chi^2 > 2\lambda) = \frac{1}{n!}\int_{\lambda}^{\infty} e^{-y} y^n dy = e^{-\lambda}\sum_{r=1}^{n} \frac{\lambda^r}{r!}$$

   (b) Let $X_1, X_2, \ldots X_n$ be a random sample of size n from $N(\mu, \sigma^2)$ population. Define

   $\overline{X}_k$: Mean of first k observations; S_k^2: Variance of first k observations.

   $\overline{X}_{n-k}$: Mean of last $n-k$ observations; S_{n-k}^2: Variance of last $n-k$ observations.

   $\overline{X}$: Mean of all observations; S^2: Variance of all observations.

   Find the sampling distributions of

   i. $\frac{(k-1)S_k^2 + (n-k-1)S_{n-k}^2}{\sigma^2}$

   ii. $\frac{\overline{X}_k + \overline{X}_{n-k}}{2}$

   iii. $\frac{S_k^2}{S_{n-k}^2}$

iv. $\dfrac{\sqrt{\dfrac{n}{n-1}}\,(X_1-\overline{X})}{\left[\dfrac{(n-1)S^2-\dfrac{n}{n-1}(X_1-\overline{X})^2}{n-2}\right]^{\frac{1}{2}}}$ 6+(2+2+2+3)

10. (a) Having a paired random sample of size n from a bivariate normal population with parameters $(\mu_1, \mu_2, \sigma_1, \sigma_2, \rho = 0)$, find the sampling distribution of the sample correlation coefficient.

(b) Let $(X, Y) \sim N_2(0, 0, 1, 1, \rho)$. Find the distribution of $U = \dfrac{X}{Y}$. 10+5

11. (a) If $X \sim F_{n_1,n_2}$, show that the distribution of $Y = n_1 X$ approaches $\chi^2_{n_1}$, as $n_2 \to \infty$.

(b) Find the mean and mode of F_{n_1,n_2} distribution and hence comment on the skewness of this distribution.

(c) Write a short note on student's t-statistic and its uses. 6+6+3

12. (a) Let X be a random variable with p.d.f.

$$f(x) = \begin{cases} 2(1-x) & \text{if } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Find the p.d.f. of the sample range.

(b) Prove that $E(X_{(p+1)} - X_{(p)}) = \dfrac{n}{p}\displaystyle\int_{-\infty}^{\infty} (F(x))^{n-p}\,(1-F(x))^p\,dx$

(c) Let X_1 and X_2 be i.i.d. random variables having the common probability density function

$$f(x) = \begin{cases} e^{-x} & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Define $X_{(1)} = \min(X_1, X_2)$ and $X_{(2)} = \max(X_1, X_2)$. Then which one of the following statements is false?

i. $\dfrac{2X_{(1)}}{X_{(2)} - X_{(1)}} \sim F_{2,2}$

ii. $2(X_{(2)} - X_{(1)}) \sim \chi^2_2$

iii. $E(X_{(1)}) = \dfrac{1}{2}$

iv. $P(3X_{(1)} < X_{(2)}) = \dfrac{1}{3}$ 5+4+6

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Four Year Undergraduate Programme
Sem-III Examination 2024

Subject: Statistics Major
Paper: MJST06T
Linear Algebra (Theory)

Answer any **four** out of the following six questions of equal marks.
Nonations are of usual meanings.

Full Marks: 60 | Time: 3 hours

1. (a) Define linear independence of vectors in a vector space. Prove that if $v_1, v_2, \ldots, v_n$ is a linearly dependent set, then at least one vector in the set can be expressed as a linear combination of the others.

   (b) Define the basis of a vector space. Prove that the number of vectors in the basis of a vector space is unique.

   (c) Let $S = \{(x, y, z) \in R^3 : 3x - y - z = 0\}$. Show that S is a subspace of R^3.

   5+5+5

2. (a) What do you mean by orthogonal vectors? Given a set of linearly independent vectors, describe how you are going to convert those into a set of orthonormal vectors.

   (b) Define the inverse of a matrix. Prove that if a matrix has an inverse, then the inverse is unique.

   (c) Show that if A and B are invertible, then AB is invertible. Also, find the inverse of the product AB.

   8+3+4

3. (a) What do you understand by rank of a matrix? For a matrix A show that $Rank(A) = Rank(A')$.

   (b) Find the rank of the matrix

$$\begin{pmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{pmatrix}$$

(c) Judge whether the following matrix is orthogonal

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & cos\theta & sin\theta \\ 0 & -sin\theta & cos\theta \end{pmatrix}$$

for all values of θ.

5+5+5

4. (a) Show that

$$\begin{vmatrix} a & b & c & d \\ -b & a & -d & c \\ -c & d & a & -b \\ -d & -c & b & a \end{vmatrix} = (a^2 + b^2 + c^2 + d^2))^2$$

(b) Show that

$$\begin{vmatrix} x & a & a & \dots & a \\ a & x & a & \dots & a \\ a & a & x & \dots & a \\ \dots & \dots & \dots & \dots & \dots \\ a & a & a & \dots & x \end{vmatrix} = (x-a)^{n-1}[x + (n-1)a]$$

7+8

5. (a) What conditions must λ, μ, v satisfy in order that the system of equations

$$x + y + 2z = \lambda$$
$$x + z = \mu$$
$$2x + y + 3z = v$$

may be consistent?

(b) Prove that a square matrix is a non-singular if and only if all its eigenvalues are non-zero.

(c) Prove that to each eigenvector of a matrix A there corresponds a unique eigenvalue.

6+6+3

6. (a) Define a quadratic form. State the conditions of different definiteness of a quadratic form.

(b) Comment with reasons on definiteness of the quadratic form

$$x^2 - 2y^2 + z^2 - 4xy - 8xz + 4yz$$

.

(c) Define $g - inverse$ of a matrix. Does it always exist? If G is a $g - inverse$ of a matrix A, then comment with reasons whether AG and GA are idempotent or not.

5+4+6

Four Year Undergraduate Programme
Sem-III Examination 2024

Subject: Statistics Major
Paper: MJST06P
Linear Algebra (Practical)

Answer the following five questions of equal marks. Symbols are of usual meanings.

