B.Sc. (Hons.) Examination 2024
Semester V
Subject: Statistics
Paper: CC-11 [Stochastic Process and Queuing Theory (Theory)]

Full Marks: 40 Time: 3 Hrs.

Answer any four questions (*Symbols have their usual meaning*)

1. a) Derive the MGF of a standard normal distribution. Hence find $E(X^3)$.
   b) Derive the PGF of poisson distribution with parameter λ. (5+2)+3

2. a) The transition probability matrix (TPM) of a Markov Chain (MC) $\{X_n,\ n = 1, 2, \cdots\}$ having five states $1, 2, 3, 4$ and 5 is

$$P = \begin{array}{c} X_{n+1}= \\ X_n=1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0.1 & 0.1 & ? & 0.3 & 0.2 \\ 0 & 0.3 & 0.3 & 0 & ? \\ 0 & 0.2 & 0.1 & 0.2 & ? \\ ? & 0.2 & 0.1 & 0 & 0.4 \\ 0.4 & 0.2 & ? & 0.4 & 0 \end{bmatrix} \end{array}$$

   . Represent the TPM as a stochastic graph?
   b) Prove that the Chapman-Kolmogorov equation can be used to compute higher-order transition probabilities in a Markov process. 4+6

3. a) Write a brief note on weak stationarity and strict stationarity.
   b) Let $\{X_n, n \geq 0\}$ be a MC has three states $1, 2, 3$ with TPM is $P = \begin{pmatrix} 0.2 & 0.5 & 0.3 \\ 0.5 & 0 & 0.5 \\ 0.4 & 0.2 & 0.4 \end{pmatrix}$ and initial distribution $P(X_0 = i) = \frac{1}{3}$, $i = 1, 2, 3$. Find the two and three step TPMs. Also find $P(X_5 = 1 \mid X_1 = 3)$, $P(X_8 = 1 \mid X_3 = 2)$. 4+6

4. Suppose a MC have the TPM, $P = \begin{pmatrix} \beta & ? \\ \gamma & ? \end{pmatrix}$, with states $0, 1$ and initial distribution $P(X_0 = 0) = p_0$. Then find $P(X_n = 1)$. Hence find $EX_n,\ V(X_n),\ E(X_nX_{n-1})$. 5+1+2+2

5. Discuss the birth-death process with an example. Using standard notations, derive the mathematical expression for $P_n(t)$ in this context. 4+6

6. Briefly discuss: M/M/1 System, Erlang Process, First-time reach probability, Ergodic Theorem.
   2.5×4

B.Sc. (Hons.) Examination 2024
Semester V
Subject: Statistics
Paper: CC-11B [Stochastic Process and Queuing Theory (Practical)]

Full Marks: 20 Time: 2 Hrs.

Answer all questions (*Symbols have their usual meaning*)

1. It is known from the Meteorological Department that for the month of July if today rains, the probability of rains tomorrow is 0.76. Also, if today does not rain, the probability of rains tomorrow is 0.43. Write down the TPM for this Stochastic Process. Hence find, i) Rain on 3^{rd} day given no rain on 1^{st} day. ii) Rain on 8^{th} day given no rain on 6^{th} day. iii) No rain on 4^{th} day given no rain on 1^{st} day. iv) No rain on 12^{th} day given no rain on 9^{th} day. 6

2. Let $\{X_n, n \geq 0\}$ be a Markov Chain having four states $1, 2, 3$ and 4 with transition probability matrix is $P = \begin{pmatrix} 1/3 & 2/3 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0.5 \end{pmatrix}$. Find $f_{22}^{(3)}$ and $f_{31}^{(3)}$. 6

3. Suppose the customers arrive at a service counter with mean rate of 3 per minute. Justify a suitable distribution for this process. Then in an arrival of 2 minutes, find the probability that the number of arriving customers is i) Exactly 4. ii) Less than 2. 5

4. Practical note book and viva-voce. 3

B.Sc. Examination, 2024
Semester-V
Statistics
Course: CC-12
(Stat Computing with C/C++)
Time: 3 Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Notations have their usual meanings

There are a total of 6 questions. Question 1 is compulsory. Answer **any three** from Questions 2 to 6.
Each question carries 10 marks.

1. State whether the following statements are True or False, along with a short explanation(s) of your answer. (answer any 5).

(a) In C, a structure can contain another structure as one of its members.

(b) Dynamic memory allocation in C is handled at compile time.

(c) If `int s[5]` is a one-dimensional array of integers, then `*(s+2)` refers to the third element in the array using pointer notation.

(d) Structures in C can only contain data members, not function pointers.

(e) & is an '*Address of*' operator, and it can give the location number used by the variable in memory.

(f) If there are multiple statements in if or else block they should be enclosed within a pair of { }.

(g) The output of the following program is 2.

```
    #include<stdio.h>
int main()
{       int i = 2;
#ifdef DEF
        i *= i;
#else
        printf("\n%d", i);
#endif
        return 0;
}
```

2×5

2. (a) Distinguish between "call by value" and "call by reference" with suitable examples.

(b) Given the coordinates (x, y) of the center of a circle and its radius r, write a program that will determine whether a point (P_x, P_y) lies inside the circle, on the circle, or outside the circle.

$5 + 5$

3. (a) Write a short note on Dynamic Memory Allocation in C using `malloc(), calloc()`. Give suitable examples explaining their usage.

(b) The inverse transform sampling method is used to simulate an exponential random variable. Write a C code to generate an exponential random variable starting from a uniform random variable U in (0, 1) and use the inverse of the CDF.

CONTINUED

4. (a) The Bell numbers (which count the number of ways to partition a set) can be generated by constructing the Bell Triangle. Write a C code to print the following output (Bell Triangle).

Hint: You may use the recurrence relation $B_{n,k} = B_{n,k-1} + B_{n-1,k-1}$, where $B_{n,1} = B_{n-1,n-1}$ and $B_{n,1}$ is the first element in n th row.

