### B.Sc. (Honours) Examination, 2025 Semester-VI Statistics Course: CC-13A

(Design of Experiments)

Time: Three Hours Full Marks: 40

Questions are of value as indicated in the margin Notations have their usual meanings

#### Answer any five questions

- 1.What do you understand of the terms local control and replication in designing an experiment? Are the use of analysis of covariance and confounding in factorial experiments achieved the local control? How does randomization effect in an experiment? 3+3+2 2.Give the layout of a CRD mentioning the principles considered. Give the statistical analysis of this design.
- 3. Describe the method of estimating the yield of a missing plot in RBD? Carry out the analysis of RBD after estimating the yield of missing plot.

  3+5
- 4. What is a Latin Square Design (LSD)? How is it improvement over Randomized Block Design (RBD)? Discuss the layout and analysis of LSD. 2+1+5
- 5. Define the terms main effects and interaction effects in relation to a  $2^2$  experiment and show that they are mutually orthogonal. How do you obtain the sum of squares due to main effects or interaction effects in this experiment? 3+3+2
- 6. Consider  $(2^4, 2^2)$  design. Treatment combinations belonging to a block is c, ad, bd, abc. (i) Construct the other three blocks. (ii) Find the confounded effects. (iii) Give the analysis of this experiment. 2+2+4
- 7. What is a split-plot experiment? Discuss the layout and analysis of a split-plot experiment in RBD. How will you estimate the standard error of the difference between two sub-plot treatment means at the different levels of the whole-plot treatment?

  2+4+2

4+4

8. Write short note on any two of the following:

(i) Comparison of efficiency of LSD with respect to RBD.

- (ii) Strip-plot experiments.
- (iii) Analysis of Covariance in RBD.

## **B.Sc.** (Honours) Examination, 2025 **Semester-VI Statistics**

**Course: CC-13B** 

#### (Practical on Design of Experiments) **Time: Two Hours** Full Marks: 20

Questions are of value as indicated in the margin

1. Given below is the layout plan and wheat yield in kg per plot for a randomized block design with 5 treatments A, B, C, D, E.

#### Block:

B1.82 E2.50 D2.27 C1.62 A0.91 2 C1.41 A0.95 E2.17 B1.95 D2.30 A0.77 B2.04 E2.50 C1.82 DMISSING

Obtain an estimate of the missing value and analyze the data.

2. The following table gives the plan and yields of an experiment involving three fertilizers N, P, K, each at two levels in eight blocks of four plots each.

Plan and yields of a 2<sup>3</sup>-factorial experiment in blocks of four plots.

| Block |           | Block |           |       |
|-------|-----------|-------|-----------|-------|
|       | Treatment | Yield | Treatment | Yield |
|       | (1)       | 145   | k         | 189   |
|       | pk        | 191   | р         | 272   |
|       | nk        | 300   | n         | 160   |
|       | np        | 240   | npk       | 305   |

| Block     |       | Block     |       |
|-----------|-------|-----------|-------|
| Treatment | Yield | Treatment | Yield |
| (1)       | 226   | р         | 226   |
| k         | 159   | nk        | 300   |
| npk       | 240   | pk        | 233   |
| np        | 182   | n         | 278   |

| Block     |       | Block     |       |
|-----------|-------|-----------|-------|
| Treatment | Yield | Treatment | Yield |
| р         | 186   | n         | 209   |
| npk       | 173   | k         | 93    |
| (1)       | 173   | pk        | 224   |
| nk        | 213   | np        | 201   |

| Block     |       | Block     |       |
|-----------|-------|-----------|-------|
| Treatment | Yield | Treatment | Yield |
| pk        | 182   | k         | 293   |
| (1)       | 175   | nk        | 226   |
| npk       | 156   | np        | 248   |
| n         | 183   | p         | 269   |

Analyze the data and write a report.

10

3. Practical Note Book and Viva-voce.

## B.Sc. (Honours) Examination, 2025 Semester-VI Statistics

Course: CC-14A

## Multivariate Analysis & Nonparametric Methods Time: 3 hrs Full Marks:40

Answer any **four** questions of the following.

- 1. (a) What effect is explained by  $r_{12.34\cdots p}$ ? Interpret the situation when  $r_{12.34\cdots p} = 0$ .
  - (b) Prove that  $b_{12.3} \times b_{21.3} = r_{12.3}^2$ . Notations bear their usual meanings.
  - (c) For three variables X; Y; Z such that aX + bY + cZ = k, find the value of  $r_{YZ,X}$ .

4 + 3 + 3

- 2. (a) Define multiple correlation and state its range.
  - (b) What are the null hypothesis and alternative hypothesis in Wilcoxon signed rank test? In this test, also state the rejection rule on the basis of test statistic.
  - (c) Show that for n observations measured on four variables  $x_1, x_2, x_3, x_4$ ,

$$\sum_{\alpha=1}^{n} e_{1.234\alpha}^{2} = \sum_{\alpha=1}^{n} e_{1.234\alpha} e_{1.23\alpha}$$

3+4+3

- 3. (a) Show that multiple correlation is the largest possible correlation among a dependent variable and any linear combination by k independent variables.
  - (b) For a nonsingular multinomial distribution with parameter  $(n, p_1, p_2, p_3, p_4)$ , prove that joint distribution of  $(X_1, X_2)$  is a trinomial. Hence find the covariance between  $X_1$  and  $X_2$ .

5+5

- 4. (a) State two cases where nonparametric tests are preferred.
  - (b) Why Wald-Wolfwitz run test is a nonparametric test?
  - (c) Discuss the situations when you use run test.