Full Marks: 20 Time: 3 hours

1. Show that the vectors $(1,0,0),(0,2,1),(1,1,0)$ form a basis for the real vector space R^3.

2. Reduce the following quadratic form to its normal form and find its rank and signature. Also, comment on its definiteness.

$$6x_1^2 + 3x_2^2 + 14x_3^2 + 4x_2x_3 + 18x_3x_1 + 4x_1x_2$$

3. Find the value of λ for which the following system of equation is solvable, and then solve it.

$$x + y + z = 2$$
$$2x + y + 3z = 1$$
$$x + 3y + 2z = 5$$
$$3x - 2y + z = \lambda$$

4. Find a g-inverse of the matrix

$$\begin{pmatrix} 2 & 4 & 5 \\ 4 & 8 & 10 \\ 1 & 2 & 3 \end{pmatrix}$$

5. Viva-voce and practical note-book.

B.Sc. (Honours) Examination, 2025
Semester-III
Statistics
Course: MJST-07
(Mathematical Analysis)
Time: 3 Hours **Full Marks: 80**

Questions are of value as indicated in the margin.
Notations have their usual meanings

Questions are separated into Group A, Group B, and Group C.
Group A contains only question 1.
Answer any four from Group B. Answer any two from Group C.
Questions of Group B carry ten marks each. Questions of Group C have ten marks each.

Group A

1. Answer the following questions with proper justifications. (Answer any five questions). $[4 \times 5 = 20]$

(a) State root test and ratio Test with an example.

(b) Find $\limsup$ and $\liminf$ of the sequence $x_n = (-1)^n + \frac{1}{n}$. Explain what will be the value of $\lim_{n\to\infty}(n^{1/n} + \dfrac{1}{n^{1/n}})$?

(c) Use Lagrange interpolation to estimate $f(3)$ given: $f(1) = 2$, $f(2) = 3$, $f(4) = 7$.

(d) Define a Cauchy sequence, with an example. Define the continuity of a function with an example.

(e) State the definition of the limit of a sequence with an example. Define neighbourhood of a point, open and closed set, with an example.

(f) Explain if the two series $\sum\limits_{n=2}^{\infty} \frac{(-1)^{n+1}}{n^3+1}$ and $\sum\limits_{n=1}^{\infty} (-1)^n \left(\sqrt{n+1} - \sqrt{n}\right)$ are absolutely convergent, conditionally convergent or divergent.

(g) Write down Striling's approximation of $n!$. Explain its utility with an example.

(h) Find an expression for $\Delta^4 y_0$ using E operator (in usual notations). prove or disprove: Δ and E commute, i.e., $\Delta E = E\Delta$.

Group B

Each Question carries 10 marks. Answer any four questions.

2. (a) Define limsup, liminf of a sequence with examples. Let $x_n = \frac{1}{n}\sum_{k=1}^{n}(-1)^k$. Determine $\limsup x_n$ and $\liminf x_n$.

(b) Prove or Disprove: $\Delta^n x^{(n)} = n!h^n$ where $x^{(n)}$ is the falling factorial. (The falling factorial is: $x^{(n)} = x(x-1)(x-2)\cdots(x-n+1)$)

5+5

3. (a) State and Prove the formula for Lagrange's interpolation.

(b) Evaluate $\int_{-3}^{3} g(x)\,dx$ using Weddle's Rule. Observe that $g(x)$ has two special properties that will greatly simplify the calculation, mention that, and proceed.

x	-3	-2	-1	0	1	2	3
$g(x)$	0	5	4	3	4	5	0

5+5

CONTINUED

4. (a) Test convergence of $\sum_{n=3}^{\infty} \frac{\mathbf{e}^{4n}}{(n-2)!}$, and $\sum_{n=2}^{\infty} \frac{1}{n\log n}$.

(b) Prove that every convergent sequence is bounded, but the converse is invalid.

5+5

5. (a) State Cauchy's 1st limit Theorem and Cauchy's 2nd limit Theorem.

(b) Find the limits $\lim_{n\to\infty}\left[\left(\frac{2}{1}\right)\cdot\left(\frac{3}{2}\right)^2\cdots\left(\frac{n+1}{n}\right)^n\right]^{1/n}$ and $\lim_{n\to\infty}\frac{1}{n}\left[1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}\right]$.

5+5

6. (a) Define Cauchy Sequence. Let x_n be the decimal truncations of $\sqrt{2}$ to n decimal places. Show that (x_n) is a Cauchy sequence in $\mathbb{Q}$ but does not converge in $\mathbb{Q}$.

(b) Show that, the series $1+\frac{3}{2!}+\frac{5}{3!}+\frac{7}{4!}+\cdots$ is convergent.

5+5

7. (a) Determine if the series $\sum_{n=1}^{\infty}\frac{3^{1-2n}}{n^2+1}$ converges or diverges.

(b) Determine whether the series

$$\sum_{n=2}^{\infty}\frac{1}{n(\log n)^p}$$

converges for (i) $p=1,2$ and (ii) for any $p\in\mathcal{N}, p\geq 3$. 5+5

8. (a) Write down the Taylor polynomial $P_2(x)$ of order 2 for the function $f(x)=\sqrt{1+x}$ and give an expression for the remainder $R_2(x)$ in Taylor's formula $\sqrt{1+x}=P_2(x)+R_2(x) \quad -1<x<\infty$

(b) Use Simpson's 1/3 Rule to evaluate $\int_0^6 \frac{1+x}{1+x+x^2}\,dx$ with $n=6$

5+5

Group C

Each Question carries ten marks. Answer any two questions.

9. (a) Does there exist a differentiable function $f:\Re\to\Re$ such that $f'(0)=0$ but $f'(x)\geq 1$ for all $x\neq 0$?

(b) Using Taylor's theorem or otherwise, show that $x-\frac{x^3}{3!}<\sin x<x-\frac{x^3}{3!}+\frac{x^5}{5!}$, for $x>0$.

(c) Prove that, between 2 real roots of $e^x\sin(x)+1=0$, there is atleast one root of $\tan(x)+1=0$.

2+4+4

10. (a) State Cauchy's Condensation Test. Use that to show that $\sum\frac{1}{n}$ diverges.

(b) Show that, $\lim_{n\to\infty}\frac{1}{n}\left[1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{n^2}\right]=0.$ and $\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^{n^2}=0.$

(c) If a_n be a sequence of positive terms and $\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\ell$, then prove that $\lim_{n\to\infty}(a_n)^{1/n}=\ell$.