1				
1	2			
2	3	5		
5	7	10	15	
15	20	27	37	52

(b) Temperature of a city in Fahrenheit degrees is input through the keyboard. Write a program to convert this temperature into Centigrade degrees.

$5 + 5$

5. (a) Explain the Box-Muller transform to generate random numbers from a normal distribution.

(b) Find errors, if any, in the following program statements and write short explanations with a reasonable solution of how the error can be rectified:

```
     #include<stdio.h>
struct s
{
        char ch;
        int i;
        float a;
};
void f(struct s);
void g(struct s*);
int main()
{
        struct s var = { 'C', 200, 25.55 };
        f(var);
        g(&var);
        return 0;
}
void f(struct s v)
{
        printf("\n%c %d %f", v->ch, v->i, v->a);
}
void g(struct s *v)
{
        printf("\n%c %d %f", v.ch, v.i, v.a);
}
```

$5 + 5$

CONTINUED

6. (a) Write a short note on Opening, writing, and closing a file in C.

(b) Write a program to receive the values of latitude (L_1, L_2) and longitude (G_1, G_2) in degrees of two places on the Earth and outputs the distance between them in nautical miles. The formula for the distance in nautical miles is:

$$D = 3963 \cdot \cos^{-1}\left(\sin L_1 \sin L_2 + \cos L_1 \cos L_2 \cdot \cos(G_2 - G_1)\right)$$

Where:

- L_1, L_2 are the latitudes of the two places in degrees,
- G_1, G_2 are the longitudes of the two places in degrees,
- D is the distance between the two places in nautical miles,
- The formula uses trigonometric functions (sine, cosine, and inverse cosine).

$5 + 5$

B.Sc. Examination, 2024
Semester-V
Statistics
Course: CC-12 B
(Stat Computing with C/C++ Practical)
Time: 2 Hours **Full Marks: 20**

Questions are of value as indicated in the margin.
Notations have their usual meanings

1. Write a program in C to input n and k ($k \leq n$) and output $\binom{n}{k}$. Using that (or otherwise), write a program to calculate the value of:

$$\sum_{i=1}^{5} \binom{n}{i}$$

4

2. Write a program C to input 10 integer values from the user and sort them in ascending and descending orders using the bubble sort algorithm. Also, print the range of that data. 4

3. Generate 20 random samples from a normal population (mean = your Roll number, variance = 1) using Box Muller Transformation implemented in C code. Compute the sample mean and standard deviation of these samples. Save the output in a file. 4

4. Write a program in C to: Open a file and store the following dataset:

Marks in Stats	38	65	64	50	30	80	90	79	82	81	95	79	80	78	98
Marks in Maths	45	75	72	60	40	75	85	70	80	62	85	79	70	65	85

Find the regression equation of marks in Maths (Y) on marks in Stats (X). Predict the marks in Maths for a student whose score in Stats is 89.5 using the regression. 4

5. Practical Notebook(s) and Viva-voce 4

B.Sc. (Honours) Semester V Examination 2025
Subject: Statistics
Paper: DSE-1
Time Series Analysis (Theory)

Full Marks: 40 Time: 3 hours.

Answer **any four** questions.

1. (a) Suppose for a set of time series observations, seasonal variation are decreasing. Write a relevant model for it.

   (b) For a time series curve $U_t = 7 + 0.4(0.5)^t$, what is the the ratio of the first differences of U_t and U_{t-1}?

   (c) In a time series forecasting problem, if the seasonal indices for quarters 1, 2 and 3 are 0.80, 0.90 and 0.95 respectively, what can you say about seasonal index of quarter 4?

   (d) What are the advantages of mean absolute percentage error in consideration of forecasting?

   (e) Give two examples of indirect cost occurred during forecasting.

   2X5=10

2. (a) Suggest a graphical method and a theoretical method in detecting whether forecasting errors are random.

   (b) Define a strict stationary process.

   (c) Is the process $X_t = X_{t-1} + Z_t$ where $Z_t \sim WN(0, 4)$ covariance stationary? Explain.

   4+2+4

3. (a) Show that $MA(1) \equiv AR(\infty)$.

   (b) For $Y_t = 0.5Y_{t-1} - 0.4Y_{t-1} + Z_t$ where $Z_t \sim WN(0, 1)$ (WN= White noise) find first two autocorrelations.

   (c) Write down the mathematical model of moving average of order three.

   4+4+2

4. (a) Discuss the effect of omission of data points in the time series plot.

(b) Discuss a method of estimating the parameters in fitting of a logistic curve based on n time series observations.

(c) Write the name of deterministic components of time series

3+5+2

5. (a) When will you adopt Holt Winter exponential smoothing forecast method?

(b) Write the expression of h-step ahead forecast.

(c) What will you conclude if a seasonal weight is found to be close to 0 in Holt Winter exponential smoothing analysis?

(d) Define partial autocorrelation. How many autocorrelations are there for AR(3) model?

2+3+2+3

B.Sc (Honours) Examination, 2024

Semester- V

Statistics

Course: DSE-2

(Demography and Vital Statistics)

Time: 3 hours **Full Marks: 40**

Questions are of values as indicated in the margin

Answer **any four** questions

1. i. Define a measurement of mortality which is a probability rate. Mention its disadvantages.
   ii. How will you compare the mortality experiences of two South Asian Countries viz. India and Sri Lanka? How will you incorporate the gender component in this comparison?

   5+5

2. i. What is infant mortality rate? What are the disadvantages of this rate?
   ii. Distinguish between Stable population and stationary population

   5+5

3. What is a life table? Write down the components of a life table and mention their inter-relations.

   10

4. Describe the method of fitting a Logistic curve by
   i. Rhode's method
   ii. Fisher's method 5+5

5. What is an abridged life table? Find the expression of ${}_nq_x$, the probability that a member of the cohort living at age x will die before reaching age $x+n$ using Graville's method.