3++3+4

5. (a) Let 
$$\mathbf{X}_p \sim N_p(\mu, \Sigma)$$
. Let us partition  $\mathbf{X}_{p \times 1} = \begin{pmatrix} \mathbf{X}_{(1)}_{2 \times 1} \\ \mathbf{X}_{(2)}_{p-2 \times 1} \end{pmatrix}$  and  $\mu = \begin{pmatrix} \mu_{(1)}_{2 \times 1} \\ \mu_{(2)}_{p-2 \times 1} \end{pmatrix}$  and  $\Sigma_{p \times p} = Var(\mathbf{X}) = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$ . Show that  $\mathbf{X}_{(1)}$  and  $\mathbf{X}_{(2)}$  are independent iff  $\Sigma_{12} = 0$ .

(b) Under the above set-up show that the conditional distribution of  $\mathbf{X}_{(1)}$  given  $\mathbf{X}_{(2)}$  is a bivariate normal.

5 + 5

- 6. (a) If  $r_{1i} = r, (i = 2, 3, \dots, p)$  and  $r_{ij} = r', (i, j = 2, 3, \dots, p; i \neq j)$ , then find out the value of residual variance under the multiple regression model with  $x_1$  as dependent variable and rest the independent variables.
  - (b) Define empirical cumulative distribution function.
  - (c) When do you use Kolmogorov Smirnov test? Discuss about the test procedure clearly mentioning null and alternative hypothesis.

4+2+4

#### B.Sc. Examination, 2025

Semester-VI Statistics

#### Course: CC 14B

Multivariate Analysis & Nonparametric Inference Time: 2 hrs Full Marks:25

#### Answer all questions.

- 1. Suppose for three variables  $x_1, x_2$  and  $x_3, r_{12} = 0.21, r_{13} = -0.43, r_{23} = 0.72$ 
  - (a) What percentage of total variance of dependent variable is explained due to linear prediction?
  - (b) If you eliminate the effect of the first variable what would be the association between the rest of two variables.

3 + 3

2. A new approach to prenatal care is proposed for pregnant women living in a rural community. The new program involves in-home visits during the course of pregnancy in addition to the usual or regularly scheduled visits. A pilot randomized trial with 15 pregnant women is designed to evaluate whether women who participate in the program deliver healthier babies than women receiving usual care. The outcome is the APGAR score measured 5 minutes after birth. Recall that APGAR scores range from 0 to 10 with scores of 7 or higher considered normal (healthy), 4-6 low and 0-3 critically low. The data are shown below.

| UsualCare        | 8 | 7 | 6 | 2 | 5  | 8 | 7 | 3 |
|------------------|---|---|---|---|----|---|---|---|
| New Care program | 9 | 9 | 7 | 8 | 10 | 9 | 6 |   |

Do you believe that participation in new program ensures healthier babies?

4

3. The data below represents earning in dollars for a random sample of 5 common stocks listed in New York stock exchange. 1.68, 3.35, 2.5, 6.23, 3.24. Use the most appropriate test to see if the data can be regarded as a random sample with Normal(3,1).

4

- 4. Let  $\mathbf{X}_{3\times 1} \sim N_3(\mu, \Sigma)$  where  $\mu' = (1, -1, 3), \Sigma = \begin{pmatrix} 4 & 0 & -1 \\ 0 & 5 & 0 \\ -1 & 0 & 2 \end{pmatrix}$ . Answer the following questions.
  - (a) Are  $X_1 + X_2$  and  $3X_1 4X_2 + 5$  independent?
  - (b) Find out the regression equation of  $X_2$  given  $X_1 = -3$  and  $X_3 = -3$ .
  - (c) For a matrix  $A = \begin{pmatrix} 5 & 7 & 0 \\ 1 & -1 & 2 \\ 0 & 4 & 3 \end{pmatrix}$ , find  $E(A\mathbf{X})$  and  $V(A\mathbf{X})$ .

2+3+2

5. Practical Note Book +Viva Voce

4

#### B.Sc. (Hons.) Examination 2025 Semester VI

## Subject: Statistics Paper: DSE-3 [Operations Research (Theory)]

Full Marks: 40 Time: 3 Hrs.

Group - A  $5 \times 2 = 10$ 

- 1. Answer any five of the following questions with proper justification.
  - (a) Check the statement: In  $E^2$ ,  $X = \{(x,y): x^2 = y\}$  is a convex set.
  - (b) What do you mean by a basic feasible solution?
  - (c) Explain the matrix form of a Linear Programming Problem (LPP).
  - (d) What do you mean by an extreme point? Give an example.
  - (e) Briefly write down the steps to find an initial basic feasible solution of an assignment problem for a given cost matrix.
  - (f) What do you mean by a pay-off function in the context of game theory?
  - (g) Briefly discuss the difference between dominance and modified dominance property.
  - (h) What do you mean by indirect inventories? Discuss with examples.

$$Group - B$$
 (Answer any three questions)  $3 \times 10 = 30$ 

- 2. (a) Briefly write down various phases of operations research.
  - (b) What do you mean by a convex polyhedron? Show that hyperplane with inequalities or, half-space is a convex set.