2+4+4

CONTINUED

11. (a) State L'Hospital's Rule. Explain whether L'Hospital's Rule is valid/invalid in determining $\lim_{x \to \infty} \frac{x + \sin x}{x}$, and determine the limit.

(b) Prove that, between two real roots of $e^x \sin(x) + 1 = 0$, there is atleast one root of $\tan(x) + 1 = 0$.

(c) State and prove the power series expansion of $\sin x$ using Taylor's Theorem.

2+4+4

12. (a) State Newton's Forward (OR) Newton's Backward Interpolation Formula.

(b) Given the following table, Construct the difference table. and find $f(1.5)$ using Newton's Forward interpolation.

x	1	2	3	4	5	6	7	8
f(x)	1	8	27	64	125	256	343	512

(c) Solve $y_{n+1} = 0.5y_n + 5$, given $y_0 = 5$

2+4+4

4-Year Undergraduate Program
with Statistics major
Examination 2024
Semester III
Subject: Statistics
Paper: [SECST 03]

Full Marks: 60 **Time: 3 Hrs.**

Answer the following questions with proper justification. Notations have their usual meanings. Provide code /pseudo-code wherever required.

1. Group A (Answer any five questions) **$6 \times 5 = 30$**

(a) What is a pointer (in C)? Explain its different uses with three examples.

(b) Implement a C function that multiplies two matrices. The function should take two 2D arrays representing the matrices and their dimensions as input and output the resulting matrix. Ensure that your program checks for dimension compatibility.

(c) Write a C code to demonstrate how to reverse the elements of an array.

(d) Write a C code for calculating a set of numbers mean and standard deviation.

(e) Distinguish between "call by value" and "call by reference" with suitable examples.

(f) Given the following code snippet, predict the output with proper explanations:

```
int arr[] = {10, 20, 30, 40};
int *ptr = arr;
printf("%d %d %d\n", *ptr, *(ptr + 1), *(ptr + 2));
```

(g) The Fibonacci sequence is a series of numbers in which each number is the sum of the two that precede it. Starting at 0 and 1, the sequence looks like this: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, and so on forever. Write a program in C to obtain the first 50 Fibonacci sequence numbers.

Group B (Answer any three questions) **$10 \times 3 = 30$**

2. Write a C program that defines a structure Student containing the following members: name (a string), id (an integer), and grades (an array of 5 floating-point numbers). Implement a function that takes an array of Student structures and com- putes the average grade for each student. Display the student's name, ID, and average grade

3. Write a program using conditional operators to determine whether a year entered through the keyboard is a leap year or not.

4. Write a short note for the recursion function in C, and, using that (or otherwise), write a C code for printing following Pascal's Triangle. (No need to print the arrows. Ensure the correct positions of the numbers in the triangle).

```
                1
               ↙ ↘
            1       1
           ↙ ↘     ↙ ↘
        1       2       1
       ↙ ↘     ↙ ↘     ↙ ↘
    1       3       3       1
   ↙ ↘     ↙ ↘     ↙ ↘     ↙ ↘
1       4       6       4       1
```

5. Write a C program to implement the Box-Muller transformation to generate two independent standard normal random variables from two uniformly distributed random variables in the range $[0, 1]$. The program should take two uniform random numbers U_1 and U_2 and output the corresponding normal random variables Z_1 and Z_2.

6. Show that if U is Uniform(0,1), the Geometric random variable X can be generated as:

$$X = \left\lceil \frac{\ln(1-U)}{\ln(1-p)} \right\rceil .$$

   Hence (or otherwise), write a C code to generate random samples from a Geometric distribution with success probability p.

B.Sc. (Honours) Examination 2023
Semester-III
Statistics
Course: CC-5A (Sampling Distribution (Theory))
Full Marks: 40 Time: 3 Hours

(Answer any four questions.)

1. Find the sampling distribution of the sample regression coefficient, when the joint distribution of (X, Y) is bivariate normal with parameters $(\mu_1, \mu_2, \sigma_1, \sigma_2, \rho)$. 10

2. (a) If $X \sim {\chi_n}^2$, show that the limiting distribution of $\sqrt{2X} - \sqrt{2n-1}$ is $N(0,1)$, as $n \to \infty$.

   (b) If $(X_1, X_2) \sim N_2(0, 0, \sigma, \sigma, \rho)$, find the distribution of $\dfrac{(X_1 + X_2)}{\mid X_1 - X_2 \mid}\sqrt{\dfrac{1-\rho}{1+\rho}}$. 5+5

3. (a) If $X_i \sim Geometric(p_i); i = 1, 2$, find the distribution of $\min(X_1, X_2)$.

   (b) Prove that in sampling from exponential distribution, the distribution function of $X_{(n)} - \ln n$ tends to the limiting form $e^{-e^{-x}}$ as $n \to \infty$.

   (c) Let $X_1, X_2, \ldots X_n$ be a sample of size n from $U(0, 1)$ distribution. Find the correlation coefficient between the largest and the smallest order statistic. 3+3+4

4. (a) Define convergence in probability and convergence in distribution. Give an example to show that the latter does not imply the former.

   (b) Check whether WLLN holds for the following sequence $\{X_n\}$ of random variables $P(X_n = \mp n) = \dfrac{1}{2\sqrt{n}}; P(X_n = 0) = 1 - \dfrac{1}{\sqrt{n}}$. (2+2+2)+4

5. (a) Suppose X_1, X_2, X_3, X_4 be i.i.d. $N(0, 1)$ variables. Find the sampling distribution of $\dfrac{{X_1}^2 + {X_2}^2}{{X_1}^2 + {X_2}^2 + {X_3}^2}$ (You have to prove the necessary results used).

   (b) Suppose $X \sim Beta(n_1, n_2), Y \sim Beta(n_1 + \frac{1}{2}, n_2)$. Find the distribution of $U = \sqrt{XY}$. 5+5

6. (a) Define parameter, statistic and sampling distribution of a statistic with examples.

   (b) Define Jacobian of a transformation. Find its value for an orthogonal transformation.