   3+7

6. i. Define Gross reproduction rate (GRR) and Net reproduction rate (NRR). Describe different cases when NRR =1, NRR>1 and NRR<1.
   ii. What do you mean by the statement: "NRR of a country is 2.7". Explain why NRR is always less than GRR. 5+5

B. Sc. Examination 2024
Semester: V
Statistics
Paper: DSE 2B (Practical)
(Demography and Vital Statistics)

Time: 2 Hours **Full Marks: 20**

Questions are of value as indicated in the margin

1. Calculate age specific death (ASDR) rates and standardized death rates for both the populations.

Age Group (years)	Population A	Population B	Standard Population
0-4	10,000	8,000	12,000
5-14	20,000	15,000	18,000
15-24	18,000	14,000	16,000
25-34	22,000	20,000	21,000
35-44	16,000	18,000	19,000
45+	14,000	15,000	15,000

4

2. Construct a life table using the provided data on population and deaths across different age intervals.

Age Interval (x to x+n)	Population (lx)	Deaths (dx)
0-4	100,000	7,000
5-14	93,000	2,700
15-24	90,300	1,800
25-34	88,500	1,100
35-44	87,400	700
45-54	86,700	350
55-64	86,350	180
65-74	86,170	100
75-84	86,070	50
85-94	86,020	15

8

3. Suppose we have the following data on the population of a city over 10 years. Fit a logistic curve using the method of Rhodes

Year:	1	2	3	4	5	6	7	8	9	10
Population:	50000	60000	75000	90000	105000	120000	135000	145000	150000	152000

5

4. Practical note book and viva-voce. 3

B.Sc. (Hons.) Examination 2023
Semester V
Subject: Statistics
Paper: CC-11 [Stochastic Process and Queuing Theory (Theory)]

Full Marks: 40 Time: 3 Hrs.

Answer any four questions (*Symbols have their usual meaning*)

1. a) Derive the MGF of a bivariate normal distribution i.e. $BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Hence find $E(XY^2)$.
b) Suppose you have given the MGF of $N(0,1)$ distribution. Using this how you can find the MGF of $N(\mu, \sigma^2)$? (5+2)+3

2. a) The transition probability matrix (TPM) of a Markov Chain (MC) $\{X_n,\ n = 1, 2, \cdots\}$ having five states $1, 2, 3, 4$ and 5 is

$$P = \begin{array}{c} X_{n+1}= \\ X_n=1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0.2 & ? & 0 & 0.3 & 0.2 \\ 0.2 & 0 & 0.3 & 0 & 0.5 \\ 0 & 0.3 & 0.4 & ? & 0 \\ 0.2 & 0 & 0.3 & 0 & 0.5 \\ 0.3 & 0.2 & ? & 0.4 & 0.1 \end{bmatrix} \end{array}$$

. Represent the TPM as a stochastic graph?
b) In usual notations, show that $p_{ij}^{(\alpha)} = \sum_{r=1}^{m} p_{ir}^{(\alpha-\beta)} p_{rj}^{(\beta)} = \sum_{r=1}^{m} p_{ir}^{(\beta)} p_{rj}^{(\alpha-\beta)}$, where the SP having states $1, 2, \cdots, m$ and α is a positive integer. 4+6

3. a) Create brief explanations with examples for: States that are accessible and states that are communicable.
b) Let $\{X_n, n \geq 0\}$ be a MC has three states $1, 2, 3$ with TPM is $P = \begin{pmatrix} 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \\ 0.2 & 0.6 & 0.2 \end{pmatrix}$ and initial distribution $P(X_0 = i) = \frac{1}{3}$, $i = 1, 2, 3$. Find the two and three step TPMs. Also find $P(X_5 = 2 \mid X_1 = 2)$, $P(X_8 = 1 \mid X_6 = 3)$. 4+6

4. Suppose a MC have the TPM, $P = \begin{pmatrix} \alpha & ? \\ ? & \beta \end{pmatrix}$, with states $0, 1$ and initial distribution $P(X_0 = 1) = p_0$. Then find $P(X_n = 1)$. Hence find $EX_n,\ V(X_n),\ E(X_n X_{n-1}),\ E(X_n X_{n-2})$. 10

5. Explain the Yule-Furry Process and derive the expression for $P(N(t) = n)$. Additionally, determine the mean and variance of the process. (3+5)+2

6. a) Explain M/M/s model in detail with the basic assumptions. Hence find the expression of p_n for $M/M/\infty$ model.
b) What do you mean by Doob-type martingale? In a fair coin-flipping game where you double your money on heads and lose your entire stake on tails, how does the concept of a Doob-type martingale apply, and what does the martingale property imply in this scenario? (5+2)+(1+2)

B.Sc. (Hons.) Examination 2023
Semester V
Subject: Statistics
Paper: CC-11B [Stochastic Process and Queuing Theory (Practical)]

Full Marks: 20 Time: 2 Hrs.

Answer all questions (*Symbols have their usual meaning*)

1. The transition probability matrix of a Markov Chain (MC) $\{X_n,\ n = 1, 2, \cdots\}$ having three states $1, 2$ and 3 is
$$P = \begin{array}{c} X_{n+1}= \\ X_n=1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} a & b & c \\ 0.2 & c & a \\ 0 & 0.3 & c \end{bmatrix} \end{array}.$$ Find the positive entries a, b, c and represent the transition probability matrix as a stochastic graph. 5

2. Let $\{X_n, n \geq 0\}$ be a Markov Chain having four states $1, 2, 3$ and 4 with transition probability matrix is $P = \begin{pmatrix} 2/3 & 1/3 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 0.5 & 0.5 \end{pmatrix}$. Find $f_{11}^{(3)}$ and $f_{21}^{(3)}$. 6

3. Suppose the customers arrive at a bank with mean rate of 2 per minute. Justify a suitable distribution for this process and then find the probability that the interval between two successive arrivals i) More than 2 minute ii) 3 minutes or less iii) between 1 and 2 minutes. 6

4. Practical note book and viva-voce. 3

B.Sc. Examination, 2023
Semester-V
Statistics
Course: CC-12
(Stat Computing with C/C++)
Time: 3 Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Notations have their usual meanings

There are a total of 6 questions. Question 1 is compulsory. Answer **any three** from Questions 2 to 6.
Each question carries 10 marks.