    5+5
- 3. (a) Explain the mathematical form of a simplex table at any stage of the computation. In this context also discuss the terms: admissible basis, associated cost vector and net evaluation.
  - (b) Write down the mathematical form of balanced and unbalanced transportation problem? Show that the exact number of basic variables are m+n-1 for a balanced transportation problem with m origins and n destinations. (2+3)+(2+3)
- 4. (a) Suppose f(x,y) be a real valued function of x,y defined for  $x \in A$ ,  $y \in B$ , where  $A,B \subseteq \mathbb{R}$ . Now if both  $\max_{\mathbf{x} \in \mathbf{A}} \min_{\mathbf{y} \in \mathbf{B}} \mathbf{f}(\mathbf{x},\mathbf{y})$  and  $\min_{\mathbf{y} \in \mathbf{B}} \max_{\mathbf{x} \in \mathbf{A}} \mathbf{f}(\mathbf{x},\mathbf{y})$  exist then show that  $\max_{\mathbf{x} \in \mathbf{A}} \min_{\mathbf{y} \in \mathbf{B}} \mathbf{f}(\mathbf{x},\mathbf{y}) \leq \min_{\mathbf{y} \in \mathbf{B}} \max_{\mathbf{x} \in \mathbf{A}} \mathbf{f}(\mathbf{x},\mathbf{y})$ .
  - (b) Briefly discuss the mixed strategy technique. Find the value of the following  $2 \times 2$  game by using  $R_1$   $R_2$

mixed strategies.  $A_1$  a -b with a, b, c, d > 0. 4+(2+4)  $A_2$  -c d

- 5. (a) In standard notation, suppose we change a component of the cost vector c in an LPP. Analyze the optimality of new basic feasible solution and the change in optimal cost.
  - (b) Write down the analysis of a quantity discount inventory model with uniform demand rate, non-instant replenishment and no shortage allowed.

    4+6
- 6. Write short notes on any two of the following: i) Graph theoretic approach to solve LPP, ii) MODI method, iii) Classification of inventories.

  5+5

#### B.Sc. (Hons.) Examination 2025 Semester VI

#### **Subject: Statistics**

Paper: DSE-3B [Practical on Operations Research]

Full Marks: 20 Time: 2 Hrs.

Answer all questions

1. Solve the LPP using graphical or, simplex method.

 $Minimize, z = 2x_1 + 3x_2$ 

Subject to,

$$2x_1 + 7x_2 \ge 22$$

$$x_1 + x_2 \ge 6$$

$$5x_1 + x_2 \ge 10$$

$$x_1, x_2 \ge 0. 4$$

2. Four products are produced in three machines and their margins are given by the table below with capacity  $a_i$  and requirement  $b_j$ . Find out an optimal allocation so that the profit is maximized. Also discuss about its multiple optimal solutions.

$$\left[\begin{array}{cccccccc} P_1 & P_2 & P_3 & P_4 & a_i \\ M_1 & 6 & 4 & 1 & 5 & 14 \\ M_2 & 8 & 9 & 2 & 7 & 18 \\ M_3 & 4 & 3 & 6 & 2 & 7 \\ b_j & 6 & 10 & 15 & 8 \end{array}\right]$$

3. Reduce the following game to  $2 \times 2$  game by modified dominance property and then solve it.

|       | $B_1$ | $B_2$ | $B_3$ | $B_4$ |
|-------|-------|-------|-------|-------|
| $A_1$ | 3     | 2     | 4     | 0     |
| $A_2$ | 3     | 4     | 2     | 4     |
| $A_3$ | 4     | 2     | 4     | 0     |
| $A_4$ | 0     | 4     | 0     | 8     |

4. Practical note book and viva-voce.

6

### B.Sc. (Honours) Examination, 2024 Semester-VI Statistics Course: CC-13A

(Design of Experiments)

Time: Three Hours Full Marks: 40

Questions are of value as indicated in the margin Notations have their usual meanings

#### Answer **any five** questions

- 1. What are the basic principles of design of experiments? Discuss their use in CRD, RBD and LSD. How do the size and shape of plots and blocks affect the result of a field experiment?

  3+2+3
- 2. What is a standard Latin Square Design (LSD)? How basic principles of design of experiments are used in LSD? How many different LSD's can be generated from a standard LSD by permuting its rows and columns? When two LSDs are said to be orthogonal? Give an example of two mutually orthogonal LSDs.

  1.5+2+1.5+1.5+1.5
- 3. How will you estimate the yield of a missing plot in an RBD? Discuss in detail how you will carry out the analysis of an RBD after estimating the yield of missing plot.

  3+5
- 4. Write down the ANOCOVA model for RBD with one concomitant variable. Outline a method to judge whether the inclusion of the concomitant variable is worthwhile or not. If worthwhile, give the detailed analysis.

  2+2+4
- 5. (a) What is a factorial experiment? In what respect is it different from a single-factor experiment?
  - (b) Show that in a  $2^4$ -factorial experiment with factors A, B, C and D, the effects AB and BCD are orthogonal. Is ACD also orthogonal to AB and BCD? (2+2)+(2+2)
- 6. The incomplete key block of a 2<sup>5</sup>-factorial experiment involving the factors A, B, C, D and E conducted in four blocks is given below:
  - (1), ab, cd, ace, ....

Identify the other four treatment combinations in the key block. Also obtain the treatment combinations of the other three blocks. Write down the ANOVA table of the experiment.

3+2+3

- 7. What is strip-plot experiment and how does it differ from the split-plot experiment? Discuss the layout and analysis of a strip-plot experiment in RBD. 3+5
- 8. Write short note on any two of the following:

4+4

- (i) Confounding in design of experiments.
- (ii) Series of experiments.
- (iii) Efficiency of LSD compared to RBD.