   (c) Are the means of $F(3, 4)$ and $F(2, 4)$ distributions different? Give reasons. 4+4+2

B.Sc. (Honours) Examination, 2023
Semester-III
Statistics (Practical)
Course: CC-5B (Sampling Distribution (Practical))
Full Marks: 20 Time: 2 Hours

(1) During a smallpox epidemic, the following data were collected on the basis of a survey of 222 persons vaccinated against the disease. Do you think that the standard of vaccination affects the power to resist the disease?

	Attacked with smallpox	*Not attacked*	*Total*
Well vaccinated	33	120	153
Badly vaccinated	18	51	69
Total	51	171	222

(5)

(2) An investigation of the performance of two machines, in a factory manufacturing large number of bobbins, gives the following results:

	No. of bobbins examined	*No. of bobbins found defective*
Machine 1	375	17
Machine 2	450	22

Test whether there is any significant difference in the performance of the two machines.

(4)

(3) The marks obtained by 20 students of College A and 15 students of College B in an examination are given below:

College A: 89, 71, 47, 29, 76, 82, 83, 48, 65, 97, 31, 73, 69, 89, 43, 80, 55, 52, 86, 44
College B: 80, 12, 21, 60, 53, 92, 37, 82, 76, 42, 34, 68, 50, 73, 62

Do you think that the students of College A are more proficient than the students of College B? (8)

(4) Practical Note Book and Viva-Voce. (3)

B.Sc. (Honours) Semester III Examination 2022
Subject: Statistics
Paper: CC 6A
Statistical Inference (Theory)

Full Marks: 40 | Time: 3 hours.

Answer any four of the following six questions of equal marks.
(Notations carry usual meanings)

1. (a) Let $x_i; i = 1, 2, 3, \ldots, n$ be a random sample from the probability distribution with density function

$$f_\theta(x) = \theta x^{\theta - 1}; 0 < x < 1, \theta > 0$$

Obtain a sufficient statistics for θ.

(b) Researchers have claimed that the average number of headaches per student during a semester of Statistics is 18. Statistics students believe that the average is higher. In a sample of 22 students the mean is 23 headaches with a standard deviation of 2.1. Now answer the following.

i. Name the population. Write the null and alternative hypothesis.
ii. What would be the relevant test statistics to test this?
iii. Write down the rejection rule.
iv. If the p-value of the test turns out .07, what would be your decision at 5% level of significance?

5+(2+1+1+1)

2. (a) If $x_1, x_2, \ldots, x_n$ be independently and identically distributed random variables with mean μ and unknown variance, then show that $\overline{x}$ is the minimum variance unbiased linear estimator of μ.

(b) i. Define a Most Powerful test.
ii. Suppose in an urn there are 5 balls out of which θ are white and $5 - \theta$ are black balls. You are asked to perform a test $H_0 : \theta = 3$ against $H_1 : \theta = 2$. 3 balls are drawn with out replacement. If there are more than two white balls then we accept the null hypothesis otherwise we reject. Find probability of type I error and probability of type II error.

5+1+4

3. (a) Let $x_1, x_2, \ldots, x_n$ be a random sample of size n from the distribution with the density function

$$f_\theta(x) = \frac{1}{\theta}\exp(-\frac{x}{\theta}); x > 0; \theta > 0$$

Show that $\overline{x}$ is an Minimum Variance Bound estimator of θ and has variance $\frac{\theta^2}{n}$.

(b) i. Prove that every most powerful test is necessarily unbiased.

ii. Is a test with two sided alternative uniformly most powerful ?

5+3+2

4. (a) Let T and U be independent and consistent estimators of $\gamma(\theta)$. Show that $aT+bU$, where $a+b=1$, is also consistent for $\gamma(\theta)$.

(b) For checking the software knowledge on a batch of 10 students, a special training workshop on R is implemented in that class. You are asked to design a test in this regard assuming normality of knowledge distribution among the students. Describe a test procedure clearly stating null and alternative hypothesis.

5+5

5. (a) Is Maximum Likelihood Estimator always unique? Answer with example.

(b) i. Define likelihood ratio test statistic.

ii. Write the rejection rule based on that test statistic.

iii. What is the asymptotic distribution of likelihood ratio test statistic?

5+2+1+2

6. (a) What do you understand by a Baye's estimate? Show that under absolute loss median of the posterior distribution is the Baye's estimate

(b) Describe likelihood ratio test process for testing $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 > \sigma_0^2$ when a random sample of size n is collected from $N(\mu, \sigma^2)$ where μ is unknown.

5+5

B.Sc. (Honours) Semester III Examination 2022
Subject: Statistics
Paper: CC 6B
Statistical Inference (Practical)

Full Marks: 20 Time: 2 hours.

1. A study on $n = 49$ hospital employees found that the number of latex gloves used per week by the sampled workers is summarized by $\overline{x} = 18.9$ and $\sigma = 13.5$. Let μ be the mean number of latex gloves used per week by all hospital employees. The hospital authority wants to check the claim of mean number of rubber gloves is less than 20. Answer the following.

   (a) Write null hypothesis and alternative hypothesis.

   (b) Calculate the test statistic.

   (c) Find the p-value.

   1+1+2

2. The average amount of time boys and girls aged seven to eleven spend playing sports each day is believed to be the same. A study is done and data are collected, resulting in the data in the following table. Each populations has a normal distribution. Is there a difference in the mean

	Sample Size	Average number of hours playing/day	Sample sd
Girls	9	2	.866
Boys	16	3.2	1.0

   amount of time boys and girls aged seven to eleven play sports each day? test at the 5% level of significance?

   3

3. A psychologist was interested in exploring whether or not male and female college students have different driving behaviors.The psychologist conducted a survey of average speed of a random 12 male college students and a random 14 female college students. Here is a descriptive

Male(X)	Female(Y)
$\bar{x} = 105.8$	$\overline{y} = 90.6$
$s_x = 20.1$	$s_y = 12.2$

summary of the results of her survey: Is there sufficient evidence at 5% level to conclude that the variance of the speed driven by male college students differs from the variance of the speed driven by female college students?

3

4. Let $x_1 = 3.5$; $x_2 = 7.5$ and $x_3 = 5.2$ be observed values of a random sample of size three from a population having uniform distribution over the interval $(\theta; \theta + 5)$, where $\theta \in (0; \infty)$ is unknown and is to be estimated. A student claims that "2.4 is a maximum likelihood estimate of the unknown parameter θ". Examine and comment on the truthfulness of the student's claim.