1. State whether the following statements are True or False, along with a short explanation(s) of your answer. (answer any 5).

(a) Are the following array declarations correct? (written in C):

```
int a(25);   int size = 10, b[size];
```

(b) `num[4]` refers to the 4 the element of the array.

(c) If `int s[5]` is a one-dimensional array of integers, then `*(s+2)` refers to the third element in the array using pointer notation.

(d) & is an '*Address of*' operator, and it can give the location number used by the variable in memory.

(e) If a file is opened for reading, it is necessary that the file must exist. Moreover, if a file opened for writing already exists, its contents would be overwritten.

(f) Any C program contains at least one function.

(g) The output of the following code is `5 8 1`

```
#include <stdio.h>
int main()
{
        int n[3][3]={
                 {2,4,3},{6,8,5},{3,5,1}
               };
        printf("%d %d %d\n",*(*(n)+1),n[1][1],n[2][2]);
        return 0;
}
```

(h) A pointer variable points to a data type (like `int`) of the same type and is created with the * operator.

2×5

2. (a) Write a short note on Dynamic Memory Allocation in C using `malloc(), calloc()`. Give suitable examples explaining their usage.

(b) Write a C code for printing the following output (Floyd's Triangle).

```
1
2  3
4  5  6
7  8  9  10
11  12  13  14 15
```

$5 + 5$

CONTINUED

3. (a) Write a short note on ordering an array using a suitable sorting technique (of your choice) in C.

(b) Write a short note on Box Muller transformation for sampling from Normal distribution.

$5 + 5$

4. (a) Write a program in C to print the first 30 prime numbers. Explain each step.

(b) Find errors, if any, in the following program statements and write short explanations with a reasonable solution of how the error can be rectified:

```
     #include<stdio.h>
struct s
{
        char ch;
        int i;
        float a;
};
void f(struct s);
void g(struct s*);
int main()
{
        struct s var = { 'C', 100, 12.55 };
        f(var);
        g(&var);
        return 0;
}
void f(struct s v)
{
        printf("\n%c %d %f", v->ch, v->i, v->a);
}
void g(struct s *v)
{
        printf("\n%c %d %f", v.ch, v.i, v.a);
}
```

$5 + 5$

5. (a) Write a short note on Opening, writing, and closing a file in C.

(b) Consider the following program (pseudocode). Assume that all variables are integers. Note that $x\%y$ computes the remainder after dividing x by y. The division is an integer division. For example, $1/3$ will return zero while $10/3$ will return 3. What is $g(25)$ according to the program? Explain your answer.

```
int g(n)
{  result=0; i=1;
    while(n!=0)
      { remainder=n%2;    n=n/2;
          result=result+(remainder*i);
        i=i*10;
      }
  return result;
}
```

$5 + 5$

CONTINUED

6. (a) Distinguish between "call by value" and "call by reference" with suitable examples.

(b) State whether the stated "Match the following(s)" is True/False with reference to the following program segment. Explain your answers.

```
int x[3][5]={
                            {1,2,3,4,5},
                            {6,7,8,9,10},
                            {11,12,13,14,15}
            }, *n=&x;
```

1.	*(*(x+2)+1)	11
2.	*(*x+2)+5	8
3.	*(*(x+1))	6
4.	*(*(x)+2)+1	4
5.	*(*(x+1)+3)	9
6.	*n	1
7.	*(n+2)	3
8.	(*(n+3)+1	5
9.	*(n+5)+1	7
10.	+++n	2

$5 + 5$

B.Sc. Examination, 2023
Semester-V
Statistics
Course: CC-12 B
(Stat Computing with C/C++ Practical)
Time: 2 Hours **Full Marks: 20**

Questions are of value as indicated in the margin.
Notations have their usual meanings

1. Write a program in C to input 10 integer values from the user and sort them in descending orders by using **any sorting algorithm** (mention which sorting algorithm you are using). Print the range of that data and print the median. Print the outputs in a file. 4

2. Write a program in C to calculate the **Pearson product-moment correlation coefficient** for the following dataset. 4

Students Roll No.	1	2	3	4	5	6	7	8
Marks in Statistics.	67	54	49	45	82	56.7	69	48
Marks in Mathematics.	70	53	56	67	88	62	56.5	68

Table 1: Dataset for Question 2

3. The values of X and $G(X)$ are given in Table for some points. Write a program in C to find $G(5)$ by using **Lagrange's Interpolation Formula** (up to four decimal approximations). 4

X	-4	-1	0	2	3	7	10
$G(X)$	-23	-7	1	1	2	131	340

Table 2: Table for Question 3

4. Generate 20 random samples from a Standard Normal population by using C-code. **You may employ the Box-Muller transformation or otherwise**. Compute the mean and standard deviation of these samples. Print the output in a file. 4

5. Practical Notebook(s) and Viva-voce 4

B.Sc. (Honours) Semester V Examination 2024
Subject: Statistics
Paper: DSE-1
Time Series Analysis (Theory)

Full Marks: 40 Time: 3 hours.

1. Choose out the correct alternative for the following questions. 5

   (a) The null hypothesis in Portmanteau test is
      i. time series observations are nonrandom.
      ii. residuals are nonrandom.
      iii. residuals are random observations.
      iv. residuals are autocorrelated.

   (b) The following linear process $X_t = .7X_{t-1} + .2X_{t-2} + Z_t$ is
      i. stationary but not invertible
      ii. not stationary but invertible
      iii. stationary and invertible
      iv. neither stationary not invertible

   (c) For Holt Winters method when do you think that nonseasonal model will be befitting?
      i. seasonal weight near to 0
      ii. seasonal weight near to trend weight
      iii. seasonal weight near to .5
      iv. seasonal weight near to 1.