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## B.Sc. (Honours) Examination, 2024 Semester-VI Statistics

**Course: CC-13B** 

(Practical on Design of Experiments)
Time: Two Hours Full Marks: 20

Questions are of value as indicated in the margin

A varietal trial was conducted at a Research Station. The design adopted for the same was five randomized blocks of 6 plots each. The yields in lb. per plot (of 1/20<sup>th</sup> of an acre) obtained from the experiment are as under:

| Blocks |       | Varieties |       |       |       |    |
|--------|-------|-----------|-------|-------|-------|----|
|        | $V_1$ | $V_2$     | $V_3$ | $V_4$ | $V_5$ |    |
|        | $V_6$ |           |       |       |       |    |
| I      | 39    | 23        | 34    | 25    | 20    | 13 |
| II     | 30    | 22        | 28    | 25    | 28    | 32 |
| III    | 56    | 43        | 43    | 31    | 49    | 17 |
| IV     | 38    | 45        | 35    | 36    | 32    | 30 |
| V      | 44    | 51        | 23    | 58    | 40    | 20 |

Analyze the design and comment on your findings.

2. The plan and the yield figures of the 2<sup>3</sup> experiment with the treatment combinations in the accompanying layout are given below. Analyze the data fully to test the different interactions and also for the main effects (consider rows as blocks in each replication).

Replicate-I

| D 1 |          | TT  |
|-----|----------|-----|
| Ken | lication | -11 |
|     |          |     |

| (1)<br>101 | nk<br>291 | nd<br>383 | kd<br>388 |
|------------|-----------|-----------|-----------|
| n          | 291<br>k  | 303<br>d  | nkd       |
| 106        | 265       | 312       | 450       |

| kd  | nd  | (1) | nk  |
|-----|-----|-----|-----|
| 407 | 338 | 106 | 306 |
| n   | nkd | k   | d   |
| 89  | 449 | 272 | 324 |

#### Replicate-III

| nk  | (1) | kd  | nd  |
|-----|-----|-----|-----|
| 334 | 87  | 423 | 324 |
| d   | k   | n   | nkd |
| 323 | 279 | 128 | 471 |

#### Replicate-IV

| nd  | nk  | (1) | kd  |
|-----|-----|-----|-----|
| 361 | 272 | 131 | 445 |
| k   | n   | nkd | d   |
| 302 | 103 | 427 | 331 |

3. Practical Note Book and Viva-voce.

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# B.Sc. (Honours) Examination, 2024 Semester-VI Statistics

Course: CC-14A

## Multivariate Analysis & Nonparametric Methods Time: 3 hrs Full Marks:40

Answer any **four** questions of the following.

- 1. (a) Prove that multiple correlation of any order is always positive. Hence state its range.
  - (b) Cite two instances where nonparametric testing is recommended.
  - (c) Suppose you have a data set of size 8. You want to test if the central value of the population, where the data coming from, is 16.
    - i. Write the null hypothesis of your test.
    - ii. Under this null hypothesis, deduce the expectation and variance of Wilcoxon signed rank test statistic.

3 + 3 + 4

- 2. (a) Suppose you have a data set of size 9. Discuss a test procedure briefly to check if a data of size n comes from a standard normal distribution.
  - (b) For a p order multiple regression model find the expression of  $V(x_{1.23\cdots p} 2\,e_{1.23\cdots p})$  where notations bear the usual meaning.
  - (c) If  $x_3 = ax_1 + bx_2$  for all sets of values of  $x_1, x_2$  and  $x_3$ , find the value of  $x_{23.1}$ .

4 + 4 + 2

- 3. (a) Establish the relationship  $\rho_{12.34\cdots p} = \frac{-\sigma^{12}}{\sqrt{\sigma^{11}\sigma^{22}}}$  where  $\rho_{12.34\cdots p}$  being the population partial correlation coefficient and  $\sigma^{ij}$  being the (i,j) element of  $\Sigma^{-1}$ .
  - (b) For a linear regression model coefficient of determination is 35%. Explain it.
  - (c) What is meant by the run? Write the rejection rule of run test clearly stating  $H_0$  and  $H_1$ .

5+2+3

- 4. (a) Let  $\mathbf{X}_p \sim N_p(\mu, \Sigma)$ . Let us partition  $\mathbf{X}_{p\times 1} = \begin{pmatrix} \mathbf{X}_{(1)_{2\times 1}} \\ \mathbf{X}_{(2)_{p-2\times 1}} \end{pmatrix}$  and  $\boldsymbol{\mu} = \begin{pmatrix} \mu_{(1)_{2\times 1}} \\ \mu_{(1)_{p-2\times 1}} \end{pmatrix}$  and  $\boldsymbol{\Sigma}_{p\times p} = \begin{pmatrix} \Sigma_{112\times 2} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$ . Show that the conditional distribution of  $\mathbf{X}_{(2)}$  given  $\mathbf{X}_{(1)}$  follow a multivariate normal of order p-2.
  - (b) Let  $\mathbf{X} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \sim N_3 \begin{bmatrix} (1, -1, 3)', \Sigma = \begin{pmatrix} 40 & 0 & -1 \\ 0 & 5 & 0 \\ -1 & 0 & 2 \end{bmatrix} \end{bmatrix}$ . Check if  $X_1 X_2$  and  $X_1 + 3X_2 2X_3$  are independent.

6 + 4

- 5. (a) For a k th order multinomial distribution, prove that the regression equation of the the third variable over the other variables is linear.
  - (b) Suppose for a set of 5 observations  $(x_1, x_2, \dots, x_5)$ ,  $r_{1,j} = r$ ,  $\forall j = 2(1)5$  and  $r_{i,j} = r' \, \forall i \neq 1$  and  $i \neq j$ . Deduce  $r_{12.345}$ .