4

5. For a data set of size 10 extracted from a distribution having p.d.f. $f(x) = \lambda e^{\lambda x}$ where $X > 0$ find the maximum likelihood estimate and moment estimate of the parameter involved. The observations are as follows.
9.9,2.3, 6.5, 3.9, 9, 11, 13.2, 4, 5.2, 8.

3

6. Practical note book+Viva voce

3

B.Sc. Examination, 2023
Semester-III
Statistics
Course: CC-7
(Mathematical Analysis)
Time: 3 Hours **Full Marks: 60**

Questions are of value as indicated in the margin.
Notations have their usual meanings

Questions are separated into Group A, Group B, and Group C.
Group A contains only question 1.
Answer any five from Group B. Answer any two from Group C.
Questions of Group B carry six marks each. Questions of Group C have ten marks each.

Group A

1. Answer the following questions with proper justifications. (Answer any five questions). [$2 \times 5 = 10$]

(a) State the definition of the limit of a sequence.

(b) State the ratio test for determining the convergence/divergence of a series.

(c) Determine if the series $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n^3+1}$ is absolutely convergent, conditionally convergent or divergent.

(d) Write down Striling's approximation of $n!$.

(e) Give an example to show that bounded sequences may not converge.

(f) Find an expression for $\Delta^4 y_0$ using E operator (in usual notations).

(g) Show that, $\lim_{n\to\infty} \frac{1}{n}\left[1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}\right] = 0$.

(h) State Maclaurin‘s Theorem.

Group B

(Each Question carries six marks. Answer any five questions)

2. (a) State with proof that the following series is convergent/divergent:

$$\sum_{n=3}^{\infty} \frac{\mathbf{e}^{4n}}{(n-2)!}$$

(b) Prove that every convergent sequence is bounded, but the converse is invalid.

3+3

3. (a) Show that, the sequence $a_n = \left(1+\frac{1}{n}\right)^{(n+1)}$ is strictly monotone decreasing.

(b) Suppose you have four data points: $(1,4), (2,7), (3,11)$, and $(4,16)$. Apply Lagrange's interpolation to find the polynomial $P(x)$ that passes through these points. Evaluate $P(x)$ at $x = 2.5$

3+3

CONTINUED

4. (a) State Cauchy's 1st limit Theorem and Cauchy's 2nd limit Theorem.

(b) Write down the Taylor polynomial $P_2(x)$ of order 2 for the function $f(x) = \sqrt{1+x}$ and give an expression for the remainder $R_2(x)$ in Taylor's formula $\sqrt{1+x} = P_2(x) + R_2(x) \quad -1 < x < \infty$

3+3

5. (a) Show that, $\lim_{n\to\infty} \frac{1}{n}\left[1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2}\right] = 0.$

(b) Find the limit $\lim_{n\to\infty} \left[\left(\frac{2}{1}\right)\cdot\left(\frac{3}{2}\right)^2 \cdots \left(\frac{n+1}{n}\right)^n\right]^{1/n}.$

3+3

6. (a) Show that, the sequence $\{x_n\}$ defined by

$$x_n = \left[\frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \cdots + \frac{1}{(n+n)^2}\right]$$

converges to 0.

(b) Does there exist a differentiable function $f : \Re \to \Re$ such that $f'(0) = 0$ but $f'(x) \geq 1$ for all $x \neq 0$? (Hint: you may use mean value theorem if you wish)

3+3

7. (a) Determine if the series $\sum_{n=1}^{\infty} \frac{3^{1-2n}}{n^2+1}$ converges or diverges.

(b) Determine if the series $\sum_{n=0}^{\infty} \frac{(2n)!}{5n+1}$ converges or diverges.

3+3

8. (a) State Lagrange's Interpolation formula.

(b) Using Lagrange's interpolation formula find $f(10)$ from the following table:

x	5	6	9	11
$f(x)$	12	13	14	16

3+3

Group C

Each Question carries ten marks. Answer any Two questions.

9. (a) State Taylor's Theorem.

(b) Using Taylor's theorem or otherwise, show that $x - \frac{x^3}{3!} < \sin x < x - \frac{x^3}{3!} + \frac{x^5}{5!}$, for $x > 0$.

(c) Apply Mean value theorem to prove that $\frac{x}{1+x} < \log(1+x) < x, \quad \forall\, x > 0.$

2+4+4

10. (a) Define the limit of a function f at $x = a$.

(b) State and prove the power series expansion of $\sin x$ using Taylor's Theorem.

(c) If $a < c < b$ and $f''(x)$ exists on $[a, b]$, then show that, there exists $\xi \in (a, b)$, such that

$$\frac{f(a)}{(a-b)(a-c)} + \frac{f(b)}{(b-c)(b-a)} + \frac{f(c)}{(c-a)(c-b)} = \frac{1}{2}f''(\xi). \quad (1)$$

2+4+4

11. (a) State Newton's Forward Interpolation Formula.

(b) Given the following table, Construct the difference table. and find $f(1.5)$ using Newton's Forward interpolation.

x	1	2	3	4	5	6	7	8
f(x)	1	8	27	64	125	256	343	512

(c) Compute $f(7.5)$ using the above table by Newton's Backward formula.

2+4+4

12. (a) State Sandwich Theorem.

(b) Show, $\left(\frac{n}{n+1}\right)^{n^2} \to 0$ as $n \to \infty$.

(c) If a_n be a sequence of positive terms and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \ell$, then $\lim_{n\to\infty}(a_n)^{1/n} = \ell$.

2+4+4

B.Sc. (Hons.) Examination 2023
Semester III
Subject: Statistics
Paper: [SECC-1] Statistical Data Analysis Using R

Full Marks: 25 Time: 2 Hrs.

Group – A $5 \times 2 = 10$

Group – A (Answer any ten questions) $10 \times 1 = 10$

1. Answer the following questions with proper justification.
   (a) How to find the number of elements of a numeric vector using R?
   (b) Which function do we use to draw Q-Q plot in R?
   (c) Round off $\sqrt[5]{7}$ upto 5 decimal point.
   (d) Using R, find the value of $\log_2(3) + \sin(2 + 3e^{0.6})$.
   (e) Suppose you have an array of numbers starting from 10 to 1000. Using R, how to find the numbers lies between 200 to 500?
   (f) Create a square matrix A of order 4 and compute A^5.
   (g) Input six names and sort them in ascending alphabetical order.
   (h) Write down the steps to import a dataset in R.
   (i) Create a vector x with elements $0, 0.2, 0.4, 0.6, .., 50$.
   (j) How to print a character string in R?
   (k) Create a function in R that takes two user defined input x, y and shows $x + 2y$ as output.
   (l) Using R, draw the graph of $y = \sin(x^2)$ for $x \in [-2, 2]$.
   (m) In R, how to remove some columns from a dataframe?