   (d) Trend value of a time series of each time point is not available in use of the method of
      i. graphical approach
      ii. semi averages
      iii. least squares
      iv. moving averages

   (e) Which measure of the following is unit free?(the abbreviations bear the usual meaning)
      i. MAD
      ii. ME
      iii. MAPE

iv. SE

2. Answer any **five** of the following. 5×2

   (a) When do you use ratio to moving average method for deseasonalization?

   (b) What is indirect cost occurred during forecasting method? Cite an example.

   (c) For a time series curve $U_t = 7 + 1.1(.5)^t$, what is the ratio of first difference of U_t and U_{t-2}?

   (d) How does the autocorrelation function of $Y_t = .5X_t - .8X_{t-1} + Z_t$ where $Z_t \sim WN(0, 1)$ look like? (WN:white noise)

   (e) Why don't we use regression technique in time series analysis ?

   (f) How will you decide that modified exponential curve is to be fitted for a time series data?

3. For the generalized linear process $X_t = Z_t + .7X_{t-1} + .2Z_{t-1}$, find out first two π weights and two ψ weights, where Z_t be the white noise process with 0 mean and variance 1.

5

4. Establish mathematically that $AR(1)$ is equivalent to $MA(\infty)$ and $MA(1)$ to $AR(\infty)$.

5

5. Answer any **three** of the following. 3×5

   (a) Construct Yule Walker equations for AR(2) model. Hence find the first two autocorrelations.

   (b) What are link relative indices? How will you use these in deseasonalization?

   (c) Describe a method of estimation of the parameters involved in a logistic curve.

   (d) For a time series model having only level and trend component, which forecasting method will you use? Discuss it with the corresponding smoothing equations.

1.Calculate the theoretical autocorrelations of $Y_t=a_t+.6a_{t-1}+.2a_{t-2}$ where a_t being the white noise process with mean 0 and variance 1. Also draw the simulated autocorrelation function based on 100 observations drawn from the above model. 5

2.Extract the time series data from R on average monthly temperature of Nottingham using the code "nottem". Now answer the following.

(i) What type of data is it? Give a very brief detail of the data (as described in R)**clearly mentioning the variable of interest.**

(ii) Plot the data. Is it stationary?

(iii) Is there seasonality? If so, what type of seasonality is it?

(iv) What type of exponential smoothing would you consider here and why?

(v) Interpret on the value of smoothing parameter due to seasonality.

(vi) Make a plot on residuals due to predictions of the given observations. Does the plot show random fluctuations or any other nonrandom pattern?

(vii) Forecast the temperature for the years from1940 to 1942.
7

3. Below are given the figures of production of a sugar factory in tons:

Year:	1975	1976	1977	1978	1979	1980	1981
Production:	7.7	8.8	8.5	9.0	9.1	9.8	9.4

Fit an exponential curve. 5

Viva Voce+ Practical copy 3

B.Sc (Honours) Examination, 2023

Semester- V

Statistics

Course: DSE-2

(Demography and Vital Statistics)

Time: 3 hours **Full Marks: 40**

Questions are of values as indicated in the margin

Answer **any four** questions **two** from each group

1. i. Define a measurement of mortality which is a probability rate. Mention it's disadvantages.
   ii. How will you compare the mortality experiences of two states, West Bengal and Kerala? How will you incorporate the gender component in this comparison?
   iii. Define comparative mortality factor.
   3+5+2
2. i. What is infant mortality rate? What are the disadvantages of this rate?
   ii. If the distributions of infant deaths occurring in a year are available by the year of birth, write three different alternative modes of determining the rate of infant mortality.
   5+5
3. What is a life table? Write down the components of a life table and mention their inter-relations.
   10
4. Describe the method of fitting a Logistic curve by
   i. Rhode's method
   ii. Fisher's method 5+5
5. What is an abridged life table? Find the expression of ${}_nq_x$, the probability that a member of the cohort living at age x will die before reaching age $x+n$ using Graville's method.
   3+7
6. i. Define Gross reproduction rate (GRR) and Net reproduction rate (NRR). Describe different cases when NRR =1, NRR>1 and NRR<1.
   ii. Show that for any community the NRR is necessarily less than GRR.

B. Sc. Examination 2023
Semester: V
Statistics
Paper: DSE 2B (Practical)
(Demography and Vital Statistics)

Time: 2 Hours **Full Marks: 20**

Questions are of value as indicated in the margin

1. Calculate age specific death (ASDR) rates for the following data:

Age group (in years)	Population (′000)	Number of Deaths
Below 10	25	50
10−30	30	90
30−45	40	160
45−70	20	100

4

2. A part of a life table is given below with most of the entries missing. On the basis of the available figures, supply the missing ones:

Age (x)	l_x	d_x	q_x	L_x	T_x	e_x^0
10	74600		0.00350			
11			0.00338			
12			0.00361			
13			0.00420			
14			0.00517			
15			0.00530			
16			0.00538			
17			0.00544			
18			0.00549			
19			0.00554			
20			0.00560		2535675	

(symbols having their usual meaning) 8

3. Calculate total fertility rate (TFR) for the following table:

Age of the woman	Female population	Number of live births
15-19	593,262	36,784
20-24	587,076	81,213
25-29	505,362	65,236
30-34	424,186	37,506
35-39	385,749	17,532
40-44	325,105	4,929
45-49	266,575	512

5

4. Practical note book and viva-voce. 3

B.Sc. (Honours) Examination, 2022
Semester-V
Statistics
Course: CC-11
(Stochastic Process and Queuing Theory (Theory))

Time: 3 Hours **Full Marks: 40**

Questions are of value as indicated in the margin
Notations have their usual meanings

Answer any four questions $4 \times 10 = 40$

1. a) Find the moment generating function of $N(\mu, \sigma^2)$ distribution. 5
   b) Using moment generating function or otherwise, find the first three raw moments of the geometric distribution with pmf $P(X = x) = pq^x,\ x = 0, 1, 2, \cdots;\ p+q = 1$. 5