6 + 4

- 6. Write short note on any two of the following.
  - (a) Partial correlation coefficient
  - (b) Mann Whitney U test
  - (c) Multivariate moment generating function

5+5

#### B.Sc. Examination, 2021

Semester-VI Statistics

#### Course: CC 14B

Multivariate Analysis & Nonparametric Inference Time: 2 hrs Full Marks:25

#### Answer all questions.

1. Consider a Phase II clinical trial designed to investigate the effectiveness of a new drug to reduce symptoms of asthma in children. A total of n = 10 participants are randomized to receive either the new drug or the old drug. Participants are asked to record the number of episodes of shortness of breath over a 1 week period following receipt of the assigned treatment. The data are shown below.

| old drug | 7 | 5 | 6 | 4 | 12 |
|----------|---|---|---|---|----|
| new drug | 3 | 6 | 7 | 4 | 1  |

Is there a difference in the number of episodes of shortness of breath over a 1 week period in participants receiving the new drug as compared to those receiving the placebo?

4

- 2. From the data relating to the yield of dry bark $(X_1)$ , height  $(X_2)$  and girth  $(X_3)$  for 18 cinchona plants, the following correlation coefficients were obtained:  $r_{12} = .74$ ,  $r_{13} = .43$  and  $r_{23} = .53$ . Answer the following questions.
  - (a) What percentage of total variance is explained due to linear prediction?
  - (b) If you eliminate the effect of the third variable what would be the association between the first two variables?

2 + 3

3. New recruits to a call centre are given initial training in answering customer calls. Following this training they are independently assessed on their competence, and are rated on a score of 1 to 10, 1 representing 'totally incompetent' to 10 'totally competent'. It is usual for the trainees' scores to be symmetrically distributed about a median of 6. A new trainer has been appointed and the scores of her first 15 trainees are:

 $6\; 5\; 6\; 9\; 7\; 3\; 4\; 6\; 7\; 2\; 9\; 8\; 9\; 4\; 5$ 

Is there evidence at the 5% level that the new trainer has made any difference?

4

4. Given below are the first 14 decimal places on  $\pi$ . Test for randomness in the way that **odd** and **even** digits occur in the sequence.

 $1\ 4\ 1\ 5\ 9\ 6\ 5\ 3\ 5\ 8\ 9\ 7\ 9\ 3.$ 

4

- 5. Let  $\mathbf{X}_{3\times 1} \sim N_3(\mu, \Sigma)$  where  $\mu' = (1, -1, 3), \Sigma = \begin{pmatrix} 4 & 0 & -1 \\ 0 & 5 & 0 \\ -1 & 0 & 2 \end{pmatrix}$ . Answer the following questions.
  - (a) Find out the regression equation of  $X_1$  given  $X_3 = -2$  and  $X_2 = -4$ .
  - (b) For a matrix  $\begin{pmatrix} 5 & 7 & 0 \\ 1 & -1 & 2 \end{pmatrix}$ , find  $V(A\mathbf{X})$ .

3+2

6. Viva voce+Practical Copy

#### B.Sc. (Hons.) Examination 2024 Semester VI

## Subject: Statistics Paper: DSE-3 [Operations Research (Theory)]

Full Marks: 40 Time: 3 Hrs.

Group - A  $5 \times 2 = 10$ 

- 1. Answer any five of the following questions with proper justification.
  - (a) Check the statement: In  $E^2$ ,  $X = \{(x,y): x^2 + y^2 = 1\}$  is a convex set.
  - (b) What are the steps to find out the optimal solution of a transportation problem (TP)?
  - (c) Write down the fundamental theorem of linear programming problem (LPP).
  - (d) Explain the mathematical form of TP.
  - (e) What do you mean by a two person zero sum game?
  - (f) In the context of LPP, briefly discuss the restrictions of graphical solution.
  - (g) What are cost components in an inventory model?
  - (h) Give a real life example of Pareto's Principle.

$$Group - B \ (Answer \ any \ three \ questions)$$
  $3 \times 10 = 30$ 

- 2. (a) Briefly write down various types of operations research problems.
  - (b) What do you mean by a convex set? Show that intersection of a finite number of convex sets is a convex set. Discuss whether union of two convex sets is a convex set or, not. 4+(1+4+1)
- 3. (a) What do you mean by basic feasible solution? Explain with an example.
  - (b) Solve (if possible) the LPP using algebraic method

$$Maximize, z = 5x_1 + 2x_2 + 2x_3$$

Subject to,

$$x_1 + 2x_2 - 2x_3 \le 30$$
  
 $x_1 + 3x_2 + x_3 + x_4 \le 36$   
 $x_1, x_2, x_3 \ge 0$ .

(2+2)+6

- 4. (a) If the  $k^{th}$  constraint of a primal be an equation, then show that  $k^{th}$  dual variable will be unrestricted in sign.
  - (b) Briefly explain which kind of problem we solved using TP? What are the basic assumptions of TP? In usual notations, show that the exact number of basic variables are m+n-1 for a balanced TP with m origins and n destinations. 3+(2+2+3)
- 5. (a) Briefly discuss the use of post-optimality analysis in the context of LPP.
  - (b) Give a real-life example of assignment problem and formulate its mathematical form.
  - (c) Formulate and analyze an economic order quantity model with uniform demand rate, non-instant replenishment and no shortage allowed type. 2+2+6
- 6. Write short notes on any two of the following: i) Limitations of ABC analysis, ii) Modified dominance property, iii) Mixed Strategy.

  5+5

#### B.Sc. (Hons.) Examination 2024 Semester VI

#### Subject: Statistics

Paper: DSE-3B [Practical on Operations Research]

Full Marks: 20 Time: 2 Hrs.