Group – B (Answer any three questions) $3 \times 5 = 15$

2. Using R, find the raw and central moments up to order 4 for the data (x): 21 33 32 16 26 32 38 25 10 33 15 30 34 27 24 27 38 14 15 26. 5

3. Fit a 2^{nd} degree polynomial regression curve based on the data: X: 107 110 121 124 129 132 131 153 142 151 158 152 164 152 162 156 175 178 172 177 174 183 172 176; Y: 99 82 102 81 89 87 59 112 74 91 116 91 96 85 131 136 162 121 137 82 165 159 91 193. Discuss the summary statistics and hence find the correlation index of order 2 i.e. r_2. Draw the regression line over the scatter plot. 5

4. Let X be a random variable with pdf of X is $f(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta},\ x > 0$; α is shape and β is rate i.e. $X \sim Gamma(\alpha, \beta)$. Using R, plot the multiple graphs of gamma distribution from $Gamma(2, 0.5)$, $Gamma(2, 2.5)$ and $Gamma(1.2, 2)$. 5

5. A company has offices in 50 cities across the country (all with roughly the same number of employees in similar roles). Due to travel restriction, it is not possible to visit every office to collect the data. Now using R, find 10 office (stage 1 cluster). Then from these 10 offices select 5 employees from each office (stage 2 cluster). 5

6. Generate 500 random samples from N(1,1) and U[0,2]. Also draw the respective histograms. 5

B.Sc. Examination, 2022
Semester-III
Statistics
Course: CC-5
Sampling Distribution (Theory)
Time: 3 Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Notations have their usual meanings

There are a total of 7 questions. Answer **any four**. Each question carries 10 marks.

1. (a) Define Sample space and sampling distribution. State and prove: Markov inequality.

(b) Find the Distribution of smallest order statistics and the Distribution of sample range. Consider random variable $Z \sim \chi_n^2$. Show that $\sqrt{2Z} - \sqrt{2n-1} \xrightarrow{d} N(0,1)$ as $n \to \infty$.

$5 + 5$

2. (a) Define F distribution. Let $F \sim F_{m,n}$ (F-distribution). Prove that, $\frac{1}{F} \sim F_{n,m}$ and $mF \to \chi_m^2$ as $n \to \infty$.

(b) Let $Y \sim \chi_n^2$ If $p = P(Y > a)$ then show that $a = 2\log_e(1/p)$.

$5 + 5$

3. (a) Define Chi-square distribution and t distribution. Comment on skewness, Kurtosis, and asymptotic behavior (for large degrees of freedom) of standard chi-square distribution of n degrees of freedom.

(b) State Weak Law of Large Number (WLLN) or Strong Law of Large Number (SLLN). Decide with reason whether the WLLN holds for independent random variables with distributions as follows $(n = 1, 2, \cdots)$

$$P(X_n = \sqrt{n}) = P(X_n = -\sqrt{n}) = \frac{1}{2n} \quad \text{and} \quad P(X_n = 0) = 1 - \frac{1}{n}$$

$5 + 5$

4. (a) Let $X \sim N(0,1)$. Find the distribution of $Y = e^X$.

(b) Let $X, Y \sim N(0,1)$ (iid). Find the pdf of $\frac{X+Y}{|X-Y|}$

$5 + 5$

5. (a) Let $(X_1, X_2, \cdots, X_n)$ be random sample from a normal distribution $N(\mu, \sigma^2)$. Let $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ and $S^2 = \frac{1}{n-1}\sum_{i=1} n(X_i - \bar{X})^2$ be sample mean and sample variance (respectively). Show that $\bar{X}$ and S^2 are independently distributed, $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$, and $\frac{(n-1)S^2}{\sigma^2} \sim \chi_{n-1}^2$.

(b) (using the result proved in part (a) of this question or otherwise) Show that $\frac{\sqrt{n}(\bar{X} - \mu)}{S} \sim t_{n-1}$.

$7 + 3$

CONTINUED

6. (a) State Central Limit Theorem, state the Delta method and obtain the expression starting from Central Limit Theorem.

(b) Define order statistics of a random sample of size n. Let $X_{(n)}$ be the largest order statistic in a random sample of size n from the distribution with p.d.f: $f(x) = \frac{1}{\theta}, \quad 0 < x < \theta$, (where $\theta > 0$). Show that $X_{(n)}$ tends in distribution to a random variable which is degenerate at θ.

$4 + 6$

7. (a) (i) Define bivariate normal distribution. What will be the distribution of the correlation coefficient between $(X_i, Y_i) : i = 1, 2, \cdots, n$ assuming (X_i, Y_i) are i.i.d bivariate normal?

(ii) Derive mean and variance expression for a chi-square distribution with degrees of freedom n.

(b) Let $X_1, X_2, \cdots, X_n$ be i.i.d $uniform(0, 1)$. Find the distribution of sample range.

$(2 + 3) + 5$

B.Sc. Examination, 2022
Semester-III
Statistics
Course: CC-5 B
Sampling Distribution (Practical)
Time: 2 Hours **Full Marks: 20**

Questions are of value as indicated in the margin.
Notations have their usual meanings

1. Consider the following candy flavor count data for a bag containing 100 candies of different flavors.

Flavor	Number of Pieces of Candy
Apple	18
Lime	24
Mango	14
Orange	24
Grape	20

Suppose we wish to hypothesize about uniform distribution for the candy flavor counts for the candy bag and want a statistical answer to whether our hypothesis is true, based on given data. Perform a Pearson Chi-Square Goodness of Fit test (using R and manually). 2+3

2. Consider the following data:

Smoker	83	90	129	70
patients	86	93	136	82

We are interested in testing the following hypothesis about proportions of smokers:

H_0 : (The null hypothesis) the four populations from which the patients were drawn have the same true proportion of smokers.

H_A : (The alternative) this proportion is different in at least one of the populations.