2. a) The transition probability matrix of a Markov chain $\{X_n,\ n = 1, 2, \cdots\}$ having three states $0, 1$ and 2 is

$$P = \begin{array}{c} X_{n+1}= \\ X_n=0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0 & 0.3 & 0.7 \\ 0.8 & 0 & 0.2 \\ 1 & 0 & 0 \end{bmatrix} \end{array}.$$

   Represent the two step transition probability matrix as a stochastic graph. 3
   b) Define closed, absorbing and transient state 3
   c) Let $\{X_n, n \geq 0\}$ be a Markov chain has three states $1, 2, 3$ with TPM is $P = \begin{pmatrix} 0.3 & 0.5 & 0.2 \\ 0.2 & 0.4 & 0.4 \\ 0.6 & 0.3 & 0.1 \end{pmatrix}$ and initial distribution $\Pi_0 = (0.3, 0.4, 0.3)$. Find $P(X_2 = 2),\ P(X_8 = 3 \mid X_6 = 2)$. 4

3. a) State first entrance theorem. In usual notations, how to find $f_{ij}^{(3)}$ if P matrix is given? Write down the detailed steps. 5
   b) Discuss the postulates of a Poisson process. Show that the interval between two successive occurences of a Poisson process $\{N(t),\ t > 0\}$ having parameter λ has a negative exponential distribution with mean λ^{-1}. 2+3

4. a) Define a martingle process. Suppose that a gambler with one unit of money plays a series of fair games betting an amount a times $(0 < a < 1)$ the amount of fortune at hand. If we define Y_r is the fortune after r^{th} game, then show that $\{Y_r,\ r \geq 0\}$ is a martingle. 5
   b) Define homogeneous and non-homogeneous Markov chain. Show that Markov chain is completely defined by initial and transitional probabilities. 2+3

5. Briefly discuss: Service time, Taffic intensity, M/M/1 system, General ergodic theorem. 2.5×4

6. Discuss birth-death process with an example. Using usual notations, derive the mathematical expression of $P_n(t)$ in this context. 4+6

B.Sc. (Honours) Examination, 2022
Semester-V
Statistics
Course: CC-11B
(Practical on Stochastic Process and Queuing Theory)
Time: 2 Hours Full Marks: 20

Questions are of value as indicated in the margin
Notations have their usual meanings

Answer **all** questions

1. The transition probability matrix of a Markov chain $\{X_n, n = 1,2,...\}$ having four states 1,2,3,4 is

$$P = \begin{pmatrix} 0.2 & 0.3 & 0 & 0.4 & a \\ 0.2 & 0 & a+b & 0 & 0.5 \\ 0 & 0.3 & a+b+c & 0.3 & 0 \\ b & 0 & b+c & 0 & a+2b \\ 0.3 & a+c & 0 & 0.4 & c \end{pmatrix}.$$

Find the positive entries a,b,c and represent the transition probability matrix as a stochastic graph. 5

2. Let {Xn, n=1,2,...} be a Markov chain having three states 1,2,3 with transition probability matrix is $P = \begin{pmatrix} 0.3 & 10a^2 & 0.3 \\ ab & 0.5 & 0.3 \\ 0.2 & bc & 0.1 \end{pmatrix}$ and initial distribution (0.3,0.5,0.2). Find positive entries a,b,c then two and three step transition probability matrices. Hence find $P(X_8 = 3 \mid X_6 = 1)$, $P(X_7 = 2, X_6 = 2 \mid X_4 = 1)$, $P(X_3 = 3)$. 7

3. a) What is the distribution of inter-arrival times of a Poisson process $\{N(t), t \geq 0\}$ with parameter 'p'? Write down its density and distribution function.
   b) Suppose that customers arrive at a bus ticket counter in accordance with a Poisson process with mean rate of 10 per 2 minutes. Then find the probability of interarrival-times less than 3 minutes. 3+2

4. Practical note book and viva-voce. 3

B.Sc. Examination, 2022
Semester-V
Statistics
Course: CC-12
(Stat Computing with C/C++)
Time: 3 Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Notations have their usual meanings

There are a total of 6 questions. Question 1 is compulsory. Answer **any three** from Questions 2 to 6.
Each question carries 10 marks.

1. State whether the following statements are True or False, along with a short explanation(s) of your answer. (answer any 5).

(a) If a is an integer variable, a = 11/2; will store 6 in a, and the expression (written in C), `a = 5000*3+6754;` would evaluate to 21755.

(b) Bubble sort is one of the simplest sorting techniques, which is also called an exchange sort.

(c) & is an '*Address of*' operator, and it can give the location number used by the variable in memory.

(d) Any C program contains at least one function.

(e) A pointer variable points to a data type (like `int`) of the same type and is created with the $*$ operator.

(f) Functions can be called either by value or reference and can have more than one return.

(g) The following C code will compile successfully and the output printed is `STAT COMPUTING`.

```c
#include<stdio.h>
#include<stdlib.h>
int main()
{        int i = 0; i++;
        if(i <= 5)
        {        printf("STAT COMPUTING\n");
                exit(0);
                main();
        }
        return 0;
}
```

2×5

2. (a) Distinguish between "call by value" and "call by reference" with suitable examples.

(b) Write a C code for printing the following output (Pascal's Triangle).

```
                1
              1   1
            1   2   1
          1   3   3   1
        1   4   6   4   1
      1   5  10  10   5   1
    1   6  15  20  15   6   1
  1   7  21  35  35  21   7   1
```

$5 + 5$

CONTINUED

3. (a) Write a short note on Dynamic Memory Allocation in C using `malloc(), calloc()`. Give suitable examples explaining their usage.

(b) (i) What is a pointer (in C)?

(ii) What will be the output of the following program in C? Write short explanations explaining each line of the following code.

```c
#include <stdio.h>
int main()
{
        int a[5] = { 5, 1, 15, 20, 25 };
        int i, j, k = 1, m;
        i = ++a[1];
        j = a[1]++;
        m = a[i++];
        printf("\n%d , %d , %d", i, j, m);
        return 0;
}
```

$5 + (2 + 3)$

4. (a) Explain each line of the following C code and also state what would be the output of the following programs:

```c
#include <stdio.h>
int main()
{
        int n[3][3] = {
                2, 4, 3,
                6, 8, 5,
                3, 5, 1
        };
        int i, j;
        for (i = 0; i <= 2; i++)
                for (j = 0; j <= 2; j++)
                        printf("\n%d %d", n[i][j], *(*(n + i) + j));
        return 0;
}
```

(b) (i) Distinguish between array and Linked-list in C.