Answer all questions

1. Solve the LPP using simplex method.

Maximize,  $z = 5x_1 + 2x_2 + 2x_3$ 

Subject to,

$$x_1 + 2x_2 - 2x_3 \le 30$$

$$x_1 + 3x_2 + x_3 \le 36$$

$$x_1, x_2, x_3 \ge 0.$$

3

2. Obtain the IBFS to the following transportation problem by matrix (cost) minima method and corresponding cost of the transportation.

|       | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $a_i$ |
|-------|-------|-------|-------|-------|-------|
| $O_1$ | 5     | 4     | 6     | 14    | 15    |
| $O_2$ | 2     | 9     | 8     | 6     | 4     |
| $O_3$ | 6     | 11    | 7     | 13    | 8     |
| $b_i$ | 9     | 7     | 5     | 6     |       |

3. Reduce the following  $2 \times n$  game into a  $2 \times 2$  game using graphical method and hence solve it algebraically.

|       | $B_1$ | $B_2$ | $B_3$ | $B_4$ |
|-------|-------|-------|-------|-------|
| $A_1$ | 2     | 2     | 3     | -1    |
| $A_2$ | 5     | 3     | 2     | 6     |

4. Practical note book and viva-voce.

### B.Sc. (Honours) Examination, 2023 Semester-VI Statistics Course: CC-13A

(Design of Experiments)

Time: Three Hours Full Marks: 40

Questions are of value as indicated in the margin Notations have their usual meanings

#### Answer any five questions

- 1.What do understand the terms local control and replication in designing an experiment? How local control is achieved through the use of analysis of covariance and confounding in factorial experiments? What is the role of randomization in an experiment? 3+3+2 2.Give the layout of an RBD mentioning the principles considered. Give the statistical analysis of this design. 3+5
- 3. How will you estimate the yield of a missing plot in RBD? Discuss in detail how you will carry out the analysis of RBD after estimating the yield of missing plot.

  3+5
- 4. What is a Latin Square Design (LSD)? How is it improvement over Randomized Block Design (RBD)? Discuss the layout and analysis of LSD. 2+1+5
- 5. Define the terms main effects and interaction effects in relation to a  $2^3$  experiment and show that they are mutually orthogonal. How do you obtain the sum of squares due to main effects or interaction effects in this experiment? 3+3+2
- 6. Consider (2<sup>4</sup>, 2<sup>2</sup>) design. Treatment combinations belonging to a block is a, b, cd, abcd. (i) Construct the other three blocks. (ii) Find the confounded effects. (iii) Give the analysis of this experiment.

  2+2+4
- 7. What is a split-plot experiment? Discuss the layout and analysis of a split-plot experiment in RBD. How will you estimate the standard error of the difference between two sub-plot treatment means at the different levels of the whole-plot treatment?

  (2+4+2)
- 8. Write short note on any two of the following:

(i) Uniformity trial experiment.

- (ii) Series of experiments.
- (iii) Analysis of Covariance in RBD.

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4+4

## B.Sc. (Honours) Examination, 2023 Semester-VI Statistics

**Course: CC-13B** 

## (Practical on Design of Experiments) Time: Two Hours Full Marks: 20

#### Questions are of value as indicated in the margin

1. Given below is the layout plan and wheat yield in kg per plot for a randomized block design with 5 treatments A, B, C, D, E.

#### Block:

- 1 E2.50 D2.27 C1.62 B1.82 A0.91
- 2 C1.41 A0.95 E2.17 B1.95 D MISSING
- 3 A0.77 B2.04 E2.50 C1.82 D2.50

Obtain an estimate of the missing value and analyze the data.

2. The following table gives the plan and yields of an experiment involving three fertilizers N, P, K, each at two levels in eight blocks of four plots each.

Plan and yields of a 2<sup>3</sup>-factorial experiment in blocks of four plots.

| Block           |     |           | Block |     |  |
|-----------------|-----|-----------|-------|-----|--|
| Treatment Yield |     | Treatment | Yield |     |  |
|                 | (1) | 145       | k     | 189 |  |
|                 | pk  | 191       | р     | 272 |  |
|                 | nk  | 300       | n     | 160 |  |
|                 | np  | 240       | npk   | 305 |  |

| Block           |     | Block     |       |  |
|-----------------|-----|-----------|-------|--|
| Treatment Yield |     | Treatment | Yield |  |
| (1)             | 226 | р         | 226   |  |
| k               | 159 | nk        | 300   |  |
| npk             | 240 | pk        | 233   |  |
| np              | 182 | n         | 278   |  |

| Blo             | ock | Block     |       |  |
|-----------------|-----|-----------|-------|--|
| Treatment Yield |     | Treatment | Yield |  |
| p               | 186 | n         | 209   |  |
| npk             | 173 | k         | 93    |  |
| (1)             | 173 | pk        | 224   |  |
| nk              | 213 | np        | 201   |  |

| Blo             | ock | Block           |            |  |
|-----------------|-----|-----------------|------------|--|
| Treatment Yield |     | Treatment Yield |            |  |
| pk              | 182 | k               | 293        |  |
| (1)             | 175 | nk              | 226        |  |
| npk             | 156 | np              | 226<br>248 |  |
| n               | 183 | p               | 269        |  |

Analyze the data and write a report.

10

3. Practical Note Book and Viva-voce.

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## B.Sc. (Honours) Examination, 2023 Semester-VI Statistics

Course: CC-14A

## Multivariate Analysis & Nonparametric Methods Time: 3 hrs Full Marks:40

Answer any **five** questions of the following.