Perform the above Hypothesis Test (for difference of proportions) and interpret the result. You may use R. 3

3. Consider the following R code for simulated data for mice weight (`micewt`):

```
  set.seed(100)
      micewt <- data.frame(
 name = paste0(rep("M_", 10), 1:10),
 weight = round(rnorm(10, 20, 2), 1)
)
```

(a) Consider the data `micewt$weight`. Perform a one-sample t-test for the null hypothesis: true mean weight is equal to 25 g against the alternative hypothesis: true mean weight is not equal to 25 g. State the assumptions (if any).

(b) Perform a Hypothesis test of whether the mean weight of mice for the above-simulated data is greater than 25g (one-tailed test).

2+2

4. Consider the following R code for obtaining copies of the built-in R data sets `mtcars` and `ToothGrowth`:

```
cardata <- mtcars
twdata  <- ToothGrowth
```

(a) Perform a Pearson Correlation test between mpg variable (`cardata$mpg`) and wt variable (`cardata$wt`) under 95 percent confidence level under suitable assumptions (if any).

(b) `ToothGrowth` data variables (equivalently, `twdata` data, which is copy of original `ToothGrowth` data) are: `len, supp, dose`. Perform **F test to compare two variances of len by supp** under suitable assumptions (if any).

2.5+2.5

5. Viva voce 3

B.Sc. (Honours) Semester III Examination 2022
Subject: Statistics
Paper: CC 6A
Statistical Inference (Theory)

Full Marks: 40 Time: 3 hours.

Answer any four of the following six questions of equal marks.
(Notations carry usual meanings)

1. (a) Let $x_i; i = 1, 2, 3, \ldots, n$ be a random sample from the probability distribution with density function
$$f_\theta(x) = \theta x^{\theta-1}; 0 < x < 1, \theta > 0$$
Obtain a sufficient statistics for θ.

   (b) Researchers have claimed that the average number of headaches per student during a semester of Statistics is 18. Statistics students believe that the average is higher. In a sample of 22 students the mean is 23 headaches with a standard deviation of 2.1. Now answer the following.
      i. Name the population. Write the null and alternative hypothesis.
      ii. What would be the relevant test statistics to test this?
      iii. Write down the rejection rule.
      iv. If the p-value of the test turns out .07, what would be your decision at 5% level of significance?

5+(2+1+1+1)

2. (a) If $x_1, x_2, \ldots, x_n$ be independently and identically distributed random variables with mean μ and unknown variance, then show that $\overline{x}$ is the minimum variance unbiased linear estimator of μ.

   (b) i. Define a Most Powerful test.
      ii. Suppose in an urn there are 5 balls out of which θ are white and $5 - \theta$ are black balls. You are asked to perform a test $H_0 : \theta = 3$ against $H_1 : \theta = 2$. 3 balls are drawn with out replacement. If there are more than two white balls then we accept the null hypothesis otherwise we reject. Find probability of type I error and probability of type II error.

5+1+4

3. (a) Let $x_1, x_2, \ldots, x_n$ be a random sample of size n from the distribution with the density function

$$f_\theta(x) = \frac{1}{\theta}\exp(-\frac{x}{\theta}); x > 0; \theta > 0$$

Show that $\overline{x}$ is an Minimum Variance Bound estimator of θ and has variance $\frac{\theta^2}{n}$.

(b) i. Prove that every most powerful test is necessarily unbiased.

ii. Is a test with two sided alternative uniformly most powerful ?

5+3+2

4. (a) Let T and U be independent and consistent estimators of $\gamma(\theta)$. Show that $aT+bU$, where $a+b=1$, is also consistent for $\gamma(\theta)$.

(b) For checking the software knowledge on a batch of 10 students, a special training workshop on R is implemented in that class. You are asked to design a test in this regard assuming normality of knowledge distribution among the students. Describe a test procedure clearly stating null and alternative hypothesis.

5+5

5. (a) Is Maximum Likelihood Estimator always unique? Answer with example.

(b) i. Define likelihood ratio test statistic.

ii. Write the rejection rule based on that test statistic.

iii. What is the asymptotic distribution of likelihood ratio test statistic?

5+2+1+2

6. (a) What do you understand by a Baye's estimate? Show that under absolute loss median of the posterior distribution is the Baye's estimate

(b) Describe likelihood ratio test process for testing $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 > \sigma_0^2$ when a random sample of size n is collected from $N(\mu, \sigma^2)$ where μ is unknown.

5+5

B.Sc. (Honours) Semester III Examination 2022
Subject: Statistics
Paper: CC 6B
Statistical Inference (Practical)

Full Marks: 20 Time: 2 hours.

1. A study on $n = 49$ hospital employees found that the number of latex gloves used per week by the sampled workers is summarized by $\overline{x} = 18.9$ and $\sigma = 13.5$. Let μ be the mean number of latex gloves used per week by all hospital employees. The hospital authority wants to check the claim of mean number of rubber gloves is less than 20. Answer the following.

   (a) Write null hypothesis and alternative hypothesis.

   (b) Calculate the test statistic.

   (c) Find the p-value.

1+1+2

2. The average amount of time boys and girls aged seven to eleven spend playing sports each day is believed to be the same. A study is done and data are collected, resulting in the data in the following table. Each populations has a normal distribution. Is there a difference in the mean

	Sample Size	Average number of hours playing/day	Sample sd
Girls	9	2	.866
Boys	16	3.2	1.0

amount of time boys and girls aged seven to eleven play sports each day? test at the 5% level of significance?

3

3. A psychologist was interested in exploring whether or not male and female college students have different driving behaviors.The psychologist conducted a survey of average speed of a random 12 male college students and a random 14 female college students. Here is a descriptive

Male(X)	Female(Y)
$\bar{x} = 105.8$	$\overline{y} = 90.6$
$s_x = 20.1$	$s_y = 12.2$

summary of the results of her survey: Is there sufficient evidence at 5% level to conclude that the variance of the speed driven by male college students differs from the variance of the speed driven by female college students?

3

4. Let $x_1 = 3.5$; $x_2 = 7.5$ and $x_3 = 5.2$ be observed values of a random sample of size three from a population having uniform distribution over the interval $(\theta; \theta + 5)$, where $\theta \in (0; \infty)$ is unknown and is to be estimated. A student claims that "2.4 is a maximum likelihood estimate of the unknown parameter θ". Examine and comment on the truthfulness of the student's claim.