(ii) Find errors, if any, in the following program statements and write short explanations with a reasonable solution of how the error can be rectified:

```c
      #include <stdio.h>
int main()
{       struct
        {       char bookname[25];
                float price;
        };
        struct book b = { "Go Embedded", 240.00 };
        printf("%s %f\n", b.bookname, b.price);
        return 0;
}
```

$5 + (3 + 2)$

CONTINUED

5. (a) The Fibonacci sequence is a series of numbers in which each number is the sum of the two that precede it. Starting at 0 and 1, the sequence looks like this: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, and so on forever. Write a program in C to obtain the first 50 Fibonacci sequence numbers.

(b) What will be the output of the following program in C? Write short explanations explaining the code.

```c
#include <stdio.h>
void change(int*, int);
int main()
{
        int a[] = { 2, 4, 6, 8, 10 };
        int i;
        change(a, 5);
        for (i = 0; i <= 4; i++)
                printf("\n%d", a[i]);
        return 0;
}

void change(int *b, int n)
{
        int i;
        for (i = 0; i < n; i++)
                *(b + i) = *(b + i) + 5;
}
```

$5 + 5$

6. (a) Write a short note on Opening, writing, and closing a file in C. (or) Write a short note on ordering an array using a suitable sorting technique (of your choice) in C.

(b) Consider the following code, in which A is an array written in C, indexed from 0, and n is the number of elements in A.

```
function foo (A, n)
 {        L = 0;
         R = n - 1;
     while ( L <= R )
         {
          i = ceil (( L + R )/2);
            if ( A [ i ] < i )
            { L = i + 1;}
            else
            { if ( A [ i ] > i )
                { R = i - 1; }
            else
               { return ( i );}
            }
         }
           return ( -1);
}
```

Here, `ceil(x)` returns the smallest integer bigger than or equal to the number x. If `A = [-5, -4, -3, -2, -1, 4, 6, 8, 10, 12]`, what will `foo(A, 10)` return? Give suitable explanations for your answer.

$5 + 5$

B.Sc. Examination, 2022
Semester-V
Statistics
Course: CC-12 B
(Stat Computing with C/C++ Practical)
Time: 2 Hours **Full Marks: 20**

Questions are of value as indicated in the margin.
Notations have their usual meanings

1. Write a C code to implement **any sorting technique (of your choice)** to sort the following data (written as an array in C) and hence find the median. 5

```
int mark[15] = {19, 10, 8, 1, 17, 4, 9, 20, 5, 11, 3, 12, 18, 2, 6 };
```

2. Write a C code for finding the multiplication of two $n \times n$ matrices. Using the C code, find whether the following matrix A is idempotent.

$$A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

6

3. Suppose we want to fit a simple linear regression model for the following dataset: **Iris Setosa** (part data is taken here). The data is for a random sample of the wildflower iris setosa. In the following data:

X = sepal width (cm)
Y = sepal length (cm)

(Data Reference: Fisher, R.A., Ann. Eugenics, Vol. 7 Part II, p 179-188)

X	Y
3.5	5.1
3	4.9
3.2	4.7
3.1	4.6
3.6	5
3.9	5.4
3.4	4.6
3.4	5
2.9	4.4
3.1	4.9
3.7	5.4
3.4	4.8
3	4.3
4	5.8
4.4	5.7
3.9	5.4

Write a C program to fit a simple linear regression model. Hence or otherwise, obtain the sepal length predicted by that simple linear regression model for a sepal width of 4.8 cm. 6

4. Viva voce 3

B.Sc. (Honours) Semester III Examination 2022
Subject: Statistics
Paper: DSE-1
Time Series Analysis (Theory)

Full Marks: 40 Time: 3 hours.

Answer question 1 and any three from the rest of the following.
(Notations carry usual meanings)

1. Answer any **five** questions.

2X5=10

(a) Why can't we use least square method in estimating the parameters in growth curve model?

(b) What is indirect cost in forecasting? Cite an example too.

(c) What are the advantages of mean percentage error in consideration of forecasting?

(d) What do mean by variance stabilizing transformation? Name one such transformation.

(e) Define a chain index.

(f) In a time series forecasting problem, if the seasonal indices for quarters 1,2 and 3 are .80,.90 and .95 respectively, what can you say about seasonal index of quarter 4?

2. (a) For the model $X_t = .5X_{t-1} - .4X_{t-2} + Z_t$ where $Z_t \sim WN(0,1)$ discuss on the stationarity and invertibility.

(b) For the above model find out first two autocorrelations. How does the autocorrelation graph of this process look like?

5+5

3. (a) Suggest a graphical method and a theoretical method in detecting whether forecasting errors are random.

(b) Define a strict stationary process.

(c) Is the process $X_t = X_{t-1} + Z_t$ where $Z_t \sim WN(0,4)$ covariance stationary? Explain.

4+2+4

4. (a) Explain how you will decide whether modified exponential trend is to be fitted to a time series data.

(b) Describe a method of fitting trend by modified exponential curve.

(c) How do you determine the period of moving average in a periodic time series?

(d) How many partial correlations are obtained for AR(2) model?

2+4+2+2

5. Suppose for a set of time series observations seasonal variation are increasing. Answer the following.

   (a) Write type of exponential smoothing technique would be adopted for this?

   (b) Write the relevant model for it.

   (c) Write the equations of estimating the smoothing constants.

   (d) Write the expression of h-step ahead forecast.

   (e) What will you conclude if a seasonal weight is found to be close to 0?

1+2+3+2+2

6. (a) What is the basic of harmonic analysis in time series? explain briefly.