- 1. (a) What effect is explained by  $r_{12.34\cdots p}$ ? Interpret the situation when  $r_{12.34\cdots p} = 0$ .
  - (b) Prove that  $b_{12.34\cdots p} \times b_{21.34\cdots p} = r_{12.34\cdots p}^2$ .
  - (c) Distinguish between the terms 'distribution free' and 'nonparametric'.

3+3+2

- 2. (a) Find the value of  $r_{1.23\cdots p}$  if the independent variables are pairwise uncorrelated.
  - (b) If  $r_{1.234} = 1$ , prove that  $r_{2.134} = 1$ . What is the value of  $r_{2.134}$  if  $r_{1.234} = 0$ ?
  - (c) Show that for n observations measured on four variables  $x_1, x_2, x_3, x_4, \sum_{\alpha=1}^n e_{1.234\alpha}^2 = \sum_{\alpha=1}^n e_{1.234\alpha} e_{1.234\alpha}$

3+3+2

- 3. (a) For a p-component vector **X** prove that variance covariance matrix is nonnegative definite.
  - (b) Let  $(X_1, X_2, X_3)$  be a random vector with joint p.m.f.  $f(x_1, x_2, x_3) = \frac{1}{4}$  if  $(X_1, X_2, X_3) \in A$  where  $A = \{(1, 0, 0), (0, 1, 0), (0, 0, 1), (1, 1, 1)\}$ . Are  $X_1, X_2, X_3$  mutually independent? Are  $X_1$  and  $X_2$  pairwise independent?

4 + 4

- 4. (a) Construct a  $100(1-\alpha)\%$  confidence interval for the population median using sign test.
  - (b) Define run test statistic and find its null distribution.

4 + 4

- 5. (a) Show that  $\rho_{1.23\cdots p} = (1 \frac{1}{\sigma^{11}\sigma_{11}})^{\frac{1}{2}}$  where  $\sigma_{11}$  being the (1,1) element of  $\Sigma$  and  $\sigma^{11}$  being the (1,1) element in  $\Sigma^{-1}$ .
  - (b) Define Kruskal Wallis test statistic and state its null distribution. Also mention the critical region of the test.

4+4

6. (a) Let  $\mathbf{X} \sim Multinomial(n, p_1, p_2, \dots, p_{k-1})$ . Find its moment generating function. Hence deduce the covariance between  $X_3$  and  $X_4$ .

(b) Define sign test statistic for one sample location problem. Justify whether the test is nonparametric. Mention one of its drawbacks.

4 + 4

- 7. (a) Let  $\mathbf{X}_p \sim N_p(\mu, \Sigma)$ . Let us partition  $\mathbf{X}_{p \times 1} = \begin{pmatrix} \mathbf{X}_{(1)_{2 \times 1}} \\ \mathbf{X}_{(2)_{p-2 \times 1}} \end{pmatrix}$  and  $\mu = \begin{pmatrix} \mu_{(1)_{2 \times 1}} \\ \mu_{(1)_{p-2 \times 1}} \end{pmatrix}$  and  $\Sigma_{p \times p} = Var(\mathbf{X}) = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$ . Show that  $\mathbf{X}_{(1)}$  and  $\mathbf{X}_{(2)}$  are independent iff  $\Sigma_{12} = 0$ .
  - (b) Under the above set-up show that the conditional distribution of  $\mathbf{X}_{(1)}$  given  $\mathbf{X}_{(2)}$  is a bivariate normal.

4+4

- 8. (a) If  $r_{1i}=r, (i=2,3,\cdots,p)$  and  $r_{ij}=r', (i,j=2,3,\cdots,p; i\neq j)$ , then find out the value of residual variance.
  - (b) Define one-sample and two-sample Kolmogorov-Smirnov test statistic. Mention their uses.

4 + 4

## B.Sc. (Honours) Examination, 2022 Semester-VI Statistics

Course: CC-14B

Multivariate Analysis & Nonparametric Methods Time: 2 hrs Full Marks:20

Answer all the questions. Tables are attached at the bottom of the questions.

- 1. Comment on the consistency of  $r_{12} = 0.6$ ,  $r_{13} = -0.4$  and  $r_{23} = .7$ . (2)
- 2. Literacy rate is a reflection of the educational facilities and quality of education available in a country, and mass communication plays a large part in the educational process. In an effort to relate the literacy rate of a country to various mass communication outlets, a demographer has proposed to relate literacy rate to the following variables: number of daily newspaper copies (per 1000 population), number of radios (per 1000 population), and number of TV sets (per 1000 population). Here are the data for a sample of 10 countries:

| Country     | newspapers | radios | tv sets | literacy rate |
|-------------|------------|--------|---------|---------------|
| Slovakia    | 280        | 266    | 228     | 0.98          |
| Italy       | 142        | 230    | 201     | 0.93          |
| Kenya       | 10         | 114    | 2       | 0.25          |
| Norway      | 391        | 313    | 227     | 0.99          |
| Panama      | 86         | 329    | 82      | 0.79          |
| Philippines | 17         | 42     | 11      | 0.72          |
| Tunisia     | 21         | 49     | 16      | 0.32          |
| USA         | 314        | 1695   | 472     | 0.99          |
| Russia      | 333        | 430    | 185     | 0.99          |
| Venezuela   | 91         | 182    | 89      | 0.82          |

Write the least square regression equation of literacy rate on other explanatory variables. Explain the effect of newspaper on literacy rate.