4

5. For a data set of size 10 extracted from a distribution having p.d.f. $f(x) = \lambda e^{\lambda x}$ where $X > 0$ find the maximum likelihood estimate and moment estimate of the parameter involved. The observations are as follows.
9.9,2.3, 6.5, 3.9, 9, 11, 13.2, 4, 5.2, 8.

3

6. Practical note book+Viva voce

3

B. Sc. Examination 2022
Semester: III
Subject: Statistics
Paper: C-303 (Mathematical Analysis)

Time: 3 Hours **Full Marks: 60**

Questions are of value as indicated in the margin

Group A (Answer any five questions) **2 × 5**

1. Is the intersection of infinite collection of open sets always open?
2. Give an example of a set which open as well as closed.
3. State the completeness axiom of $\boldsymbol{R}$.
4. Find an interior set of $\{1,2,3\}$.
5. Find the number of significant figures in the following numbers: 0.002406, 26100.
6. Evaluate $\frac{\Delta^2}{E} x^4$.
7. State Newton-Cotes formula for numerical quadrature.

Group B (Answer any five questions) **6 × 5**

1. Define open set. Show that (1,2) is open in $\boldsymbol{R}$. Is ϕ an open set in $\boldsymbol{R}$?

2. Find the derived set of $\boldsymbol{S} = \{\frac{\boldsymbol{1}}{\boldsymbol{n}} : \boldsymbol{n} \in \boldsymbol{N}\}$**.**

3. Show that 1 is boundary point of $[0,1)$ but 0 is not a boundary point of $[-1,1)$

4. Check the convergence of the following series

$$\sum_{n=1}^{\infty} \frac{3}{n^2+10}, \sum_{n=1}^{\infty} \frac{1}{n^2-\frac{1}{2}}$$

5. Obtain the value of the following integral $\int_1^3 \frac{dx}{\sqrt{3+2x-x^2}}$ by Simpson's one-third rule and hence obtain the value of π.

6. State Lagrange's interpolation formula. Show that the sum of the coefficients of the entries $y_r, r = 1, 2, \ldots n$, in the Lagrange interpolation formula is unity.

7. Prove that $\Delta^n O^m = n\,(\Delta^{n-1} O^{m-1} + \Delta^n O^{m-1})$, symbols having their usual meanings.

Group C (Answer any two questions) 10 ×2

1. State the Archimedean property of real numbers. Using Archimedean property, show that for any real number a

$$lim_{n\to\infty} \frac{a}{n} = 0$$

Also Show that,

$$\sqrt{2} \in R$$

2. Prove that every convergent sequence in R is bounded. Is the converse true? If a sequence of real numbers $\{a_n\}$ converges to 0, prove that

$$\lim_{n\to\infty} \frac{a_1 + a_2 + \cdots + a_n}{n} = 0$$

3. State Euler-Maclaurin's summation formula. Hence derive the approximate expression of n! for large n

B.Sc. (Honours) Examination, 2022
Semester-III
Statistics
Course: SECC-1
(Statistical Data Analysis Using R)

Time: 2 Hours **Full Marks: 25**

Questions are of value as indicated in the margin
Notations have their usual meanings

Group – A (Answer any ten questions) $10 \times 1 = 10$

1. Answer the following questions with proper justification.

(a) In R Studio, how do you access an installed package?

(b) How to write the expression $\sqrt{sin^{-1}(\pi/4)}$ in R?

(c) Write down the R code for creating an identity matrix of dimension 8.

(d) How to create a vector with elements $1, 1.2, 1.4, 1.6, \cdots, 120$? Also use the function to find its length.

(e) Suppose we have a numeric vector V of length 100. How to reduce the vector to its elements lies between 2 and 5.

(f) Create a '.csv' file contains the data: 76 66 39 44 34 84 85 24 66 18. Import it in R, sort decreasing order and store in a variable say 'y'.

(g) Suppose you have a data A: $1, 2, 1, 3, 1, 3, 4, 3, 5, 5, 6, 6, 6, 2, 1, 3, 7, 1, 7, 7, 4, 3, 5, 7, 1, 2, 3, 4, 8$. Using R, how to find the frequency of a particular number?

(h) In which situation we use box plot? How we will use it in R?

(i) Using R, find the mean and sd for the data (x): 11 27 29 66 60 77 36 59 84 39 with frequencies (f): 10 4 6 8 3 9 12 5 21 9.

(j) Which R package generates automated reports? How to install it?

(k) Using R, generate 100 random samples from χ^2_3.

(l) Suppose there are 80 people numbered from 1 to 80. You want select 25 among them to give movie ticket. Use SRSWR (one people may get multiple tickets) and SRSWOR to select these lucky people.

(m) Write down a measure of kurtosis. Create a hypothetical data of length 8 and compute your mentioned measure using R.

(n) In R, how you rename x and y axis of a plot?

Group – B (Answer any three questions) $3 \times 5 = 15$

2. Describe stem-leaf plot and histogram. Generate a sample of size 250 from a $Poi(4)$ distribution and draw its histogram. Also discuss about the histogram. 5

3. Suppose we have a bivariate data of 10 students, Marks in School (x): 219 316 611 566 465 260 136 176 245 344 and Marks in College (y): 471 310 245 327 348 513 313 519 616 210. Fit a regression line X on Y and discuss the summary statistics. Draw the regression line over the scatter plot. 5

4. Using R, Plot the graph of normal distribution with parameters $\mu = 0.4, sd = \sigma = 1.2$ i.e. $N(0.4, 1.2)$. Here, X is a random variable with probability density function of X is $f(x) = \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2\sigma^2}(x-\mu)^2}, \quad -\infty < x < \infty$; μ is mean and σ is sd. Here we say that $X \sim N(\mu, \sigma)$. Hence add multiple graphs of normal distribution from $N(0.4, 0.6)$ and $N(0.4, 1.8)$. 5

5. a) Suppose there are 1000 trees with numbering. Using R, select 140 of these by using circular systematic sampling. b) Suppose there are 1200 students in Zoology. Among them 800 are girls and 400 are boys. You want to select 60 among them for a conference with same proportion reflects the population. Write down the R code for stratified sampling to select these lucky people. 2+3