   (b) Let us consider a time series without trend and seasonal component such that $U_t = acos\frac{2\pi}{\lambda}.t + \epsilon_t$ where ϵ_t is the error with zero mean and constant variance. For this series discuss a method of finding out the true intensity of the series.

4+6

B.Sc. (Honours) Semester III Examination 2022
Subject: Statistics
Paper: DSE-1
Time Series Analysis (Practical)
Full Marks: 20 Time: 2 hours.

1. Calculate seasonal indices by the ratio to moving average method from the following data.

Quarter Year	1980	1981	1982	1983
Q1	75	86	90	100
Q2	60	65	72	78
Q3	54	63	66	72
Q4	59	80	85	93

Which season has the biggest effect on the observation? 5

2. Extract the time series data from R on annual water flow of river Nile using the code "Nile". Now answer the following.

(i) Give a very brief detail of the data (as described in R) **clearly mentioning the variable of interest.**

(ii) Plot 4 pt moving averages for all observations.

(iii) Present a subset of data extracted from the main data, taking from 1895 to 1965. Plot it.

(iv) What type of exponential smoothing would you consider here and why?

(v) Interpret on the value of smoothing parameter due to trend, if any.

(vi) Make a plot on predicted figures along with forecasted figures of next two years.

(vii) Present the 95% upper limit and lower limit of your prediction.

(viii) Plot ACF and PACF function for logarithm of the original series. Can you make any guess regarding the model from it? 10

3. Note Book+Viva Voce 5

B.Sc. (Honours) Examination, 2022
Semester-V
Statistics
Course: DSE-2A
(Demography and Vital Statistics)
Time: Three Hours Full Marks: 40

Questions are of value as indicated in the margin
Notations have their usual meanings

Answer **any five** questions

1. Explain the need of standardization in comparing death rates. Describe the methods of direct and indirect standardization of death rates. 2+6

2. In what way do total fertility rate (TFR), gross reproduction rate (GRR) and net reproduction rate (NRR) differ from one another as measures of reproduction? Does TFR strictly conform to our ideas of a measure of reproduction? How does NRR indicate the growth of population? Interpret the situations when for a society
(i) GRR=NRR and (ii) NRR=1 3+1+2+2

3. With appropriate assumption and usual notation, show that

$$ {}_nq_x = \frac{2n.{}_nm_x}{2 + n.{}_nm_x + \frac{n^2}{6}\left\{ {}_nm_x^2 - \frac{d}{dx}\left({}_nm_x\right)\right\}}. $$ 8

4. Write short notes on (i) Whipple's index, and (ii) Age dependency ratio. 5+3

5. Define CBR, GFR and ASFR, and indicate why each is considered an improvement on the preceding measure of fertility. 2+3+3

6. How does an abridged life table differ from a complete life table? Describe a method of constructing an abridged life table. 4+4

7. Describe few sources of raw data in Demography. Distinguish between errors of coverage and errors of response in connection with errors in census and registration data. 2+6

8. Discuss how will you compare mortality situations (i) in two different communities and (ii) at two different periods of time for the same community. 4+4

B.Sc. (Honours) Examination, 2022
Semester-V
Statistics
Course: DSE-2B
(Practical on DSE-2A)
Time: Two Hours Full Marks: 20

Questions are of value as indicated in the margin

1. Complete the following life table:

Age(x)	l_x	d_x	$1000\,q_x$	L_x	T_x	e_x^0
25	78046					
26	77614	400				
27						
28	76723		6.06			
29						
30	75523				2750943	

Hence determine the probability that a person of age 25 lbd will die before reaching age 30 lbd. 4+1

2. With the help of the following data, calculate the standardised death rates for Poland and Sweden, taking the ISI population as standard and comment. 3

Age	Death rate (per thousand)		Number in ISI standard million
	Poland	Sweden	
0-4	18.870	4.348	119900
5-14	0.759	0.465	206900
15-24	1.385	0.767	183200
25-34	2.048	1.075	147900
35-44	3.326	1.882	120500
45-54	7.006	4.669	93900
55-64	18.111	12.477	70800
65-74	45.795	34.060	40500
75 and above	124.258	116.433	16400

3. Age-groups (in years) of mothers, female population (in ’000) and number of live-births (in’00) to the mothers are given for a small country with total population 2286×10^3.

Age-group (in year)	Female population (in ’000)	Number of live-births (in’00)
15-19	85	23.4
20-24	70	145.4
25-29	73	167.4
30-34	76	102.4
35-39	75	51.3
40-44	72	14.2
45-49	67	1.0

Obtain (a) the crude birth rate, (b) the general fertility rate and (c) the total fertility rate for the country. If the crude rate of natural increase is 10 (per 1000), what is the crude death rate for the country? 5

4. Compute a summary index of age preference of the following table in ending by "0" using Whipple index in the range 25-76. Also calculate the age dependency ratio. 4

Age(in years)	Number	Age(in years)	Number	Age(in years)	Number
10	841356	37	242462	64	34381
11	581400	38	316210	65	102440
12	796786	39	225207	66	26445
13	619293	40	434156	67	35311
14	596592	41	128632	68	40711
15	565714	42	217881	69	20921
16	566942	43	169167	70	136771
17	538891	44	151142	71	13000
18	651318	45	319118	72	28017
19	491441	46	160329	73	16662
20	565801	47	160855	74	14490
21	494895	48	237287	75	50558
22	515823	49	155094	76	15010
23	456892	50	313636	77	11878
24	425212	51	78534	78	23353
25	522203	52	128935	79	9212
26	358549	53	93279	80	73791
27	376221	54	95715	81	5532
28	395766	55	163093	82	9331
29	300610	56	87754	83	5653
30	535924	57	71828	84	5089
31	333086	58	93049	85	18604
32	318481	59	72206	86	4803
33	246260	60	275436	87	5617
34	233700	61	31299	88	4388
35	401936	62	49634	89	4000
36	242659	63	40154		

5. Practical Note Book and Viva-voce. 3