Predict literacy rate for a country that has 200 daily newspaper copies (per 1000 in the population), 800 radios (per 1000 in the population), and 250 TV sets (per 1000 in the population). (3+2+2)

3. Use an appropriate nonparametric test to judge whether the square roots of the following numbers are coming from a U(0,1) distribution. (4)

0.0123, 0.1039, 0.1954, 0.2621, 0.2802, 0.3217, 0.3645, 0.3919, 0.4240, 0.4814, 0.5139, 0.5846, 0.6275, 0.6541, 0.6889, 0.7621, 0.8320, 0.8871, 0.9249, 0.9634

4. A firm is advertising that it has been successful in designing a new home automatic clothes washer which is more effective in removing dirt than the most popular washer now in use. In support of its claim, it is also displaying the following data of the dart removed (in suitable units) by the most popular washer and the new washer for 16 equally-sized and equally-soiled loads of clothes which were

washed with the same soap and for the same length of time, 7 loads being washed by the popular washer:

Dirt Removed by

**Popular washer:** 13, 10, 9, 12, 11, 10, 8 **New washer:** 10, 11, 12, 13, 9, 11, 14, 12, 13.

Do you have reasons to believe that the firm's claim is genuine? Use two different nonparametric test procedures for the justification. (4)

5. Practical Note Book +Viva Voce (3)

#### B.Sc. (Hons.) Examination 2023 Semester VI

## Subject: Statistics Paper: DSE-3 [Operations Research (Theory)]

Full Marks: 40 Time: 3 Hrs.

Group - A  $5 \times 2 = 10$ 

- 1. Answer any five of the following questions with proper justification.
  - (a) Check the statement: In  $E^2$ ,  $X = \{(x,y): y^2 > x\}$  is a convex set.
  - (b) What do you mean by a non-degenerate basic feasible solution?
  - (c) Write down a linear programming problem with four variables and two constraints with one variable showing no restriction in sign. Hence convert the primal into its dual problem.
  - (d) Explain the mathematical form of assignment problem.
  - (e) What do you mean by a pure strategy in the context of game theory?
  - (f) Briefly discuss the idea behind modified dominance property.
  - (g) What are the components of an inventory model?
  - (h) Briefly write down the steps to find an initial basic feasible solution of a transportation problem (TP) for a given profit matrix.

$$Group - B$$
 (Answer any three questions)  $3 \times 10 = 30$ 

- 2. (a) Briefly write down various phases of operations research.
  - (b) What do you mean by a convex set? Show that convex polyhedron i.e.  $X = \{x : x = \sum_{i=1}^{k} \lambda_i x_i, \text{ with } \sum_{i=1}^{k} \lambda_i = 1\}, \lambda_i \geq 0 \text{ is a convex set.}$
- 3. (a) Explain the mathematical form of a simplex table at any stage of the computation. In this context also discuss the terms: admissible basis and net evaluation.
  - (b) What are the basic assumptions of a TP? Show that the exact number of basic variables are m+n-1 for a balanced TP with m origins and n destinations. (2+2)+(2+4)
- 4. (a) What do you mean by pay-off matrix? How is it related with the pay-off function? Each of two players A and B shows one, two and three fingers simultaneously. The player B pays to A an amount equal to the two times total number of fingers shown, on the other hand A pays to B equal to the product of the number of fingers shown. Form the pay-off matrix.
  - (b) Solve the following  $2 \times 4$  game graphically. Here A is the row player having strategies  $A_1, A_2$  and B is the column player having strategies  $B_1, B_2, B_3, B_4$ .

$$\begin{bmatrix} B_1 & B_2 & B_3 & B_4 \\ A_1 & 2 & 2 & 3 & -1 \\ A_2 & 5 & 3 & 2 & 6 \end{bmatrix}$$
 (1+1+3)+5

- 5. (a) In standard notation, suppose we change a component of the cost vector c in an LPP. Analyze the optimality of new basic feasible solution and the change in optimal cost.
  - (b) Write down the analysis of a quantity discount inventory model with uniform demand rate, instant replenishment and no shortage allowed.
- 6. Write short notes on any two of the following: i) ABC analysis, ii) Hungarian method, iii) Advantages of duality.

  5+5

#### B.Sc. (Hons.) Examination 2023 Semester VI

#### Subject: Statistics

Paper: DSE-3B [Practical on Operations Research]

Full Marks: 20 Time: 2 Hrs.

#### Answer all questions

1. Solve the LPP using graphical method and discuss about multiple optimal solutions.

Maximize,  $z = 500x_1 + 400x_2$ 

Subject to,

 $10x_1 + 8x_2 \le 800$ 

 $x_1 \le 60$ 

 $x_2 \ge 75$ 

 $x_1, x_2 \ge 0.$ 

2. Solve the following assignment problem and find the optimal assignment profit from the profit matrix below. 7

|   | A  | B  | C  | D  | E  |
|---|----|----|----|----|----|
| 1 | 5  | 7  | 6  | 8  | 7  |
| 2 | 10 | 7  | 9  | 11 | 7  |
| 3 | 5  | 12 | 11 | 13 | 7  |
| 4 | 11 | 9  | 15 | 10 | 7  |
| 5 | 5  | 11 | 8  | 11 | 11 |

3. Reduce the following game to  $2 \times 2$  game by modified dominance property and then solve it. 6

|       | $B_1$ | $B_2$ | $B_3$ | $B_4$ |
|-------|-------|-------|-------|-------|
| $A_1$ | 3     | 2     | 4     | 0     |
| $A_2$ | 3     | 4     | 2     | 4     |
| $A_3$ | 4     | 2     | 4     | 0     |
| $A_4$ | 0     | 4     | 0     | 8     |

4. Practical note book and viva-voce.

4